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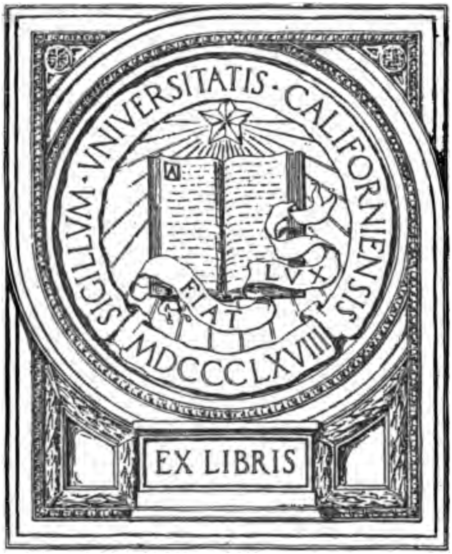

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HYDRAULICS OF PIPE LINES

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THE GLASGOW TEXT BOOKS.
EDITED BY G. MONCUR.

HYDRAULICS OF PIPE LINES

UNIV. OF CALIFORNIA

BY

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With 134 *Illustrations*

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AUTHOR'S PREFACE

This book is intended to give, in a reasonably adequate engineering form, a discussion of the more important hydraulic problems which arise in connection with pipe lines and pipe line flow. No attempt has been made to cover the subject from the structural or descriptive viewpoints. In Chapter V the treatment is partly structural and descriptive, but rather as incidental to the main purpose of the work.

Chapter I presents briefly the elementary principles relating to pipe line flow with some special emphasis on the subjects of pipe friction and secondary losses due to miscellaneous turbulence. Special attention may also, perhaps, be directed to the series of relations and expressions in Section 11 and to the treatment of network systems in Section 21.

Chapter II presents the subject of surge, not with analytical detail but in such manner as to place before the engineer a variety of means for dealing with this important problem. Attention may be directed to the special application of the principles of similitude as applied to this problem, thus bringing it within the range of laboratory investigation.

In Chapter III is given a reasonably full analytical treatment of the subject of water ram or shock. This seems to be justified by the absence, so far as the author is aware, of any measurably adequate discussion of this subject in the English language. The method employed starts with the fundamental principles as first developed by Joukovsky. The details of the development are however, largely independent of other sources. Special attention may be called to the discussion of other proposed formulæ for shock, showing the necessary limitations which must surround their use, and in particular to that of Allievi which has been so commonly employed without a proper appreciation of its necessary limitations. Attention may also be called to the extension of this method to include the hypothesis of partial or imperfect reflection at the valve—and of the need of experimental work to serve as a

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basis for determining what degree of reflection may be expected under various operating conditions.

Chapter IV presents, with some new material, the general subject of stresses in pipe lines, due either to static pressure or under conditions of flow.

Chapter V presents some descriptive and structural material, especially with reference to materials, joints, fastenings, fittings, etc. It gives likewise some discussion of the problem of economic design and of a number of special problems connected with the installation and equipment of pipe lines.

Chapter VI presents a brief discussion of oil pipe lines, or more broadly of any pipe line intended especially for the carriage of viscous fluids. Particular attention may be directed to the method of treatment involving the use of the general equation of flow as discussed briefly in Appendix I. There seems good reason to believe that if the frictional resistance is thus determined, as a function of diameter, velocity, density and viscosity, with a proper allowance for the physical condition of the flow surface of the pipe, the results will be entirely reliable and the design of such pipe lines may be undertaken with the same degree of confidence as in the case of those for the flow of water.

Any work of the character of this volume must be, in large parts, a compilation and adaptation of material drawn from many sources. It has been the intention to give credit whenever direct use has thus been made. The reader will find, however, a considerable amount of material which is either new or which has not been commonly presented in the form here given.

It is hoped that the work may prove of some direct help to engineers in dealing with the hydraulic problems of pipe line flow, and also that it may serve as a stimulus to the further study of many phases of these problems regarding which our knowledge is still entirely too fragmentary.

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PRINCIPAL NOTATION EMPLOYED

A—Area of pipe or conduit.
a—Area of nozzle or through discharge valve.
B—Summation of terms following first in equation (42) Chapter III.
b—Head due to atmosphere.
b—Expression L/C^2r.
C—Chézy coefficient.
c—Expression $L/C^2r+1/2g$.
D—Diameter of pipe or conduit.
d—Diameter of pipe or conduit.
e—Efficiency of riveted joint.
F—Area of surge chamber.
f—Efficiency of nozzle.
f—Friction coefficient in Darcy formula.
H—Total head above atmosphere or special pressure datum.
h—Lost head due to friction and turbulence.
h—Change of pressure head (increase or decrease) attendant on closure or opening of valve at discharge end of line.
i—Hydraulic gradient.
i—Time x/S, section (38).
K—Coefficient of elasticity of water (cubical), Chapter III.
k—Ratio m/T in case of valve closure or opening, as in section (42).
L—Length of pipe line or conduit.
M—Bending moment.
M—Momentum.
m—Ratio of area of nozzle or through valve to area of pipe or ratio between areas of different parts of a complex pipe line.
n—Kutter's roughness coefficient.
p—Pressure intensity in general.
q—Pressure change (increase or decrease) attendant on closure or opening of valve at discharge end of line.
r—Mean hydraulic radius.
S—Velocity of propagation of acoustic wave.
s—Change in velocity v attendant on opening or closure of a valve in delivery end of line.
T—Tensional stress.
T—Time of valve movement (opening or closing) at delivery end of pipe line.
t—Time in general.
t—Thickness of pipe wall.

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u—Velocity at mouth of nozzle.
V—Volume in general, usually in f3.
v—Velocity of flow in pipe line or conduit, usually in fs.
w—Density of water.
x—Variable length in general.
y—Head due to some special pressure.
Z—Total head, including atmosphere or special pressure.
z—Elevation above datum level.
z—Time $2L/S$ required for an acoustic wave to travel twice the length of the line L.

a—Ratio S/g or velocity of acoustic wave divided by gravity.
μ—Coefficient of viscosity.
σ—Density of steel, or (Chapter VI) density in general.

Other notation employed as convenient and as defined in connection with special problems.

NOTATION FOR UNITS OF MEASURE

In order to facilitate representation in typographical form the following notation is employed:

Feet	(f)
Inches	(i)
Square feet	(f2)
Square inches	(i2)
Pounds per square foot	(pf2)
Pounds per square inch	(pi2)
Pounds per cubic foot	(pf3)
Seconds	(s)
Feet per second	(fs)
Cubic feet per second	(f3s)

Note that these designations in Roman type are placed in parentheses. This will serve to distinguish them from other quantities which might be denoted by the same letters.

Note also that throughout the text the oblique line is used as the sign of division, thus a/b, dv/dt, etc.

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HYDRAULICS OF PIPE LINES

CHAPTER I

GENERAL HYDRAULIC PRINCIPLES—STEADY MOTION

1. Energy of a Flowing Stream

As shown in the elementary theory of hydraulics the total energy of a flowing stream must be considered under three heads. Specifically in referring to the energy of a stream we mean the energy of one pound of the liquid at a definite location in the line. In hydraulics the word *head* is very commonly used instead of energy. It must be remembered, however, that the word head thus employed means in effect the energy of one pound of the liquid contents of a flowing stream. With this understanding the three components of the energy or head are as follows:

(*a*) Pressure energy or pressure head $\frac{p}{w}$

(*b*) Kinetic energy or velocity head $\frac{v^2}{2g}$

(*c*) Potential energy or gravity head z.

Where p=pressure, absolute (pf2).
w=weight (pf3).
v=velocity (fs).
z=elevation of given point above reference datum (f).

In addition to these three primary forms of energy we must recognize two incidental or secondary forms.

(*d*) The kinetic energy of eddies, vortex motion and turbulence generally.

(*e*) Heat energy.

The entire theory of hydraulics, with special reference to the dynamics of stream and pipe line flow, is based on certain fundamental propositions which will be briefly stated, referring the reader to elementary textbooks on the subject for a more detailed treatment.

1. The three fundamental forms of energy (*a*), (*b*) and (*c*) are mutually convertible into each other and into mechanical work.

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2. Any and all of the three fundamental forms (*a*), (*b*) and (*c*) are convertible into either of the secondary forms (*d*) (*e*).

3. Neither of the forms (*d*) or (*e*) is convertible into (*a*), (*b*) or (*c*), or into mechanical work. A vortex or eddy or turbulence of any kind once formed can never be untangled or transformed back into any of the three available forms. Heat once formed as a result of the deformation of stream line flow cannot be transformed directly back into any of the available forms (*a*), (*b*) or (*c*).

4. Form (*d*) inevitably degenerates into (*e*) and becomes dissipated as such.

5. Any action in a stream which results in the formation of energy in either of the forms (*d*) or (*e*) involves inevitably the ultimate dissipation of such energy as heat, and therefore the irreversible transfer of energy from an available form (*a*), (*b*) or (*c*) to a form absolutely unavailable in a mechanical sense.

One of the most important of hydraulic problems is concerned with the loss of total available energy or head occasioned by or incidental to the transfer of a liquid through a pipe line or other form of conduit.

Let Z_0 denote the total available head or energy of one pound of water at the origin or starting point O. At any other point P in the line let h denote the amount of energy which has been transferred from forms (*a*), (*b*) and (*c*) into forms (*d*) (*e*).

Let Z denote the total available energy at P.

Then from the conservation of energy we must have

$$Z_0 = Z + h$$
$$\text{or } Z = Z_0 - h.$$

Again, if we represent Z by the sum of the three components (*a*), (*b*), (*c*) we have

$$Z_0 = \frac{p}{w} + \frac{v^2}{2g} + z + h \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

All quantities on the right refer to the point P. The first three comprise the total available energy at P while the fourth represents the loss in available energy between the origin O and P. The quantity h thus transferred irreversibly from the available energy to the unavailable form is called the *loss* of *head*.

In this fundamental equation, as noted, p denotes the total or absolute pressure at any given point in the stream. In hydraulic formulæ generally, p is more commonly used to denote rather the pressure above the atmosphere, and similarly p/w to denote the pressure head in excess of the atmospheric head. Care should be exercised in all cases to note the exact sense in which the symbol for pressure is employed.

If h is neglected in equation (1) we have the well-known Bernouilli equation for steady flow without loss of head. Including the term h, (1) is to be considered as the general energy equation for

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one pound of water at any point in the line and in which the first three terms denote the total available energy of one pound of water at the point P, while h represents the work done against various forms of resistance (and hence the energy transferred from the available forms (*a*), (*b*), (*c*) into the unavailable forms (*d*), (*e*)) in carrying one pound of water from O to P.

More briefly h may be viewed as the measure of the work done against secondary and viscous forces in carrying one pound of water from O to P.

Loss of head or loss of energy may thus be viewed directly as the energy equivalent of the work involved in various accidental phenomena connected with stream line flow. Thus wherever the lines of stream flow are abruptly diverted or redistributed, or whenever the stream undergoes abrupt changes in size, or whenever stream flow undergoes abrupt change in geometrical character generally, there is always, in a slightly viscous liquid such as water, a tendency toward the formation of eddies, vortices and confused turbulence. Such confused turbulence is also formed at the surface of separation between a flowing stream and the containing conduit, or generally at the surface of separation of a solid and a liquid in relative motion. The confused turbulence thus formed at the surface of the containing conduit is due partly to the action of roughness and irregularity of surface and partly to the action of adhesive forces acting between the solid and the liquid.

In all cases, and no matter how produced, the formation of eddies, vortices and turbulence in general requires the expenditure of work which can only be furnished by transfer from the available forms (*a*), (*b*), (*c*). Such formation always involves, therefore, a transformation of available energy into forms (*d*) and (*e*); primarily into (*d*) and ultimately all into (*e*).

For convenience of discussion we may classify these losses as follows:

(1) Loss of head due to turbulence formed at or near the surface of the containing conduit and due to roughness of surface and to the action of adhesive forces.

(2) Loss of head due to all other causes involving miscellaneous turbulence and redistribution of stream line flow.

Loss (1) is commonly referred to as due to *friction*, though the actual phenomena involved have little in common with those characterizing the frictional resistance between two solids.

Loss (2) we shall refer to as due to miscellaneous turbulence.

Before proceeding with the discussion of these various forms of loss of head it will be well to note at this point that in Appendix I will be found a brief discussion of the general theory of pipe line flow as developed from the principle of dimensions, and including the influence due to the viscosity and density of the liquid, and with due regard to their dependence in turn on temperature. It appears

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that there are in general two modes of flow for a fluid moving in a pipe or conduit: (1) stream line and (2) turbulent, but that in all cases arising in ordinary engineering practice, the flow of water occurs under the *turbulent* mode.

With regard to viscosity, the variation in the case of water, at least over ordinary working temperatures, is relatively small, and viscosity as a factor in the problem is of relatively small importance in the turbulent mode of flow. For these reasons, in ordinary hydraulic problems, commonly no attempt is made to include this factor.

With regard to density, the variation with temperature is relatively greater, but the influence of this factor on lost energy measured in terms of head is relatively small in the case of turbulent flow, and it is therefore commonly neglected. If the loss is expressed in terms of pressure, however, the density enters directly as a factor and care must be taken in transforming head into pressure that the proper value of the density as effected by the temperature is employed.

On the other hand, in the case of crude petroleum oil and such-like liquids, the influence of viscosity as dependent on density and temperature commonly plays a controlling part and must therefore be included in any determination of the value of the lost head.

For a discussion of formulæ and methods of computation where the influence of viscosity must be included, see Appendix I and Chapter VI on oil pipe lines. In the present chapter, dealing with water conduits primarily, the usual practice will be followed and the relatively small influence due to variations of viscosity and density with temperature will be neglected.

We shall now proceed with the discussion of certain formulæ commonly used for the computation or estimation of these various forms of loss of head.

In general we shall designate all such losses by h. The context will always show whether special reference is intended to loss (1) as above (skin effect) or loss (2) (turbulence) or to the sum of the two.

2. Loss of Head. Chézy Formula

The loss of head due to so-called "fluid friction" in a conduit with liquid flowing under steady conditions depends, aside from viscosity and density, on the following factors:

(*a*) The velocity of flow.
(*b*) The character of the surface.
(*c*) The length of the conduit.
(*d*) The transverse dimensions and form of the conduit.

Various formulæ have been proposed for the purpose of relating the factors in this problem. The best known and most commonly employed is the so-called Chézy formula.

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Let v=vel. (fs).
C=coefficient.
i=hydraulic gradient=h/L.
r=hydraulic mean radius=A/P.
h=friction head (f).
L=length of conduit (f).
A=cross section area (f2).
P=wetted perimeter (f).

We then have the following formulæ :

$$v=C\sqrt{ir} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\text{or} \quad h=\frac{Lv^2}{C^2r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In this formula for h, which is admittedly not fully rational, especially as regards the index of v, the coefficient C must include some influence due to factors (a), (b) and (d) above. With values drawn from experience, however, and representing these factors, the formula will give reliable results within the range of values covered by the experimental basis.

It results that the values of C in this formula must be selected with reference to *velocity*, *roughness*, and *size* of conduit.

In the use of the Chézy formula or any of its derivatives, care must be taken to distinguish between the hydraulic gradient i and the actual gradient on which the pipe is laid. In the case of a pipe running under pressure, there is no necessary relation between the two.

3. Loss of Head. Kutter's Formula

As an aid in the selection of a value of C for the Chézy formula, Kutter's formula is frequently employed, though it should not be forgotten that this formula was orginally developed for the discussion of the problem of the flow in open channels. The formula is furthermore empirical rather than rational in its relation of the three controlling variables, *hydraulic gradient*, *roughness* and *geometry of conduit*. The use of this formula for the determination of the coefficient C in the Chézy formula (equation (3)), should therefore be made with some reserve, and preferably as based on observation for closely similar conditions.

Let n=roughness coefficient.

Then in English units Kutter's formula is as follows :

$$C=\frac{1{\cdot}81\left[23+\frac{{\cdot}00155}{i}+\frac{1}{n}\right]}{1+\frac{1{\cdot}81\,n}{\sqrt{r}}\left[23+\frac{{\cdot}00155}{i}\right]} \quad \ldots\ldots\ldots\ldots (4)$$

In this formula the hydraulic gradient i is intended to represent the velocity, r represents the transverse dimensions or cross section

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of the conduit and n represents the character and condition of the surface.

In any given case i and r will be known and n must be assumed by judgment in accordance with the character of the surface. Typical values will be found in Sec. 7.

The computation of values from (4) is somewhat tedious, and examination shows further that the variation over the range of ordinary values of i is relatively small, and in consequence by taking i constant at a mean value, the formula may be simplified in marked degree. Thus if i be taken constant at 1 : 1000 the formula reduces to

$$C=\frac{\frac{1{\cdot}8}{n}+45}{1+\frac{45n}{\sqrt{r}}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The simpler form of (5) may be further justified by the consideration that the presumable error due to the use of (5) rather than (4) will be small in comparison with the uncertainty which will inevitably attach to the arbitrary selection of a value for the roughness coefficient n, or to the use of the formula itself for finding the value of C.

4. Loss of Head. Exponential Formula

Reference has been made in Sec. 2 to the fact that the Chézy formula is not quite rational in taking resistance or loss of head to vary with the square of the velocity.

Experiments with varying velocities in conduit flow as well as a vast amount of research in the related field of ship resistance show that, with the other factors constant, the resistance or loss of head due to friction varies with the speed according to an index which may range about 1·83, but which, in practically all cases, is less than 2. In a more fully rational formula for loss of head, the velocity v should, therefore, have an index approximately 1·83 or 1·85. Several formulæ of this character have been proposed. Of these one of the best known is that proposed by Williams and Hazen, as follows:

Let r=mean hydraulic radius=A/P as in Sec. 2.
i=hydraulic gradient=h/L as in Sec. 2.
B=coefficient.
C=coefficient in Chézy formula.

Williams and Hazen Formulæ:

$$v=Br^{0.63}i^{0.54} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and $$h=\frac{Lv^{1.85}}{B^{1.85}r^{1.17}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

or $$v=1{\cdot}318Cr^{\cdot 63}\,i^{\cdot 54} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and $$h=\frac{Lv^{1.85}}{1{\cdot}668C^{1.85}r^{1.17}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

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In (6) and (7) B is a coefficient which is supposed to depend on roughness alone and which may be selected on the basis of the same features as for n in Kutter's formula. In (8) and (9) the numerical factor is introduced in order to relate B of (6) and (7) to the coefficient C of the Chézy formula. For average or typical values of $i=\cdot001$, and $r=1$, it is readily seen that $B=(\cdot001)^{-\cdot04}C=1\cdot318C$. Hence in the form of (8) or (9) C may be selected for the given character of surface and assuming a hydraulic gradient of ·001 and a mean hydraulic radius of 1. This value in (8) or (9) will then give the proper results with the actual gradient i and actual radius r. The significance of this form of the coefficient in (8) and (9) lies in the fact that many engineers prefer to estimate directly the value of C in the Chézy formula, and, with nearly standard conditions as to hydraulic gradient and radius, are able to do so with quite satisfactory accuracy. There has accumulated in the literature of this subject, furthermore, a large amount of material bearing directly on values of C for the Chézy formula, and thus serving as a basis for the immediate selection of this coefficient for various conditions. If it is understood that the C of (8) and (9) is the C of the Chézy formula for $r=1$ and $i=\cdot001$, a large amount of this data becomes immediately available for purposes of selection.

The principal practical drawback to the use of exponential formulæ lies in the more complicated numerical procedure which is involved. As an aid in making such computations effective use may be made of logarithmic cross section paper, and as an extension of the same idea a special slide rule has been devised for the direct computation of the terms in equation (8).

5. Loss of Head. Darcy's Coefficient

The following formula is given in all elementary works on hydraulics :

$$h=f\frac{L}{D}\frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where f=coefficient as below:

L=length.
D=diameter.
v=velocity (fs).

Note that in the above formula L and D must both be measured in terms of the same unit.

In this formula f represents a roughness or friction coefficient which decreases with increase in either D or v. The variation with v is, however, slight and may usually be neglected. For variation with diameter Darcy recommends a formula which may be expressed in the form

$$f=\cdot02+\frac{\cdot00166}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where D is diameter in feet.

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This value of f is recommended for clean pipe. For old pipe a suitable increase up to double value should be made.

Comparison with the Chézy formula in Sec. 2 develops immediately the following relations between f and C.

$$f=\frac{8g}{C^2}=\frac{257{\cdot}3}{C^2}$$

$$C=\sqrt{\frac{8g}{f}}=\frac{16{\cdot}05}{\sqrt{f}}$$

From the above value of f we readily find

$$C=\sqrt{\frac{257{\cdot}3D}{{\cdot}02D+{\cdot}00166}} \qquad (12)$$

Corresponding values of f and C are given in Table I.

6. Loss of Head. Volume Flow Formula

Let L=length of conduit (f).
v=velocity of flow (fs).
V=volume rate of flow (f3s).
B=constant depending on form of cross section.
C=coefficient in Chézy formula as in Sec. 2.
r=hydraulic mean radius (f).
A=area of section of conduit (f2).

Then we may put

$$A=Br^2$$

and the Chézy formula

$$h=\frac{Lv^2}{C^2r}$$

is readily put in the form

$$h=\frac{LV^2}{B^2C^2r^5} \qquad (13)$$

For a circular cross section $B=4\pi$, $r=D/4$, and we have

$$h=\frac{64LV^2}{\pi^2C^2D^5}=\frac{6{\cdot}48LV^2}{C^2D^5} \qquad (14)$$

$$\text{or} \quad V=\sqrt{\frac{{\cdot}154C^2D^5h}{L}} \qquad (15)$$

7. Suggestions Regarding Practical Values of n and C

The value of the roughness coefficient n of Kutter's formula, in cases arising in practice, is commonly found between ·010 and ·015.

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Following are values relating to pipe line or conduit surfaces as assigned by Kutter:

Material.	*n.*
Well-planed timber	·009
Neat cement	·010
Cement with one-third sand	·011
Unplaned timber	·012
Ashlar and brickwork	·013
Unclean surfaces in sewers and conduits	·015

Later observations seem to indicate further values as follows:

Concrete conduits with smooth plastered surface when new	·011 to ·012
The same conduits with ageing and a gradually acquired gelatinous or slime-covered surface	·013 to ·014
Iron and steel pipes	·013 to ·015
Wood stave pipe	·011 to ·012

There is, of course, no definite upper limit for the measure of roughness and with flaking and scaling of cement-lined surfaces or with corrosion and tuberculation of iron and steel pipe the values of n may rise to ·020 or more.

Turning now to the direct estimate of the value of C in the Chézy formula, the following suggestions are given:

For new cast-iron pipe with specially smooth surface, C may rise to values approaching 140. For average new cast-iron pipe a value of about 130 may be anticipated. With corrosion and fouling the value of C will fall, according to the degree of roughness, to values approximating 100 or less. Where the capacity of a cast-iron pipe line after some period of years is in question, values of C from 100 to 110 may usually be employed.

For cast-iron pipe 4 inches diameter to 60 inches diameter, Williams and Hazen* estimate that beginning with 130 when new the value of C will decrease to 100 in from 13 to 20 years and to 80 in from 26 to 47 years. In each case the shorter range of years is for the 4-inch size and the larger range for the 60-inch size. Intermediate ranges are for intermediate sizes, the range for any one value of C increasing at first rapidly and then more slowly in going from the smaller to the larger sizes. Again for each size of pipe the decrease in C is at first rapid and then more gradual with increasing age, the values for small pipes falling off more rapidly than those for large. These relations between the value of C, size of pipe and age are expressed in tabular form by the authors above mentioned, not as exact or precise results to be anticipated in all cases, but as a generalization based on wide observation and careful judgment. The authors are careful to state that the ranges of years stated are

* "Hydraulic Tables," John Wiley and Sons, New York. Chapman and Hall, London.

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intended to apply primarily to soft and clear but unfiltered river waters. Some waters will corrode cast-iron pipes much more rapidly than such a standard, while in other cases, especially for hard waters, the rate may be much slower. In all cases, therefore, careful judgment must be used.

For riveted-steel pipe, when new, values of C may range about 110 and upward. With age the value will drop to a range usually from 95 to 100, Hazen and Williams* state as a broad generalization, that steel pipe of a given size and age will carry the same quantity of water as cast-iron pipe of the same size and ten years older. With butt-strap circumferential joints, approximately flush riveting on the inside and special care to realize good hydraulic conditions, C for new pipe will rise to values about 120, falling with advancing age to values ranging from 100 to 110.

For new welded-steel pipe with care at the joints to give, as nearly as may be, a smooth continuous surface, the value of C may be taken 120 to 130. With advancing age this will drop to values ranging from 100 to 110.

Pipe of lead, brass, tin, glass, or of other like material giving a smooth semi-polished surface, will give values of C up to 140. A very slight roughening, almost imperceptible to the eye, will serve to reduce these values to 130 or 120 or less.

For smooth-wood or wood-stave pipe, values of C from 120 to 130 have been noted in several cases.

For masonry or concrete conduits with smooth cement-plastered surface, values of C from 130 to 140 may be anticipated when new, falling with age and the development of a slime-covered surface to values ranging from 120 to 130, and to still lower values if accompanied by flaking or scaling. Ultimate values of 120 or less will also be appropriate if the surfaces show slight waves or irregularities, as is often the case with ordinary contract work.

For vitrified pipe a value of about 110 may be employed, thus allowing for some loss at each joint, due to roughness or a slight sudden expansion in cross-sectional area.

In the preceding discussion regarding the values of the coefficient C, no specific reference has been made to the dependence of C upon size of pipe and hydraulic gradient or velocity. In so far, however, as the general indications of Kutter's formula are applicable to pipe flow, it appears that for a given assumed degree of roughness, the values of C increase with the hydraulic mean radius r and with the hydraulic gradient i or otherwise with the hydraulic mean radius and with the velocity. For large pipes and high velocities we may therefore anticipate relatively high values of C and for small pipes and low velocities, smaller values.

The form of the Chézy formula, when compared with the known experimental facts regarding surface fluid resistance, shows, further-

* *loc. cit.*

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more, that we should anticipate such a dependence of C on the other factors, especially velocity or hydraulic gradient.

Two practical questions then arise:

1. For what sizes and hydraulic gradients are the values of C, as above indicated, intended to be applicable?

2. What is the general character of the change in the value of C with varying hydraulic mean radius or varying hydraulic gradient?

In answer to the first of these it may be stated that, broadly, the values given have been derived by observation from cases where for the most part the hydraulic gradient approximates ·001, and the hydraulic mean radius ranges, say from ·2(f) to 1(f). That is, the very small sizes of pipe are excluded from the range intended. As a middle range we may assume the values of C as stated, primarily applicable to cases approximating as follows:

$$i = \cdot 001$$
$$r = \cdot 50\text{(f) or}$$
$$D = 2\text{(f)}.$$

We may then pass to the second query. Kutter's formula is, of course, intended to answer just this question, or more exactly to give C for any assumed value of the roughness coefficient n with any given value of i and r.

We shall prefer, however, to take the direct results of pipe flow observations, as on the whole more satisfactory than the numerical values of the Kutter formula, which, as previously noted, was based primarily on the flow in open channels.

A suitable analysis of the Williams and Hazen formula (see Sec. 4), and into the details of which we need not enter here, serves to show that if we denote by C_0 the value of the coefficient C, which is properly applicable to a standard value of the hydraulic mean radius r_0 and a standard value of the hydraulic gradient i_0, then the value C properly applicable to any value of r and i will be given approximately by the formula:

$$C = C_0 \left(\frac{r}{r_0}\right)^{\frac{1}{8}} \left(\frac{i}{i_0}\right)^{\frac{1}{25}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

That is, starting from standard values the value of C varies as the eighth root of r and the twenty-fifth root of i. It follows that the variation with i is much slower than with r.

Assuming then any value of C_0 as suited approximately to the values of $r_0 = \cdot 5$ and $i_0 = \cdot 001$, we may from (16) obtain an indication of the suitable value of C for any other value of r and i, as desired.

It should be here noted that the tabular values of C given by Williams and Hazen are stated to apply primarily to values $r_0 = 1$ and $i_0 = \cdot 001$. The law of variation is, however, the same, and the

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values $r_0 = \cdot 5$ and $i = \cdot 001$ are here chosen as on the whole better suited to the general range of values of C above mentioned.

It may also be noted that a suitable analysis of Kutter's formula shows likewise a very similar though not equally regular law of variation of C with r and i.

Value of C derived from f in Darcy's Formula.—A check on the value of C may be obtained by the use of the relation between the two coefficients of the Darcy and Chézy formulæ. This relation has been noted in Sec. 5. In Table I corresponding values of f and C are given against diameter. It is seen that the table is not carried beyond a diameter of 2 feet. This is for the reason that this formula gives values too small for large pipe. The Darcy formula with the value of f proposed for clean pipe should not be employed for large pipes for this reason. It is readily seen that according to the formula showing the relation between f and C, the maximum value of C, no matter what the diameter, will be $\sqrt{400g} = 113 \cdot 4$. This value for large clean pipe is entirely too small, while on the other hand, for pipe up to perhaps 1 to 2 feet in diameter the values agree well with general experience.

It may be noted also that Darcy recommends the value for f as stated in Sec. 5 to be applied to new clean pipe, with an increase in f up to 100 per cent for old and corroded pipe. This is evidently equivalent to a decrease in the value of C in the ratio 1·00 to 1·41. For intermediate states of roughness or corrosion, intermediate values will naturally be taken.

TABLE I

d	$\cdot 00166/d$	f	$8g/f$	C
·1	·01660	·03660	7030	84
·2	·00830	·02830	9092	95
·3	·00553	·02553	10080	100
·4	·00415	·02415	10650	103
·5	·00332	·02332	11030	105
·6	·00277	·02277	11300	106
·7	·00237	·02237	11500	107
·8	·00207	·02207	11660	108
·9	·00184	·02184	11780	109
1·0	·00166	·02166	11880	109
1·5	·00111	·02111	12190	110
2·0	·00083	·02083	12350	111

Hamilton Smith's Coefficients.—As the result of an extended examination of pipe flow data, Hamilton Smith has deduced a series of coefficients, varying with diameter and velocity. In the development of these coefficients a very large amount of data was

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critically examined and great care was taken to eliminate doubtful results and to develop a set of values for the coefficient C, based on reliable and consistent observations. These values have, in consequence, been widely accepted as presumably the most reliable present expression of the results of actual experience.

It should be noted that these coefficients do not contain allowance for varying degrees of roughness. It is understood that the observations on which they are based relate broadly to cast-iron and riveted-steel pipe with clean and smooth interior surfaces; that is, to what may be taken as substantially new pipe.

Smith's observations are reported in the original paper* in the form of tables giving values for C against varying values of diameter and velocity. In order to compare with other authorities giving C against diameter and hydraulic gradient, and for direct use when the hydraulic gradient is given rather than the velocity, it is convenient to transform these values into an equivalent set giving C against diameter and hydraulic gradient.

In Table II the coefficients are given in the original form, and in Table III the equivalent values are given in terms of diameter and hydraulic gradient.

TABLE II

Smith's Coefficients

Values of C *in Chézy Formula, for Clean Cast-Iron and Riveted-steel Pipe*

Diameter of pipe	Velocities in feet per second										
	1	2	3	4	5	6	8	10	12	16	20
Feet											
·05	—	78	82	86	88	89	91	91	91	91	—
·1	80	89	94	97	99	101	103	105	105	105	—
1	96	104	109	112	114	116	119	121	123	124	124
1·5	103	111	116	119	121	123	126	129	130	132	133
2	109	116	121	124	127	128	132	135	136	138	—
2·5	113	120	125	128	131	133	136	137	141	143	—
3	117	124	128	132	134	136	140	143	145	147	—
3·5	120	127	131	135	137	139	142	146	149	151	—
4	123	130	134	137	140	142	146	150	152	153	—
5	128	134	139	142	145	147	150	155	—	—	—
6	132	138	142	146	148	154	155	—	—	—	—
7	135	141	145	148	151	—	—	—	—	—	—
8	138	143	148	151	153	—	—	—	—	—	—

* "The Flow of Water through Orifices, over Weirs and through Open Conduits and Pipes." J. Wiley and Sons, New York, 1886. Also "Trans. Am. Soc. C.E., 1883."

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TABLE III

Smith's Coefficients

Values of C *in Chézy Formula, for Clean Cast-iron and Riveted-steel Pipe*

These values are transformed from those of Table II in such manner as to make diameter and hydraulic gradient the determining variables.

Hydraulic gradient parts in	Diameter, feet												
1000	·05	·10	1·0	1·5	2·0	2·5	3·0	3·5	4·0	5·0	6·0	7·0	8·0
·2					110	116	121	124	129	135	140	144	148
·4			96	104	114	120	125	129	133	140	144	148	152
·6			99	108	116	122	127	131	135	142	146	151	
·8			101	110	118	124	128	133	136	144	148		
1·0			102	112	119	125	130	135	138	145	150		
2·0			106	116	124	130	135	139	143	150			
4·0			111	120	128	134	140	144	149				
6·0			113	122	130	136	142	147	151				
8·0			114	124	132	138	144	149	152				
10·0			116	126	134	139	145	150	153				
15·0			118	128	136	142							
20·0		89	119	129	137								
30·0		92	122	131									
40·0		94	123	133									
50·0	78	95	124										
100·0	82	99											
200·0	87	102											
300·0	89	104											
400·0	90	105											
500·0	90	105											
1000·0	91	105											

Between these two tables a value is readily selected for any combination of the variables within the range which they are intended to cover.

Value of C derived from Ship Resistance Experiments.—Extended and refined investigations have determined to a high degree of accuracy the value of the coefficient of friction for the resistance of ships. This is usually expressed by a formula:

$$R=kAv^n.$$

where R=resistance (p).
A=area (f2).
v=velocity (knots or fs).
n=an index usually taken at about 1·85.
k=coefficient.

Since the loss of head h is measured by the work done in carrying one pound against friction the length of the line (see Sec. 11), we

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readily derive an expression for h in terms of the above formula for resistance as follows:

$$\text{Length of pipe occupied by 1 pound} = \frac{4}{w\pi D^2}$$

$$\text{Wetted surface for 1 pound} = \frac{4}{wD}$$

$$h = R\ (\text{for 1 pound}) \times L = \frac{4kLv^n}{wD}$$

Since hydraulic radius $r = D/4$ this becomes

$$h = \frac{kLv^n}{wr} \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

Comparing this with the Chézy formula it appears that if $n=2$ we should have

$$C^2 = \frac{w}{k}$$

Since, however, $n=1{\cdot}85$, it is readily seen, in order that the Chézy formula with its index 2 may give the same value of h as (17) with index 1·85, we must put

$$C^2 = \frac{wv^{\cdot 15}}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Now abundant experiment has shown that for smooth iron and steel plates and taking v in (fs), the value of k may be taken as follows:

Fresh water	$k = {\cdot}0037$
Salt water	$k = {\cdot}0038$

This gives values of C as follows:

v	C
2	137
4	144
6	149
8	152
10	154

It will be noted that since w and k both vary directly with density, the value of C is independent of density.

These values are undoubtedly applicable to large pipes with smooth surfaces. They furnish, moreover, a confirmation of values directly derived from pipe line observations, as noted previously. For small pipes where there is mutual interference between the filaments of flow and where the conditions between a flat plate and an indefinite body of water in relative motion cannot be realized, the resistance becomes greater and the value of C less, as previously indicated. Also where the surfaces are not smooth, due to corrosion, scaling or fouling, the coefficient of resistance will increase and C will decrease as before noted.

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8. Total Friction Head in Pipe Made up of Sections of Different Diameters

Let A_1, A_2, A_3, etc., denote respectively the cross sectional areas of the various sections of pipe. Let L_1, L_2, L_3, etc., denote the corresponding lengths. Let A_1 be taken as the reference area, and let $A_1=m_2A_2=m_3A_3$, etc.

Let v_1=velocity in section of area A_1.

Then m_2v_1=velocity in section of area A_2.

m_3v_1=velocity in section of area A_3, etc.

h=total friction head.

Then from (3) we have

$$h=\frac{L_1v_1^2}{C_1^2r_1}+\frac{m_2^2L_2v_1^2}{C_2^2r_2}+\frac{m_3^2L_3v_1^2}{C_3^2r_3}+\text{etc.},$$

$$\text{or } h=\Sigma\left(\frac{m^2L}{C^2r}\right)v_1^2 \qquad (19)$$

This expresses h in terms of a single velocity v_1 and the various ratios m with the other characteristics of the sections.

Again from (13) we have similarly

$$h=V^2\left(\frac{L_1}{B_1^2C_1^2r_1^5}+\frac{L_2}{B_2^2C_2^2r_2^5}+\text{etc.}\right)$$

$$\text{or } h=\Sigma\left(\frac{L}{B^2C^2r^5}\right)V^2$$

For a circular cross section $B=4\pi$ and $r=D/4$ and we have

$$h=\frac{64V^2}{\pi^2}\Sigma\left(\frac{L}{C^2D^5}\right) \qquad (20)$$

If the variation in diameter is moderate the coefficient C may be taken as constant at an average value and we have

$$h=\frac{64V^2}{\pi^2C^2}\Sigma\left(\frac{L}{D^5}\right) \qquad (21)$$

Let D_0 denote any standard or reference diameter and L_0 the length of a conduit of this diameter which would have the same total value of h. Then

$$\Sigma\left(\frac{L}{D^5}\right)=\frac{L_0}{D_0^5} \text{ or}$$

$$L_0=\Sigma\, L\left(\frac{D_0}{D}\right)^5 \qquad (22)$$

Thus for illustration:

Given $L_1=500$, $D_1=4$.
$L_2=800$, $D_2=3{\cdot}5$.
$L_3=1000$, $D_3=3$.

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What is the equivalent length reduced to a uniform diameter of 3 feet.

$$\left(\frac{D_0}{D_1}\right)^5=\left(\frac{3}{4}\right)^5 = \cdot 2373 \text{ and } 500\times\cdot 2373 = 118\cdot 7$$

$$\left(\frac{D_0}{D_2}\right)^5=\left(\frac{3}{3\cdot 5}\right)^5 = \cdot 4627 \text{ and } 800\times\cdot 4627 = 370\cdot 2$$

$$\left(\frac{D_0}{D_3}\right)^5=\left(\frac{3}{3}\right)^5 = 1 \text{ and } 1000\times 1 = 1000\cdot 0$$

$$L_0 = 1489\cdot 0$$

Thus a uniform conduit, 3 feet in diameter and 1489 feet long, will have the same frictional loss as the actual conduit of varying diameters and 2300 feet long.

In Table IV will be found a table of fifth powers for use in connection with formulæ such as those of Sections 6 and 8.

TABLE IV

Fifth Powers of Numbers

·1	·00001	2·7	143·49
·2	·00032	2·8	172·10
·3	·00243	2·9	205·11
·4	·01024	3·0	243·00
·5	·03125	3·1	286·29
·6	·07776	3·2	335·54
·7	·16807	3·3	391·35
·8	·32768	3·4	454·35
·9	·59049	3·5	525·22
1·0	1·0000	3·6	604·66
1·1	1·6105	3·7	693·44
1·2	2·4883	3·8	792·35
1·3	3·7129	3·9	902·24
1·4	5·3782	4·0	1024·0
1·5	7·5937	4·1	1158·6
1·6	10·486	4·2	1306·9
1·7	14·199	4·3	1470·1
1·8	18·895	4·4	1649·2
1·9	24·760	4·5	1845·3
2·0	32·000	4·6	2059·6
2·1	40·841	4·7	2293·5
2·2	51·536	4·8	2548·0
2·3	64·363	4·9	2824·8
2·4	79·626	5·0	3125·0
2·5	97·656	5·1	3450·3
2·6	118·81	5·2	3802·0

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5·3	4182·0	7·7	27068
5·4	4591·7	7·8	28872
5·5	5032·8	7·9	30771
5·6	5507·3	8·0	32768
5·7	6010·9	8·1	34868
5·8	6563·6	8·2	37074
5·9	7149·2	8·3	39390
6·0	7776·0	8·4	41821
6·1	8446·0	8·5	44371
6·2	9161·3	8·6	46043
6·3	9924·4	8·7	49842
6·4	10737	8·8	52773
6·5	11603	8·9	55841
6·6	12523	9·0	59049
6·7	13501	9·1	62403
6·8	14539	9·2	65908
6·9	15640	9·3	69569
7·0	16807	9·4	73390
7·1	18042	9·5	77378
7·2	19349	9·6	81537
7·3	20731	9·7	85873
7·4	22190	9·8	90392
7·5	23731	9·9	95099
7·6	25355	10·0	100000

9. Minor Losses of Head

Under this general head we include the various losses due to miscellaneous turbulence and deformation of stream line flow, as discussed in Sec. 1.

(*a*) **Loss of Head at Entrance.**—When water passes from a reservoir into the open end of a pipe line, there is a loss of head depending on the velocity and on the rapidity of the acceleration from rest to normal velocity within the pipe.

This is called the *entrance* loss and may be expressed by the formula:

$$h=k\frac{v^2}{2g}, \qquad (23)$$

Where v=velocity (fs).
k=coefficient depending on form of entrance.

Elementary hydraulic theory with experimental observation serves to furnish approximate values of k as follows:

		k
End of pipe flush with reservoir	(*a*) Fig. 1	·50
Pipe projecting into reservoir .	(*b*) Fig. 1	·93
Conical or bell mouth .	(*c*) (*d*) Fig. 1	·15 to ·04.

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As the value of v rarely exceeds 10 (fs) the value of h with an entrance as at (a) would not exceed ·75 (f), while with a suitably tapering entrance as at c or d it may be readily reduced to ·15 (f) or to an amount usually negligible.

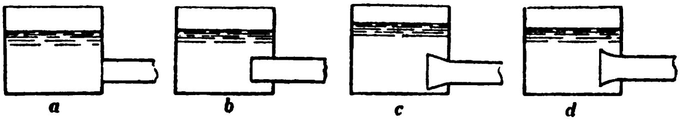

FIG. 1.—FORMS OF INLET.

(b) **Loss Due to Abrupt Expansion in Size.**—If the pipe line is made up of sections of varying diameters and the transition is sudden from one size to another, there will be corresponding sudden changes in velocity and resultant losses of head at these points of transition.

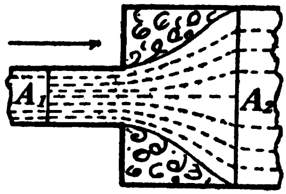

FIG. 2.—LOSS DUE TO ENLARGEMENT.

For a sudden expansion (Fig. 2), elementary hydraulic theory furnishes the approximate formula:

$$h=\left(1-\frac{A_1}{A_2}\right)^2\frac{v_1^2}{2g}=\left(\frac{A_2}{A_1}-1\right)^2\frac{v_2^2}{2g}\ldots\ldots\ldots\ldots(24)$$

Where A_1 and A_2 are the two areas
v_1 and v_2 are the two velocities.
Subscript 1 relates to the smaller size.
Subscript 2 relates to the larger size.

Putting this value of h in the form

$$h=k\frac{v_1^2}{2g}\text{ or }k\frac{v_2^2}{2g}$$

we have for the value of the coefficient:

$$k=\left(1-\frac{A_1}{A_2}\right)^2\text{ or }\left(\frac{A_2}{A_1}-1\right)^2\ldots\ldots\ldots\ldots(25)$$

according as h is related to v_1 or v_2.

(c) **Loss Due to Abrupt Contraction in Size.**—In such case elementary hydraulic theory shows that the loss is due primarily

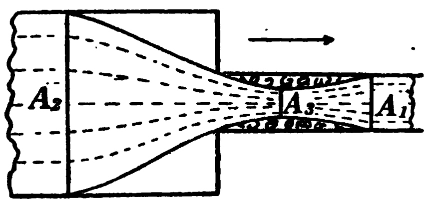

FIG. 3.—LOSS DUE TO CONTRACTION.

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to the expansion beyond the contracted vein which is formed in the entrance to the smaller size; see Fig. 3. The loss of head will therefore be determined by the relative areas A_3 and A_1. But the ratio A_3/A_1 depends on the ratio A_1/A_2 in a manner which experiment alone can determine. We may therefore express this loss by the formula:

$$h=k\frac{v_1^2}{2g}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (26)$$

Where k has values, as in the following table, derived from experimental results by Weisbach.

TABLE V

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
k . .	·362	·338	·308	·276	·221	·164	·105	·053	·015	

Regarding the same loss, Merriman* gives a formula for the contraction ration A_3/A_1. Taking the nearest two significant figures this formula is

$$\frac{A_3}{A_1}=\cdot 58+\frac{\cdot 042}{1\cdot 1-\sqrt{A_1/A_2}}$$

Using this formula we find values as in Table VI for the coefficient k in (26). Note that in this formula the loss is referred to the velocity v_1, Fig. 3.

TABLE VI

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
k . .	·333	·306	·275	·243	·208	·168	·123	·076	·028	

Bellasis† quotes values for the contraction ratio A_3/A_1 which lead to the same values of k as quoted from Weisbach.

(*d*) **Loss of Head Due to an Obstruction.**—In the case of a pipe

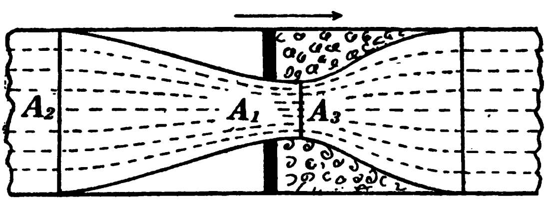

FIG. 4.—LOSS DUE TO OBSTRUCTION.

with an obstruction, as in Fig. 4, it may be assumed that the loss of head is primarily due to the sudden expansion beyond the con-

* "Treatise on Hydraulics." John Wiley and Sons, New York. Chapman and Hall, London.

† "Hydraulics." Rivingtons, London.

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tracted vein which is formed just beyond the reduced opening. Let subscripts 1, 2, 3 refer respectively to the reduced opening, to the full size and to the contracted vein, and as above let A denote area and v velocity. Then applying the formula of (b) for abrupt expansion we have for the loss due to the obstruction :

$$h=\left(\frac{A_2}{A_3}-1\right)^2\frac{v_2^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The ratio A_2/A_3 depends upon A_2/A_1. Rankine gives the following empirical formula for the ratio A_3/A_1 :

$$\frac{A_3}{A_1}=\frac{\cdot 618}{\sqrt[4]{1-\cdot 618\,(A_1/A_2)^2}}$$

From which we readily find

$$\frac{A_2}{A_3}=\frac{\sqrt{(A_2/A_1)^2-\cdot 618}}{\cdot 618} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

Putting the above value of h in the form

$$h=k\frac{v_2^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

we find by substitution in (27) and (28) the following values of k :

TABLE VII

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
k . .	229	49	18	8·1	3·9	1·9	·86	·33	·07	

In connection with the same loss Bellasis* gives on experimental basis, values for the coefficient of contraction A_3/A_1 as follows :

TABLE VIII

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
A_3/A_1	·624	·632	·643	·659	·681	·712	·755	·813	·892	1·0

These are the coefficients referred to in connection with the loss due to contraction, and differ but slightly from those given by Merriman's formula. The product of the two ratios (A_1/A_2) (A_3/A_1) gives A_3/A_2. After the analogy of (25) the value of the coefficient k with reference to the lower velocity v_2 will be

$$k=\left(\frac{A_2}{A_3}-1\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Using this formula with the values in Table VIII we find k as follows :

* *loc. cit.*

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TABLE IX

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
k . .	226	48	18	7·8	3·8	2·1	·80	·29	·06	

The agreement between the two sets of values in Tables VII and IX is very close.

(e) **Loss of Head Through Valves.**—A valve may be considered as a special form of obstruction. The loss of head in flowing through partially open valves of the gate type, as in Fig. 5, or of the butterfly type, as in Fig. 6, is found to agree approximately with the values given by (27), (28), or Table VI. Therefore if we have given the area of opening and the area of pipe in the case of valves of these types, we may reach at least an approximate estimate of the loss of head by direct substitution in these formulæ.

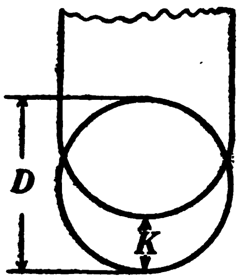

FIG. 5.—LOSS DUE TO GATE VALVE.

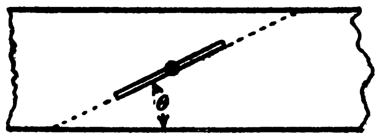

FIG. 6.—LOSS DUE TO BUTTERFLY VALVE.

Experiments on large gate valves made by Kuichling and J. W. Smith give results as below:

TABLE X

Ratio K/D (see Fig. 5)	·05	·10	·20	·30	·40	·50	·60	·70	·80
Ratio A_1/A_2	·05	·10	·23	·36	·48	·60	·71	·81	·89
k for 24″ valve	235	100	28	11	5·6	3·2	1·7	·95	—
k or 30″ valve	333	111	23	9·4	5·2	3·1	1·9	1·13	·60

If these values of k are compared with those of Table IX by plotting both sets on an axis of A_1/A_2, considerable divergence will

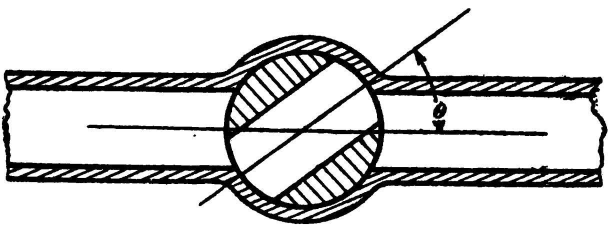

FIG. 7.—LOSS DUE TO PLUG COCK.

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be noted. The agreement, however, is perhaps quite as close as could be expected, and the divergence in any ordinary case would be unimportant.

In the case of a plug cock, as in Fig. 7, these values for the loss of head are relatively greater, as indicated in the following table :

TABLE XI

A_1/A_2	·10	·20	·30	·40	·50	·60	·70	·80	·90	1·0
k . .	430	93	35	15	6·8	3·1	1·5	·55	·11	

These various formulæ and values for loss of head through valves rest largely on Weisbach's experimental results. These experiments were carried out for the most part on relatively small sizes and extension of these results to large sizes is attended with some measure of uncertainty. Further experimental results are much needed in relation to these various problems.

(*f*) **Loss of Head due to Bends and Angles.**—When bends, curves or angles occur in a pipe line, the change in the direction of flow will result in a loss of head depending primarily on the suddenness of the curvature or on the abruptness of deviation of the stream line flow. This loss is due to the work required to bring about the readjustment of the lines of stream flow and also to some degree of abrupt contraction and expansion with eddy formation as indicated in Fig. 8.

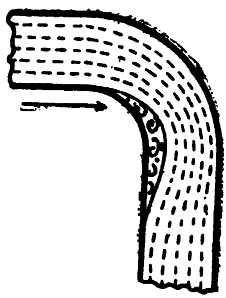

FIG. 8.—LOSS AT ELBOW.

Experimental work during the past fifteen years in particular has served to call attention to two important features connected with the flow of water around a bend or turn, as follows :

1. In the flow of water around a bend or turn, the curve showing distribution of velocity over the cross section suffers marked distortion. Instead of an approximate ellipse the curve becomes distorted with the peak of maximum velocity carried over toward the outer or convex side of the curve.

2. The influence due to a bend or turn extends far beyond the limits of the bend or turn itself. On the upstream side, at least for a little distance, the stream will show some changes in pressure and distortion of velocity curve due to the changes produced at the turn. On the downstream side, the influence of the elbow or turn extends to a very considerable distance, showing just below the turn marked distortion of the velocity curve and of the pressure distribution. This distance seems to vary with many factors, but is usually found between 50 and 100 diameters of the pipe.

It follows that the actual influence due to a bend or turn is

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distributed over a considerable length of pipe, and it is therefore necessary to define with care the meaning of the term "loss due to a bend or turn." As we shall here use this term, and with reference to the numerical values given, it will imply the difference between the *total* loss of head over the entire length of pipe influenced by the bend or turn, and the loss properly chargeable to an equal length of straight uniform pipe with the same hydraulic properties and operating with the same velocity. We may properly assume that this difference is due to the introduction of the bend with the resulting change in the direction of flow and the attendant circumstances as noted above.

Again the loss thus defined may be viewed in either of two ways:

1. In the same manner as in (*a*), (*b*), (*c*) it may be related directly to the velocity head, $v^2/2g$, through a coefficient k. In such case k will depend on the ratio of the radius of the pipe to the mean radius of the bend, on the angle covered by the bend, on the size of the pipe and doubtless on the velocity v.

2. The loss may be expressed in terms of the length of straight pipe which, under like hydraulic conditions, would show the same loss. This gives a ready expression for the loss in terms of lineal feet of straight pipe in terms of diameters of straight pipe, or in terms of the loss due to a length of straight pipe equal to the mean radius length of the bend or turn itself.

The earliest extended experiments on this subject were made by Weisbach with results as follows for the value of k in the formula:

$$h=k\frac{v^2}{2g}$$

Let D=diameter of pipe.
R=radius of curvature.

Then for an elbow or bend of 90° we have as follows:

TABLE XII

D/R	·40	·60	·80	1·00	1·20	1·40	1·60	1·80	2·00
R/D	2·50	1·67	1·25	1·00	·83	·71	·63	·55	·50
k	·14	·16	·21	·29	·44	·66	·98	1·41	1·98

Likewise for varying angles of turn, presumably for D/R approximately 1·6 we have values as follows:

TABLE XIII

Angle .	20°	40°	50°	80°	90°	100°	110°	120°
k . .	·05	·14	·36	·74	·98	1·26	1·56	1·86

Weisbach's experiments were made with relatively small pipe, and for which the Darcy coefficient may be assumed to vary from

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·025 to ·030. Recomputing the values in Table XII into terms of the equivalent length of straight pipe, we have the following:

TABLE XIV

D/R .	·40	·60	·80	1·00	1·20	1·40	1·60	1·80	2·00
R/D .	2·50	1·67	1·25	1·00	·83	·71	·63	·55	·50
L_1 .	·78	·89	1·17	1·61	2·44	3·67	5·45	7·83	11·00
L_2 .	1·87	2·13	2·80	3·87	5·87	8·80	13·07	18·80	26·40

In the above table L_1 gives the lengths of straight 2-inch pipe with Darcy coefficient=·030 equivalent to the excess loss due to the elbow, while L_2 gives the similar lengths for 4-inch pipe with Darcy coefficient=·025.

Williams, Hubbell and Fenkell,* reporting on an extended investigation on curve loss in 90° elbows in large pipes, give results for a 30-inch pipe which indicate a continuously increasing loss with increase in the ratio R/D. The general results for this size pipe are shown in the following:

TABLE XV

R/D .	24	16	10	6	4	2·4
k . .	1·4	·96	·82	·65	·27	·24
Loss .	180	123	105	83	35	31

In this table the second line gives the value of k in the formula $h=kv^2/2g$, while the third line gives the length in feet of straight 30-inch pipe equivalent to the excess loss due to elbows.

The velocities involved in the experiments leading to the results in Table XV were all low—from 1 to 3·5 feet per second.

As a result of an extended investigation on 90° elbows in 3-inch and 4-inch pipe, Brightmore† found results which he expressed in the form of curves showing values of the excess loss in terms of head. The value of C in the Chézy formula for average pipe of these sizes may be taken at 110. Using this value and converting the values drawn from Brightmore's curves we have the following:

TABLE XVI

Size of Pipe 4 inches. Velocity in (fs).

R/D	5		7·5		10	
10	·24	3·7	·20	3·1	·21	3·3
8	·26	4·0	·27	4·2	·28	4·4
6	·30	4·7	·32	4·9	·33	5·1
4	·21	3·4	·29	4·5	·28	4·4
2	·34	5·4	·38	6·0	·39	6·1
0	1·12	17·5	1·22	19·0	1·18	18·4

Values of k and of equivalent length.

* "Transactions Am. Soc. C.E., 1902."
† "Proc. Inst. C.E.," Vol. CLXIX, 1906–7.

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In the above table the first column under each velocity gives the value of k in the equation $h=kv^2/2g$, while the second column gives the length in feet of straight 4-inch pipe equivalent to the excess loss due to elbows.

TABLE XVII

Size of Pipe 3 inches. Velocity in (fs).

R/D	5		7·5		10	
14	·13	1·5	·14	1·7	·13	1·5
12	·17	2·0	·13	1·6	·12	1·5
10	·24	2·8	·19	2·2	·19	2·3
8	·30	3·5	·33	3·8	·34	4·0
6	·34	4·0	·38	4·5	·39	4·6
4	·30	3·5	·32	3·7	·30	3·5
2	·43	5·0	·42	4·9	·42	4·9
0	1·12	13·1	1·31	15·2	1·18	13·9

Values of k and of equivalent length.

In the above table the first column under each velocity gives the value of k in the equation $h=kv^2/2g$, while the second column gives the length in feet of straight 3-inch pipe equivalent to the excess loss due to elbows.

In Brightmore's experiments the elbow of ratio $R/D=0$ was represented by a Tee plugged at one outlet.

Reporting on the results of an extended investigation on 90° elbows in a 6-inch pipe line, Schoder* gives results as follows:

TABLE XVIII

Velocity in (fs).

R/D	3		5		10		16	
20	·34	9·4	·27	7·8	·19	5·6	·14	4·3
15	·15	4·2	·09	2·6	·05	1·4	·016	0·5
10	·21	6·0	·16	4·5	·11	3·3	·08	2·5
8	·28	7·8	·21	6·2	·17	5·1	·13	4·1
6	·28	7·8	·21	6·1	·17	5·1	·14	4·3
5	·14	4·0	·12	3·5	·11	3·3	·11	3·3
4	·24	6·6	·18	5·3	·16	4·7	·12	3·8
3	·21	5·8	·18	5·1	·16	4·7	·12	3·8
2·16	·22	6·2	·19	5·4	·17	5·1	·13	4·1
1·90	·25	7·0	·21	6·2	·19	5·8	·16	4·9
1·76	·24	6·8	·24	6·9	·23	6·8	·29	8·8
1·34	·39	10·8	·33	9·6	·30	8·9	·26	8·1

Values of k and of equivalent length.

* "Trans. Am. Soc, C.E.," Vol. LXII.

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In the above table the first column under each velocity gives the value of k in the equation $h=kv^2/2g$, while the second column gives the length in feet of straight 6-inch pipe equivalent to the excess loss due to elbows.

In converting from one set of values to the other the following values of the coefficient C are employed, being those given by Schoder for the particular pipe and velocities employed.

v	C
3	120
5	122
10	124
16	126

In connection with the same investigation Schoder reports measurements on certain lengths of 8-inch pipe line containing slight bends, one of 3·8°, and a reverse curve of 2·18° in one direction and 2·81° in the other, all of which failed to indicate any measurable loss due to the bends.

In discussing Schoder's results, Davis* gives values of the loss due to 90° elbows in a 2-inch line for a number of different values of R/D. These are given in terms of loss in head in feet. From these results we may derive the following:

TABLE XIX

Size of Pipe 2 inches. Velocity in (fs).

R/D	5		15	
10	·31	2·4	·43	3·4
5	·19	1·5	·40	3·1
2·5	·26	2·0	·40	3·1
1·15	·46	3·6	·60	4·7
·00†	1·65	12·9	1·60	12·5

Values of k and of equivalent length.

In the above table the first column under each velocity gives the value of k in the equation $h=kv^2/2g$, while the second column gives the length in feet of straight 2-inch pipe equivalent to the excess loss due to elbows. In converting from one set of values to the other a typical value of $C=110$ was employed.

Experiments on a wide variety of small and medium sized elbows and fittings have given indications as below for the value of k in the equation $h=kv^2/2g$.

* *loc. cit.*

† Represented by a Tee plugged at one outlet.

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TABLE XX

Description	Velocities	k		
¾″ Black mall. elbow (old)	2, 5, 10	·82	·76	·72
¾″ Galvan. mall. elbow (new)	2, 5, 10	·57	·53	·50
1″ Black mall. elbow (old)	2, 5, 10	·76	·70	·67
1″ Cast-iron elbow (old)	2, 5, 10	1·02	·95	·90
2″ Mall. iron elbow	2, 5, 10	·74	·72	·69
3″ Cast-iron elbow	5, 10, 25	·54	·54	·53
4″ Cast-iron elbow	5, 10, 25	·61	·58	·54
6″ Cast-iron elbow	5, 10, 25	·50	·48	·48
2″ Cast-iron Tee and plug (water leaving branch)	2, 5, 10	1·85	1·91	1·88
Same Tee as above (water entering branch)	2, 5, 10	1·43	1·55	1·63
3″ Cast-iron Tee and plug (water entering branch)	5, 10, 25	1·45	1·43	1·37
4″ Cast-iron Tee and plug (water entering branch)	5, 10, 25	1·55	1·41	1·22
Same Tee as above (water leaving branch)	5, 10, 25	1·24	1·17	1·10
4″ Cast-iron Tee filled in to make square elbow	5, 10, 25	1·10	1·07	1·07

Throughout these various values of the loss due to 90° elbows, there is found considerable divergence among the various experiments and abundant evidence of irregularity and inconsistency in many of the results. These are doubtless due, for the most part, to the difficulty of eliminating the influence of obscure and secondary causes which have no representation in the scheme of the investigation or in the final formulæ deduced. The trend of the investigations indicates, as would be expected, a general decrease in the loss with increasing value of the radius of the bend. The results of Brightmore and Schoder all agree, however, in indicating an increasing value for the loss, to a local maximum, for values of R/D from 6 to 8 or 10, and followed by diminishing values for greater values of R/D. The results of Davis indicate a similar condition, but were not carried far enough to determine the decreasing values of the loss for values of R/D greater than 10. The lowest values of the loss for values of R/D less than 6 or 8 seems to be indicated for values from 2 to 4. Schoder's low values for $R/D=15$ and high values for $R/D=20$ seem to be abnormal and inconsistent with each other and with the other results. The results of Williams, Hubbell and Fenkell stand alone in indicating a continuously increasing value of the loss for values of R/D from 2·4 to 24. Either some undetected source of error is involved in these results, or the value of the loss is gradually increasing toward a maximum at some value of R/D beyond 24, after which it will decrease. It is obvious that as the ratio R/D indefinitely increases,

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the excess loss, as compared with an equal length of straight pipe, must approach zero and hence the values of the loss cannot indefinitely increase with increase of R/D.

These irregularities and inconsistencies indicate two things:

1. The experimental evidence at hand is neither sufficient in amount nor consistent enough in character to permit the development of a satisfactory general formula for curve loss.

2. The influence of secondary features, such as irregularities in form or surface condition, or irregular and turbulent conditions of flow which occasionally develop, seem sufficient in importance to entirely mask the general trend of the system of phenomena expressed in terms of velocity, size and value of R/D alone.

General Considerations.—The following general considerations may be noted in connection with these various minor losses of head.

The loss due to an abrupt enlargement is usually more serious than that due to an abrupt contraction. In either case, as noted, the loss is due primarily to the eddy formation resulting from an abrupt enlargement. When the ratio A_1/A_2 is small and the loss is correspondingly large, the loss for abrupt expansion will be the larger of the two. When the ratio A_1/A_2 approaches 1 and the loss itself becomes relatively insignificant, the loss for contraction will become the larger of the two. These results are readily verified from the formulæ and tables of (*b*) and (*c*).

The flow of a liquid past a sharp edge is always a fruitful source of eddy formation and of loss of head. The presence of a sharp edge projecting into a flowing stream means an abrupt change in the cross-sectional area and a corresponding abrupt enlargement or contraction or both.

In order to minimize these various sources of loss it is clear that all abrupt changes in stream line flow must be avoided, and in particular any abrupt enlargement of cross-sectional area, any flow over or across sharp edges, or any abrupt change in the general direction of flow.

10. General Resumé of Loss of Head in Pipe Line Flow

It thus appears, in connection with the flow of water through a pipe, that there are a considerable number of sources of loss of head. These may be classified as follows:

1. Loss of head due to friction denoted by h as discussed in Secs. 2–5.
2. Loss of head due to miscellaneous turbulence caused by abrupt changes in direction or velocity as discussed in Sec. 9 and denoted also by h. These various items of loss due to turbulence are all expressed in the general form

$$h=k\frac{v^2}{2g}$$

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where k is a coefficient to be determined according to the details of the case, and as discussed in Sec. 9.

It appears that the values comprising loss (2) are usually small in comparison with those of the frictional loss (1), and with suitable dispositions they may in many cases be rendered negligible. In any case taking the values comprising loss (1) proportional to v^2, the total loss of head may be expressed in the form

$$h=\frac{Lv^2}{C^2r}+\Sigma k\frac{v^2}{2g} \qquad (31)$$

Where Σk denotes the sum of all the coefficients k for the various secondary losses discussed in Sec. 9. This may be put in the form

$$h=\left(\frac{L}{C^2r}+\frac{\Sigma k}{2g}\right)v^2 \qquad (32)$$

In this equation $\Sigma k/2g$ may be considered as a supplementary term modifying the principal term L/C^2r. In most cases, as noted above, the supplementary term will be small compared with the principal term, and in consequence it is very often neglected; or otherwise is considered as included within the margin of uncertainty which must attach to the selection of a value of C, or of the roughness coefficient in Kutter's formula from which C is determined.

In dealing with practical problems it is therefore customary to select from judgment either C direct or a value of the roughness coefficient which will determine C, and to assume that within the necessary margin of uncertainty concerning these values, the resulting value of h or friction loss, will adequately represent all loss of head throughout the pipe.

It should not be forgotten, however, that the items comprising loss (2) may require special recognition, and in such case suitable estimates or allowances must be made. Thus in case the flow passes through a partially closed valve the loss of head may be very considerable, and in such case due allowance must be made for the value of such loss independent of the estimates for determining the value of the friction loss by the usual formulæ.

11. Special Relations

The following relations which will be found of frequent value in the discussion of various problems may be conveniently assembled at this point.

(*a*) Friction head h=loss of energy per pound of water in traversing distance L.

(*b*) $\frac{Lv^2}{C^2r}$=loss of energy as in (*a*)=work done in carrying one pound of water through distance L against frictional resistance in pipe.

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(c) $\frac{v^2}{C^2r}$=frictional or skin resistance per pound of water in pipe.

(d) wAL=weight of water in pipe in pounds.

(e) $\frac{wALv^2}{C^2r}=wAh$=total frictional or skin resistance for pipe as a whole.

(f) wAv=rate of flow (pounds per second).

(g) $1/wAv$=time for flow of one pound past a given section.

(h) $\frac{wALv^3}{C^2r}$=rate of work against friction for entire pipe.

(i) $\frac{wALv^3}{C^2r}\times\frac{1}{wAv}=\frac{Lv^2}{C^2r}=h$=work done against friction in entire pipe in the time required for the flow of one pound past a given section.

(j) $\frac{WLv^2}{C^2r}=Wh$=work done against friction in entire pipe in the time required for the flow of weight W past a given section.

(k) $\frac{wALv^2}{2g}$=total kinetic energy in pipe.

(l) $\frac{wALv}{g}\,\frac{dv}{dt}$=time rate of change of total kinetic energy.

(m) $\frac{wALv}{g}\,\frac{dv}{dt}\times\frac{1}{wAv}=\frac{L}{g}\,\frac{dv}{dt}$=change in kinetic energy in the time required for the flow of one pound past a given section.

(n) $\frac{L}{g}\,\frac{dv}{dt}$=measure of an accelerating head.

Suppose an unbalanced pressure p at one end of the pipe, acting on the cross sectional area A. Then pA=total pressure acting as an accelerating force on contents of pipe wAL. Then acceleration$=\frac{dv}{dt}=\frac{\text{Force}}{\text{Mass}}=\frac{pAg}{wAL}$ and $\frac{L}{g}\,\frac{dv}{dt}=\frac{p}{w}=$ accelerating head. That is $\frac{L}{g}\,\frac{dv}{dt}$ is the measure of an accelerating head acting over the cross section A and producing an acceleration dv/dt: or again, an unbalanced head H acting at one end of a pipe will produce an acceleration in the velocity of movement of the contents of the pipe measured by $dv/dt=gH/L$.

(o) $\frac{WL}{g}\,\frac{dv}{dt}$=change in kinetic energy in the time required for the flow of weight W past a given section.

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12. Distribution of Velocity Over Cross Section of Pipe

Pitot tube measurements have been made by many investigators over the cross section of a pipe running full. The results are not altogether concordant and the law of distribution of velocity is by no means definitely assured. Errors of observation, effects due to pulsating movements and to turbulence, and obscure influences due to velocity and roughness have doubtless contributed to this result. In Fig. 9 let the velocities at varying distances along the radius be observed and set off from *AB* parallel to the line of flow and located

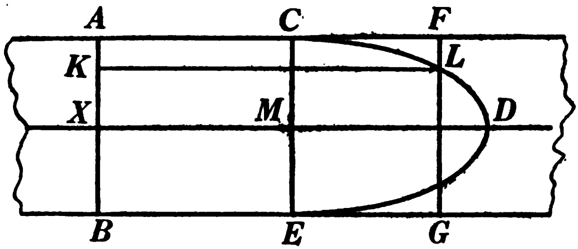

Fig. 9.—Variable Flow over Cross Section of Pipe.

at the points of observation. The general result will be a curve of the form *CDE* showing a velocity *AC* at the outer surface of the stream, a velocity *XD* at the centre and a varying distribution of velocity between the centre and the surface as shown by the curve. The chief points of interest in such a curve are the following:

1. The relation of *AC* to *XD*, the velocity at the surface to that at the centre.
2. The relation of the mean velocity to *XD*, the velocity at the centre.
3. The radius at which the actual velocity will equal the mean.

In Fig. 9 let *AF* be the mean velocity. Then the actual velocity *KL* at the radius *XK* will equal the mean velocity *AF*.

Bazin, Williams, Hubbell and Fenkell, Cole, Schoder and others have made observations on the form of such a curve and the observations generally indicate for *CDE* a curve shaped much like an ellipse. There is considerable variance in the relation between *AC* and *XD*. Williams, Hubbell and Fenkell came to the conclusion that the surface velocity was about ·50 that at the centre. Cole's measurements indicated a ratio of ·60 and more. It is clear that the mean velocity will be the height of a cylinder *AFGB* whose volume is equal to the solid of revolution formed by revolving *ACD* about *XD*. If the elliptical character of *CDE* is assumed, the latter will comprise a cylinder plus a half ellipsoid of revolution. It is

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well known in geometry that the mean height of a half sphere or ellipsoid of revolution equals 2/3 the height of the body. Hence

$$AF=AC+2/3MD.$$

Denoting AF by v_m, AC by v_s and XD by v_c, we find immediately

$$v_m=v_s+2/3(v_c-v_s)$$

or $v_m=2/3v_c+1/3v_s$.

If then $v_s=\cdot50v_c$ we have

$$v_m=\cdot833v_c.$$

If $v_s=\cdot60v_c$ we have

$$v_m=\cdot87v_c.$$

In any case it is readily seen from the properties of the ellipse, that the point where the height of the curve is 2/3 the maximum height is at $\cdot745r$. Hence if the ellipse may be assumed as a close approximation to the general character of the curve of velocity CDE, the mean velocity will be found at a point close about $\cdot75r$ from the centre.

In any measurement of importance dependence should not be placed on such a relation and one or preferably two complete traverses with the Pitot tube should be made across diameters at right angles to each other.

Bellasis has compiled from a number of sources the values given in Table XXI showing the ratio v_m/v_c for a number of different kinds and sizes of pipe, and at various values of v_m.

TABLE XXI

TABLE SHOWING VALUES OF RATIO OF MEAN VELOCITY TO CENTRE LINE VELOCITY IN PIPE LINE FLOW*

Kind of pipe	Diameter of pipe in inches	Mean velocities in feet per second						
		·78	1·5	2·5	3·5	5	8	14
Brass seamless	2	·70	·73	·77	·79	·80	—	—
Cast iron	7·5	—	—	·80	·81	·82	·83	·84
Cast iron	9·5	—	·80	·81	·82	·83	·84	·85
Cast iron with deposit	9·5	—	·81	·81	·82	·82	·83	·83
New iron coated with coal-tar	12	—	·83	·83	·84	·85	·85	·85
	16	—	·82	·83	·84	·85	—	—
	30	·75	·83	·84	·85	—	—	—
Cement	31·5	—	—	—	·85	·86	—	—
New iron coated with coal-tar	42	—	—	—	·86	—	—	—

13. MEAN VELOCITY AND MEAN VELOCITY HEAD OVER CROSS SECTION OF PIPE

The experimental investigation of the velocity over the cross section of a pipe gives a result as indicated in Sec. 12. It becomes a problem of interest to determine, from such a distribution of

* Bellasis, "Hydraulics," p. 129. Rivingtons, London.

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velocity, the volume flow and kinetic energy of the stream, or otherwise the mean velocity and the mean velocity head.

In Fig. 10 let the annular rings denote a series of elements into which the cross-sectional area may be divided, and over any one of which the velocity may be considered uniform.

Let a denote the area of any one such annular ring or element, v the velocity over this element, $\triangle V$, the corresponding element of volume flow and $\triangle K$ that of kinetic energy.

$$\text{Then } \triangle V = av$$

$$\triangle K = \frac{wav^3}{2g}$$

$$\text{And } V = \Sigma av$$

$$K = \frac{w}{2g}\Sigma av^3$$

where, as usual, Σ denotes the summation of a series of terms, all similar in character and made up by multiplying each a by its appropriate value of v in one case and v^3 in the other.

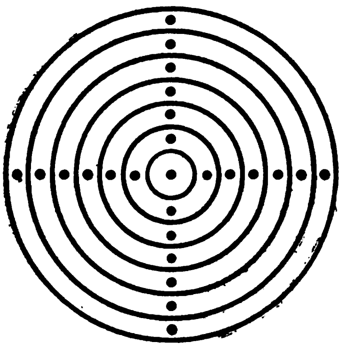

Fig. 10.—Annular Elements of Cross Section.

The general procedure may therefore be outlined in the following steps:

1. By means of Pitot tube observations obtain a series of values of v at a series of points along the diameter of the pipe, or preferably along two diameters at right angles. These points may be so chosen as to come in the middle of a series of annular rings, as in Fig. 10. These values for the two diameters may then be averaged so as to give a mean series for the mid-points along the radius.

2. The values of a are then determined in accordance with the successive values of the radius.

3. Each value of a is multiplied by the corresponding value of v and the products summed. The result will give V, the total volume flow in (f3s). This divided by the cross sectional area of the pipe will give the mean v for the entire area.

4. Each value of a is multiplied by the cube of the corresponding value of v and the results summed. This sum multiplied by $w/2g$ will give the kinetic energy passing the given section in one second of time. This divided by wV, the weight passing the section per second, will give the mean kinetic energy per pound, or otherwise the mean velocity head.

This general method is of such wide application that it will be of

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interest to develop a slightly different method for treating the Pitot tube observations.

In terms of the calculus, and assuming the width of the annular rings indefinitely small, we have

Area of annular element $2\pi r dr$. Then

$$dV = 2\pi v r dr$$

$$dK = \frac{\pi w}{g} v^3 r dr.$$

In Fig. 11 let ABC denote the values of v plotted along the radius OX. Then at a series of points along the radius take the

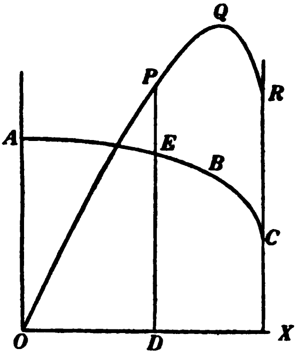

Fig. 11.—Integration for Volume Flow.

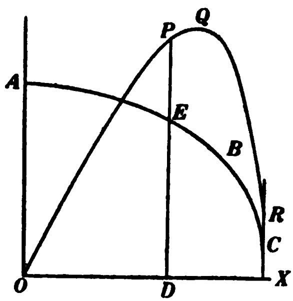

Fig. 12.—Integration for Energy Flow.

value of v, multiply it by r and set off the product as the ordinate of a new curve $OPQR$. Thus $DP = OD \times DE$ and similarly for all other points. Then the curve $OPQR$ will represent the distribution of the product vr along the radius. The area of this curve taken by planimeter, or otherwise, will give the integration of $vrdr$, and this multiplied by 2π will give the value of V.

Similarly in Fig. 12 let ABC denote the values of v^3 plotted along OX and $OPQR$, a curve derived as in Fig. 11 by multiplying each value of the ordinate as DE by the corresponding radius OD. Then the curve $OPQR$ will represent the distribution of the product v^3r along the radius and the area of this curve by planimeter or otherwise will give the integration of v^3rdr, and this multiplied by $\pi w/g$ will give the value of K.

14. Hydraulic Gradient

Writing again the general energy equation as in (1) we have

$$\frac{p}{w} + \frac{v^2}{2g} + z + h = Z_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

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In Fig. 13 let R denote the reservoir with water level at NN. Let $ACDB$ denote a pipe line with discharge at B. Let XX denote the base or datum line from which we measure gravity head z. Then within the reservoir at the surface NN we have $p/w=b$ (where $b=$ atmospheric or barometric head), $v=O$, $h=O$, $z=H_0$ and hence $b/w+H_0=Z_0$. That is the entire head Z_0 is measured by the gravity head H_0 plus the atmospheric head b/w, and the gravity head H_0 is measured by the elevation of the surface NN above the datum line XX. To simplify the present discussion suppose the pipe $ACDB$ of uniform size. Then the value of v will be uniform throughout the length, as also the value of the velocity head $v^2/2g$. Suppose this value laid off as a vertical distance from

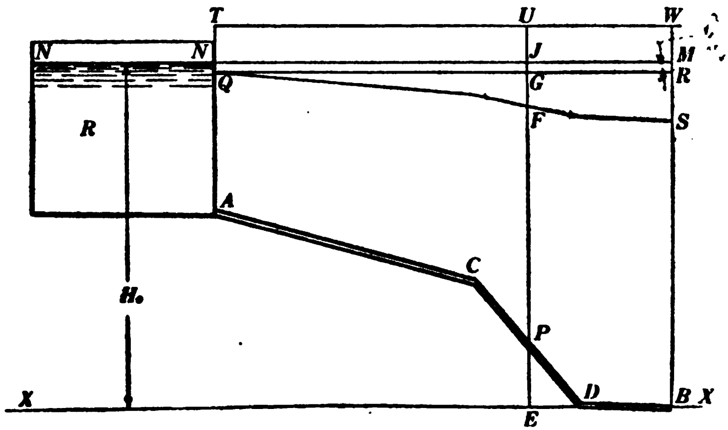

FIG. 13.—HYDRAULIC GRADE.

NM downward giving the line QR. Then the vertical intercept between NM and QR will, at any and all points, give the value of $v^2/2g$.

Suppose likewise a line TW laid off above NM at a vertical distance equal to the atmospheric head b/w.

Next suppose the value of h to be computed for the various points along the line, giving in each case the total loss of head from the entrance A to the given point P, and let the values be laid off as vertical ordinates from QR downward, giving the line QS. It may be noted that strictly speaking the line QS will start at Q a little below the velocity head line QR, the distance between the two lines at Q denoting the value of h at Q which will be measured by the entrance loss as discussed in Sec. 9.

Next it is clear that the line of the pipe $ACDB$ gives graphically at any point the value z as the vertical distance above the datum XX. Hence at any point P it follows that $z=PE$, $h=GF$, $v^2/2g=$

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JG, $b/w=JU$ and $Z_0=EU$. Hence comparing with equation (33) it follows that we must have $p/w=FP+JU$.

That is, the total pressure head within the pipe is measured by the sum of the two intercepts $FP+JU$. But the latter of these measures the atmospheric head or pressure and hence the former must measure the head or pressure above the atmosphere; or in other words, the head or pressure which would be indicated by an ordinary pressure gauge.

It follows that if an open-end vertical tube were inserted in the pipe at P the water would rise in such tube up to the level F, thus indicating directly the pressure head at P above the atmosphere. Again, if a series of such tubes were inserted along the length of the pipe from A to D the water levels would all lie on the line QS.

The line QS thus determined, is called the hydraulic gradient or hydraulic grade line. Again any point on this line as F is called "hydraulic grade" for the corresponding point P on the pipe line. The hydraulic grade line may thus be defined as a line any point of which is vertically above the pipe line a distance equal to the pressure head above the atmosphere in the pipe line at such point; or conversely as a line any point of which is vertically below the static level line a distance equal to the sum of the velocity head and lost head for the corresponding point in the pipe line.

It must not be forgotten that the pressures and pressure heads indicated by the hydraulic grade line are measured above the atmosphere and that in terms of absolute pressure the pressure in the pipe or the pressure head will be greater than the amount thus indicated by the pressure or pressure head due to the atmosphere. Thus at P (Fig. 13), the pressure head above the atmosphere is denoted by PF while the absolute pressure head is denoted by $PF+JU$.

Again, in Fig. 14 let ACB denote a pipe line with entrance at A and discharge at B. Let QFS be the hydraulic gradient and TMW

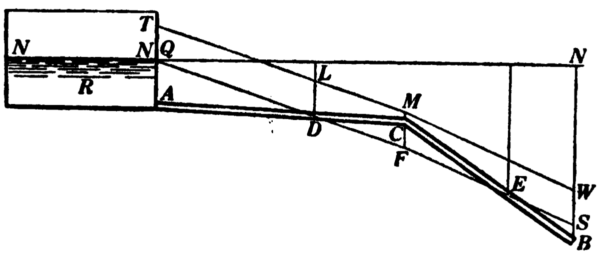

FIG. 14.—HYDRAULIC GRADE.

the line of atmospheric head above QS. Then at the points D and E the pressure in the line will just balance the atmosphere. From A to D and from E to B the pressure will be greater than the atmosphere, while between D and E the pressure will be less than the

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atmosphere. At C the pressure head will be a minimum, measured by CF below the atmosphere or CM above absolute pressure datum. It is thus clear that if any part of the pipe line crosses or rises above the hydraulic grade line the pressure within such part of the line will be less than that of the atmosphere.

If, furthermore, the line should rise sufficiently above the hydraulic grade line QS to just reach the atmospheric head line TW, then the absolute pressure at such point in the pipe will be zero; or more exactly it will be reduced to the pressure of water vapour under the temperature of the water at the given point. The pressure of water vapour at ordinary temperatures is very small and it is customary to neglect its influence on ordinary problems of pipe line flow. The water at such point would therefore be in the same physical condition as in a bell jar exhausted under an air pump to a perfect vacuum, or more exactly to a vacuum corresponding to the vapour pressure at the temperature of the water.

If again the line of the pipe should at any point rise above the line TW it would imply a negative pressure in the water, or in other words a tension instead of a compression. A tension cannot, however, be developed in a stream of water. In answer to the conditions which might tend to set up a tension the stream will break and the conditions of continuous steady flow will no longer obtain.

Actually, turbulence, unsteadiness and interruption of the conditions of steady flow will result somewhat before the pressure is reduced to zero. This is due in large measure to the liberation, under the reduced pressure, of the air which is normally held in solution in the water.

The rise of the pipe at any point above the hydraulic grade line results in a condition of unsteady flow with irregular turbulence and a tendency to set up pounding or water hammer, especially in the part beyond the point which rises above the grade line. Since the grade line varies with the velocity or with the rate of discharge, it is clear that in any pipe line, such as in Fig. 14, there will be some critical velocity which will just bring the grade line to the angle C. For lower velocities C will lie below the grade line and for higher velocities above it.

In any such case the trouble resulting from a point C lying above the grade line may be avoided by a suitable reduction of the velocity of flow with the consequent rise in the grade line.

15. General Formula for Capacity

Taking the Chézy formula for velocity we have

$$v=C\sqrt{ir}=\frac{Ci^{\frac{1}{2}}A^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

$$V=vA=\text{rate of flow (f3s).}$$

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Then $V = \dfrac{C i^{\frac{1}{2}} A^{\frac{3}{2}}}{P^{\frac{1}{2}}}$ or $V^2 = \dfrac{C^2 i A^3}{P}$ ....(34)

For a round pipe running full this becomes

$$V = \frac{C i^{\frac{1}{2}} \pi D^{\frac{5}{2}}}{8} = \cdot 3927 C i^{\frac{1}{2}} D^{\frac{5}{2}} \qquad (35)$$

Where D=diam. in feet: or solving for i we have

$$i = \frac{6{\cdot}484 V^2}{C^2 D^5} \qquad (36)$$

In Table IV will be found the fifth power of numbers which with an ordinary table of squares and square roots will serve to readily solve numerical problems involving these formulæ.

16. Carrying Capacity of Round Pipe running Partly Full

In Fig. 15 let AB denote the surface of the water. Then for the area ABC we have

$$A = \frac{D^2}{4} (\theta - \sin\theta \cos\theta),$$

$V = C\sqrt{RS}$
$V^2 = C^2 RS$
$R = \frac{V^2}{C^2 S}$

and for the wetted perimeter ACB we have

$$P = D\theta.$$

Then referring to (34), the general formula for capacity, we find by substitution and change of form:

$$\frac{V^2}{C^2 i D^5} = \frac{(\theta - \sin\theta\cos\theta)^3}{64\,\theta} \qquad (37)$$

Table XXII gives for 5° intervals the values of the right-hand side of this equation, and also of z/D. By means of this table we may readily determine the relations between V, C, i, D and θ or z. Thus if the pipe, the slope and the quantity are given and z is required, we substitute the given values in the left-hand side of (37) and find the numerical value. Then from the table we find approximately the value of θ and then the value of z. Again if z or θ, C, i and V are given, we readily find D by taking from the table the value of the right-hand member of (37) and solving the resulting equation for D.

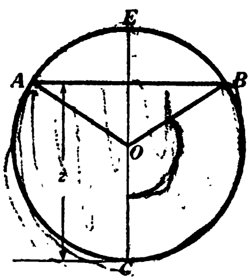

Fig. 15.—Flow in Circular Section Partially Filled.

It will be noted that the capacity is a maximum for θ approximately 155° and $z = \cdot 95D$. More exact analysis by means of the usual calculus treatment for maxima and minima shows the capacity

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a maximum for $\theta=154° 05'$, $z/D=\cdot9496$. It thus appears that a round pipe running with an open segment at the top about 5 per cent of the diameter in height, will have a capacity nearly 10 per cent more than if running full. This result arises from the fact that for a reduction in section up to this point the beneficial influence due to the relatively rapid decrease in the wetted perimeter is more influential on V than the relatively slow decrease in the area of the

TABLE XXII

θ	a	b	θ	a	b
0	·0000	—	95	·5436	1·602
5	·0019	—	100	·5868	2·015
10	·0076	—	105	·6294	2·464
15	·0171	—	110	·6710	2·932
20	·0301	—	115	·7113	3·401
25	·0469	—	120	·7500	3·854
30	·0670	·0007	125	·7868	4·272
35	·9040	·0023	130	·8214	4·639
40	·1170	·0062	135	·8536	4·944
45	·1465	·0148	140	·8830	5·178
50	·1786	·0315	145	·9096	5·334
55	·2132	·061	150	·9330	5·424
60	·2500	·111	155	·9532	5·443
65	·2887	·187	160	·9698	5·406
70	·3290	·299	165	·9830	5·324
75	·3706	·454	170	·9924	5·207
80	·4132	·659	175	·9981	5·073
85	·4564	·918	180	1·0000	4·935
90	·5000	1·233			

a=ratio of z to D, depth to diameter.
b=value of $\frac{(\theta-\sin\theta\cos\theta)^3}{64\,\theta}$ in equation (37).

section. Beyond this point, however, the influence due to the decrease of area becomes paramount and the capacity decreases for a further reduction in area.

It should be noted that these results presuppose perfectly steady conditions and a smooth water surface without waves. Actually there will be some wave formation, especially if there is any departure from uniform motion, and under such circumstances the theoretical gain for a partially filled cross section will be much reduced. Under ordinary conditions it can hardly be considered desirable to attempt to run a round pipe partially full for the sake of an increase in capacity. The same remarks apply generally to all forms of cross section, though with certain tunnel forms the relative advantage might be somewhat greater than with round pipe.

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17. General Problem of Steady Flow

Let AB (Fig. 16) denote a pipe line of varying cross section terminating in a nozzle at the discharge end B.

Let h=loss of head due to friction and turbulence in general.

Starting from the upper or entrance end of the pipe line A, the total loss h will have continuously varying values, increasing along the length of the pipe according to the circumstances affecting skin friction and turbulence.

Let a, A_1, A_2, A_3, etc., denote respectively the cross section areas of the nozzle and of the various lengths of pipe of which the line is composed, counting from the discharge end back.

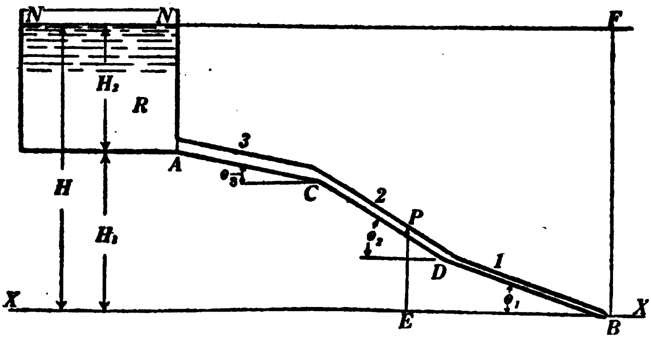

Fig. 16.—General Problem of Pipe Line Flow.

Let $a=m_1A_1=m_2A_2=m_3A_3$, etc. . . . . (38)

Where m_1, m_2, m_3, etc., form a series of coefficients relating the various areas A_1, A_2, etc., to a.

Also let u, v_1, v_2, v_3, etc.=velocities in the nozzle and in the successive sections of area A_1, A_2, etc.

Then $v_1=m_1u$, $v_2=m_2u$, etc. . . . . (39)

Let H=gravity or static head relative to the base line XX.
- h=total lost head from A to any point P.
- y_1=head due to absolute pressure (usually atmospheric) on surface of water at source.
- y_2=head due to absolute pressure at point of discharge.
- b=atmospheric head at any point P.
- p=pressure above atmosphere at any point P.
- z=gravity head at any point $P=PE$.

Usually y_1, b and y_2 are all the same. The heads y_1 and y_2 in the general case, however, may have any value. Thus we might have compressed air acting on the surface NN, or again air under a reduced pressure or partial vacuum. Likewise the head y_2 implies any pressure whatever at the point of discharge, such for example as that due to discharge into a tank with compressed air, or into

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a partial vacuum, or again discharge under water and therefore against an absolute pressure due to the water plus the atmosphere.

Usually the differences in atmospheric pressure are negligible, and in such case, and where no other pressures are involved, the terms in b and y may be omitted from the equations.

In the general case we have, as in (1), the Bernouilli equation for steady flow between A and P as follows:

$$(H+y_1-h)=\frac{p}{w}+b+\frac{v^2}{2g}+z \quad \dots\dots\dots\dots (40)$$

$$\text{or} \quad -\Delta h=\Delta\left(\frac{p}{w}+b+\frac{v^2}{2g}+z\right) \quad \dots\dots\dots\dots (41)$$

Putting (41) into words we have:

The difference between the values of the total lost head h corresponding to any two points such as P is equal, with the opposite algebraic sign, to the aggregate change in the actual head $(p/w+b+v^2/2g+z)$ between the same two points.

In applying these equations there are five variable terms involved, of which four must be known in order to determine the fifth. In the usual procedure h is expressed as a function of v by means of the formulæ of Secs. 2–6 and 9.

At the point of discharge through a nozzle or valve, however, there is usually a special loss which must be otherwise expressed.

Taking the datum for z at the level of the valve or nozzle, the absolute pressure head just back of the valve will have the value

$$\frac{p}{w}+b=H+y_1-\frac{v_1^2}{2g}-h \quad \dots\dots\dots\dots (42)$$

where h is the total lost head between A and B.

Just beyond the nozzle, at the point of discharge, the absolute pressure head is y_2.

The difference between these two measures the net or resultant pressure head acting between the two sides of the valve or through the nozzle. This net pressure head is then $H+y_1-y_2-v_1^2/2g-h$. Adding to this the velocity head $v_1^2/2g$ we shall have the total net head available for producing the discharge velocity u at the mouth of the nozzle and hence the external velocity head $u^2/2g$. This total net head is then $H+y_1-y_2-h$.

We may then consider this total net head at the valve as transformed through the valve or nozzle into the velocity head $u^2/2g$. This transformation is accompanied by loss, and hence we may write

$$\frac{u^2}{2g}=f[H+y_1-y_2-h] \quad \dots\dots\dots\dots (43)$$

where f is the efficiency of transformation.

We shall usually call f the *nozzle coefficient* or *coefficient of discharge*. The value of f is always less than 1, approaching 1 as an upper limit. The difference $(1-f)$ denotes the fraction of the total net head lost by friction and turbulence in the valve or nozzle.

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Putting Δy for $y_1 - y_2$ and substituting for h its value as in (19) we have

$$\frac{u^2}{2g} = f\left[H + \Delta y - \Sigma\left(\frac{m^2L}{C^2r}\right)u^2\right] \quad \ldots\ldots\ldots\ldots (44)$$

$$\text{whence } u = \sqrt{\frac{(H + \Delta y)}{\frac{1}{2gf} + \Sigma\left(\frac{m^2L}{C^2r}\right)}} \quad \ldots\ldots\ldots\ldots (45)$$

Note that Δy will be $+$, $-$ or 0 according as y_1 is greater than, less than or equal to y_2.

Knowing u we may then find any other velocity from (39). The known values of h, z and v for any point in the line may then be substituted in (40), serving to determine p/w or p for such point. Thus the entire hydraulic conditions throughout the length of the line become known.

If we may neglect the loss through the nozzle, (45) becomes

$$u = \sqrt{\frac{H + \Delta y}{\frac{1}{2g} + \Sigma\left(\frac{m^2L}{C^2r}\right)}} \quad \ldots\ldots\ldots\ldots (46)$$

If in addition we have $y_1 = y_2$, (46) becomes

$$u = \sqrt{\frac{H}{\frac{1}{2g} + \Sigma\left(\frac{m^2L}{C^2r}\right)}} \quad \ldots\ldots\ldots\ldots (47)$$

If, furthermore, the pipe is of uniform diameter throughout we have

$$u = \sqrt{\frac{H}{\frac{1}{2g} + \frac{m^2L}{C^2r}}} \quad \ldots\ldots\ldots\ldots (48)$$

The value of u for any other combination of the determining characteristics is readily derived from the general form in (45).

We have, also, by combining (42) and (43),

$$\frac{u^2}{2g} = f\left(\frac{p}{w} + b - y_2 + \frac{m_1{}^2u^2}{2g}\right) \quad \ldots\ldots\ldots\ldots (49)$$

$$\text{Whence } \frac{p}{w} = \frac{u^2}{2gf} - b + y_2 - \frac{m_1{}^2u^2}{2g} \quad \ldots\ldots\ldots\ldots (50)$$

$$\frac{p}{w} + b - y_2 = \frac{u^2}{2g}\left(\frac{1}{f} - m_1{}^2\right) \quad \ldots\ldots\ldots\ldots (51)$$

If the loss through the nozzle is negligible, we have

$$\frac{p}{w} + b - y_2 = \frac{u^2}{2g}(1 - m_1{}^2) \quad \ldots\ldots\ldots\ldots (52)$$

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Equation (50) gives the gauge pressure just back of the valve, while (51) and (52) give the drop in pressure through the valve.

If, furthermore, $y_2=b$, as is the case with atmospheric discharge, we have

$$\frac{p}{w}=\frac{u^2}{2g}(1-m_1^2)\qquad\qquad(53)$$

18. Pipe Line Operating with a Free Surface Flow

In certain cases a pipe line may be operated partially full, thus developing all the hydraulic characteristics of open channel flow (see Fig. 17).

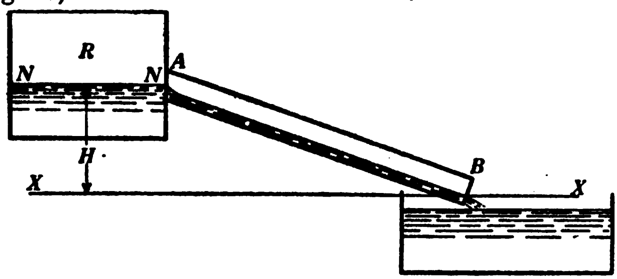

Fig. 17.—Flow in Open Channel.

Neglecting any changes in atmospheric head and taking p as the gauge pressure, we have for the general equation, as in (1):

$$H-h=\frac{p}{w}+\frac{v^2}{2g}+z.$$

At all points in the line sensibly near the surface of the stream we shall have $p=0$, and hence as the general equation of flow near the surface:

$$H-h=\frac{v^2}{2g}+z.$$

At A just inside the pipe, and where the velocity v has been developed, we shall have

$$H-h_A=\frac{v^2}{2g}+z_A\qquad\qquad(54)$$

$$\text{or } z_A=H-\left(\frac{v^2}{2g}+h_A\right)\qquad\qquad(55)$$

This means that just inside the entrance the level of the water in the pipe will have dropped below that in the reservoir a distance $(v^2/2g+h_A)$, representing the head due to the velocity v plus some entrance loss h_A, as discussed in Sec. 9.

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At the discharge end B we have $z=0$, and hence

$$H-h_B=\frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (56)$$

Subtracting (56) from (54), we have

$$h_B-h_A=z_A \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (57)$$

Expressing this in words, the loss of head in the line between A and B is the difference in level between these same points. This means that whatever the actual gradient on which AB is laid, the velocity will automatically take such a value as will consume the entire head H in doing three things, as follows:

(1) Producing the velocity v (vel. head $v^2/2g$).
(2) Overcoming loss at entrance A (lost head h_A).
(3) Overcoming losses in line (lost head $h_B-h_A=z_A$).

Or again, in a more restricted way we may say that no matter on what gradient the pipe is laid, the velocity will automatically take such a value as will consume in lost head the entire difference in level between B the point of discharge and A a point just inside the inlet and where the velocity v has been set up.

For the loss in head between A and B we have as in (3):

$$h_B-h_A=\frac{Lv^2}{C^2r}=z_A$$

or from (54):

$$H=h_A+\frac{v^2}{2g}+\frac{Lv^2}{C^2r} \quad \ldots\ldots\ldots\ldots\ldots \quad (58)$$

As in (23) we may express h_A in the form $kv^2/2g$ where k must be determined by estimate. Substituting this value in (58) and solving for v we have

$$v=\sqrt{\frac{H}{\frac{1+k}{2g}+\frac{L}{C^2r}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (59)$$

It is very common to omit $(1+k)/2g$ in comparison with L/C^2r, or otherwise to assume that its value is absorbed in the uncertainty regarding the value of C. In such case we have approximately:

$$H=\frac{Lv^2}{C^2r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (60)$$

and $v^2=\dfrac{C^2rH}{L}$

But $H/L=i$ the gradient; whence

$v=C\sqrt{ir}$, as in equation (2).

In case the gradient is very steep and the developed velocity very high, as where the pipe is used as a free surface spillway, the velocity head will be too great to permit of omission from the equation. In this case equation (59) must be employed. This

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equation will, moreover, not be exact, since there will be, at the upper end of the pipe, a certain length over which the water will be accelerating from some low velocity at the very point of inlet up to the full velocity v. The effective length of the line for velocity v, so far as friction loss is concerned, is therefore not L but something less than L. Moreover, throughout this part of the pipe, the cross sectional area of the stream will gradually contract with increasing velocity, and with corresponding change in r. Otherwise one may say that, of the total length L, a part will operate under the ultimate velocity v and a fixed r, while the remainder, at the upper end, will operate under a variable v and a variable r from the inlet to the point where the conditions are practically steady. These conditions indeed prevail in any case, but with a low gradient and low velocity both the velocity head and influence on the friction head due to the accelerating length may safely be neglected. With high velocities we may still, usually without serious error, neglect the influence on h due to the accelerating length and thus obtain from (59) a satisfactory value by taking L=length of the line and r constant.

It should also be noted that both (59) and (60) admit of direct solution in case the value of r is given. On the other hand, in case the quantity or volume flow V is given, the equations are implicit. The hydraulic mean radius is not known in advance, since it will depend on v. Such a case will call for a trial and error procedure. A value of r is assumed, the value of v found and the resulting V. Thus by successive trial a value of r will be found which will give the V desired. The relations of Table XXII will conveniently aid in handling problems of this character.

In order to illustrate the error involved in using for a case of free surface flow with high velocity, equation (2), Sec. 2, instead of (59), the following example may be noted.

Let angle of slope=45°.

Then $i = \cdot 7071$.
Let $L = 100$ f.
$C = 120$.
$r = \cdot 5$.
Then $H = 70 \cdot 7$.

Then taking $k=0$, substituting in (59) and solving we find $v=49$.

If, however, we neglect the velocity head and use equation (2) we shall find $v = 71 \cdot 4$.

The proper value of C to use in the case of such very high gradients and velocities is subject to very grave uncertainty. As has been noted in Sec. 7, there is reason to anticipate increasing values of C with increasing velocities. Actual experimental values are, however, lacking. If we attempt to deduce any indication from the results of ship resistance experiments, we find an increase approximately as the seventh root of the speed (see Sec. 7). If then a value of 110 were taken as appropriate for any given case with $v=10$ we should have

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corresponding to $v=50$, a value of $C=138$. If this value were used in (59) the result would be $v=52\cdot1$, while in equation (2) we should have $v=82$. There is need of further experimental data regarding the values of C appropriate to such velocities, and also regarding the importance of the length at the upper end affected by acceleration.

19. Pipe Line Connecting Two Reservoirs

Certain interesting problems arise in connection with the use of a pipe line connecting two reservoirs with the water at different levels. Such problems may present themselves under three principal cases.

Case 1. **Pipe running full or under pressure.** See Fig. 18. Let R and S denote the two reservoirs with water levels maintained

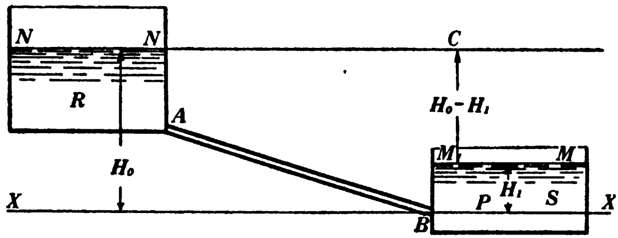

FIG. 18.—FLOW IN CONDUIT BETWEEN CONNECTING RESERVOIRS.

steadily at NN and MM, and let AB denote the connecting pipe with nozzle or valve at B. Let XX, at the level of the point of discharge B, denote the datum for measuring gravity head z, and let H_1=the excess head MB.

Then this case is covered by the general treatment of Sec. 17. Omitting any influence due to difference of atmospheric pressure between the locations of the two reservoirs, we have for Δy in equation (45), the value $-H_1$.

If in addition we may take $f=1$, we have

$$u=\sqrt{\frac{H_0-H_1}{\frac{1}{2g}+\Sigma\left(\frac{m^2L}{C^2r}\right)}} \quad \ldots\ldots\ldots\ldots\ldots(61)$$

or for a single uniform diameter of pipe

$$u=\sqrt{\frac{H_0-H_1}{\frac{1}{2g}+\frac{L}{C^2r}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(62)$$

Comparing these equations with (47) and (48) it appears, as we might expect, that in this case the velocity of discharge is the same as that for discharge into the air with a total head$=MC$, the difference in level between the two reservoirs.

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Again the head just at the point of issue is

$$H_B = H_1 + \frac{u^2}{2g}.$$

At some point P in the reservoir, on the same level as B but where the velocity has become negligible, we shall have

$$H_P = H_1.$$

This implies during the operation of bringing the stream to rest between B and P, the loss of head $u^2/2g$ as a direct result of the transfer of the energy of translation into the energy of turbulence, and its ultimate dissipation as heat.

Case 2. **Pipe running partly full or with free surface.** See Fig. 17. In this case the formulæ of Sec. 18 apply directly and no further discussion of the case is required.

Case 3. **Pipe running partly full at upper end and full at lower end.** See Fig. 19. This case will arise when the lower reservoir is

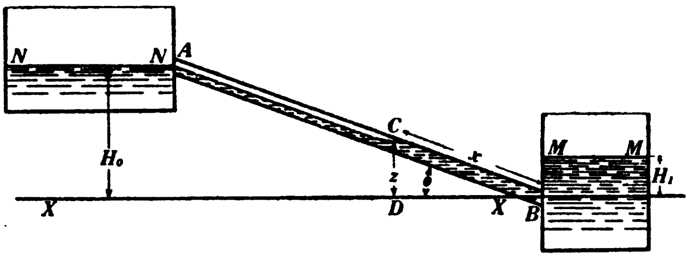

FIG. 19.—FLOW IN LINE PARTLY WITH FREE SURFACE AND PARTLY WITH FULL SECTION.

maintained with its level above the discharge end of the pipe while the upper reservoir is maintained with its level only partially covering the inlet end; or otherwise when the water admitted at the inlet end is less than the capacity of the pipe under full flow.

Under these conditions the upper end of the line will operate with free surface flow, while toward the lower end it will operate full and under pressure flow.

Let V=volume flow.

A_1=section of stream between A and C.

A_2=full section of pipe.

Assume a valve or nozzle fitted at B serving to partially close the outlet into the lower reservoir.

Let a=area of opening. This may have any value from 0 to the full area A_2.

Let v_1=velocity between A and $C = V/A_1$.

v=velocity in full pipe$=V/A_2$.

u=velocity through opening.

f=coefficient of discharge.

x=distance BC.

z=elevation CD.

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We may then consider the level C as equivalent to the level of an open surface reservoir with the pipe operating under the hydraulic conditions of Fig. 18. Referring to the general treatment of Sec. 17, and applying equation (44) to the present case, we have

$$\frac{u^2}{2g}=f\left(z-H_1-\frac{xv^2}{C^2r}\right) \qquad (63)$$

With V known, v becomes known. If then x is known or fixed, all the terms in (63) become known except u. The equation is then solved for u from which the area a is found. This gives, therefore, the area of opening which, with a given V will serve to maintain C at any desired point.

If, on the other hand, V and a are fixed then u becomes known and also v. If then the pipe BC is straight we may put $z=x \sin \theta$ and solve (63) for x finding

$$x=\frac{\frac{u^2}{2gf}+H_1}{\sin \theta-\frac{v^2}{C^2r}} \qquad (64)$$

This gives, therefore, the location of C for a given V with a fixed value of a. If the line BC comprises sections of varying diameters, then equation (63) will take the form

$$\frac{u^2}{2g}=f\left(z-H_1-\Sigma\left(\frac{Lv^2}{C^2r}\right)-\frac{xv^2}{C^2r}\right) \qquad (65)$$

where the term $\Sigma(Lv^2/C^2r)$ includes all the complete sections between B and C and x is measured from the junction next below C. The location of the section containing C must be made by inspection or trial. In each of these sections, with V fixed the value of v is known. Then with a fixed and u known, all terms in (65) become known except x and z. If the length containing C is straight we may put

$$z=z_1+x \sin \theta.$$

Where z_1 is the elevation of the junction point next below C and θ is the angle of slope of the length containing C. The equation is then solved for x.

If the length containing the point C is curved instead of straight, then z must be expressed in terms of x according to the geometry of the line, or otherwise the equation is readily solved by trial and error. A point C is assumed and the values of z and x are found and tested by substitution in equation (65).

If, on the other hand, the point C is fixed, then the right-hand side of (65) is completely known and it only becomes necessary to solve for u and then to find a. The values of v_1 and A_1 in the free surface section, AC (Fig. 18) are readily found by equation (37) or Table XXII of Sec. 16.

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20. Power Delivered at Discharge End of Line

Let V=volume rate of flow (f3s).
w=density (pf3).
P=power (fps).
H_0=static head (above atmosphere).
$h=\frac{64LV^2}{\pi^2C^2D^5}$ (Sec. 6).
H=net head$=H_0-h$.
$P=(H_0-h)Vw$.

Then assuming discharge into the air and neglecting any difference in atmospheric pressure between the entrance and discharge ends of the line, we have

$$H=H_0-\frac{64LV^2}{\pi^2C^2D^5}$$

$$\text{and } P=HVw=\left(H_0V-\frac{64LV^3}{\pi^2C^2D^5}\right)w \ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

In equation (66) it is seen that with given V and other conditions, the value of P is readily found. Conversely, however, if P is given and V required, the equation becomes a cubic in V and will usually be most conveniently solved by approximate or trial and error methods.

To determine the conditions for a maximum value of P we differentiate with reference to V, place the result$=O$ and solve for H_0 thus finding:

$$H_0=\frac{192LV^2}{\pi C^2D^5}=3h$$

$$\text{or}\quad h=H_0/3$$

Interpreted in words this means that the maximum power will be delivered at the discharge end of the pipe line when the lost head h amounts to one-third the total head H_0.

For this set of conditions we readily derive the following expressions for the values of the velocity v, the volume flow V and the maximum power P_m.

$$v=\frac{C}{2}\sqrt{\frac{DH_0}{3L}} \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

$$V=\frac{\pi CH_0^{\frac{1}{2}}D^{\frac{5}{2}}}{8\sqrt{3L}} \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

$$P_m=\frac{\pi wCH_0^{\frac{3}{2}}D^{\frac{5}{2}}}{12\sqrt{3L}} \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

These formulæ presuppose, of course, that the water called for in (68) is available. With a fixed flow of water the maximum power will, of course, be developed with the minimum lost head h.

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21. Piping Systems

Several important and interesting problems arise in connection with the flow of water through a ramifying system, as in Fig. 20. In such a system each run of pipe from one junction to the next forms a unit or element. These elements are shown by the numbers in the diagram. In any such system it is readily shown that the number of elements is equal to the number of outlets plus the number of junctions. Also that the number of junctions is one less than the number of outlets. Hence if there are n outlets there will be $(2n-1)$ elements in the system. The characteristics required to determine each element are the diameter and velocity. If neither

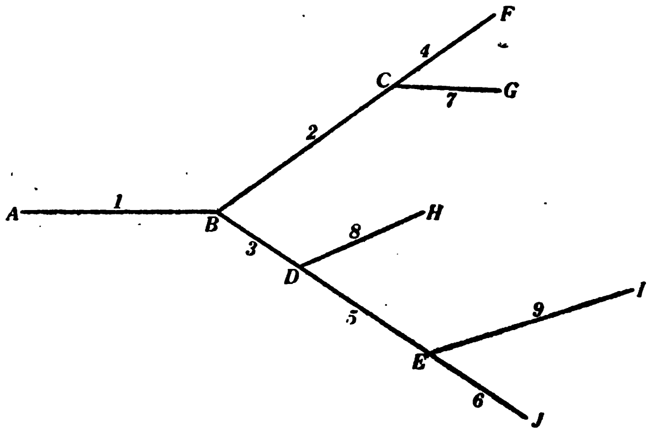

Fig. 20.—Branching System.

of these is known then the problem will comprise $2\ (2n-1)$ variables and we must have an equal number of equations in order to find a unique solution. If either the diameter or velocity is known, the number of variables is reduced to $(2n-1)$.

In connection with such a system of piping three typical problems may arise.

1. Given a definite system with fixed dimensions of pipes, with given valve openings at the discharge ends and with given coefficients of discharge, all under a fixed head H. The valve openings may be anything from 0 to full opening. Required the total discharge and the discharge at each opening.

2. Given a series of locations relative to the reservoir or source of head (pump for example) with a stated discharge at each point. Let the head at the supply point (pump or reservoir) be fixed and constant. Required a system of piping suited to accomplish these ends.

3. Same as (2) but without a fixed or given head at the source.

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Required a system of piping together with a head suited to accomplish the stated deliveries.

We shall only refer briefly to the principal features of these problems.

Problem 1.—In this problem everything is given regarding the various elements of the system except the values of v. We shall therefore, with n outlets, have $(2n-1)$ unknown velocities and shall require $(2n-1)$ equation for their determination. We obtain these as follows:

(a) One equation for each outlet, or n in all, by tracing the loss in head from source to discharge and equating the head at the point of discharge to the original head diminished by the loss.

(b) One equation for each junction, or $(n-1)$ in all, by equating the flow in the pipe leading to the junction to the sum of the flows in those leading from it.

This will furnish a system of $(2n-1)$ simultaneous equations, the solution of which will give the values of v, one for each element of the system. Naturally the actual solution becomes rapidly burdensome with increase in the number of elements, but the principle remains the same and for the problem as stated there seems no way of evading the details of the solution in this manner.

The development of such a system of equations will be sufficiently illustrated by writing those for a simple Y branch with unequal legs (see Fig. 21). We shall assume that each element is formed of pipe of uniform diameter. If this is not so the principles discussed

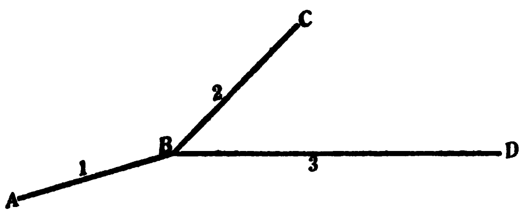

FIG. 21.—SINGLE Y BRANCH.

in Sec. 8 readily furnish a means for making provision for this feature. Let H in general denote the difference in external head between the supply point A and any other point in the line. If the flow is from an elevated reservoir at A, H=the difference in level between A and the given point. If the head at A is supplied by a pump, H is the difference between the total head furnished by the pump and the elevation at the given point, both reckoned from the same datum, the pump for example.

Let m_2, m_3 denote respectively, as in Sec. 17, the ratio between the discharge orifice area and the area of the pipe, for branches 2 and 3, and f_2, f_3 the corresponding coefficients of discharge. Then

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using in general the notation of Sec. 17 we have, for the values of the head at the discharge points C and D:

$$\frac{v_2^2}{2gm_2^2}=f_2\left[H_2-\frac{L_1v_1^2}{C_1^2r_1}-\frac{L_2v_2^2}{C_2^2r_2}\right]\dots\dots\dots\dots(70)$$

$$\frac{v_3^2}{2gm_3^2}=f_3\left[H_3-\frac{L_1v_1^2}{C_1^2r_1}-\frac{L_3v_3^2}{C_3^2r_3}\right]\dots\dots\dots\dots(71)$$

$$\text{Also,}\quad v_1A_1=v_2A_2+v_3A_3\ \dots\dots\dots\dots\dots\dots(72)$$

These three equations will serve to determine v_1, v_2, v_3, and hence the flow condition becomes completely known. From (70) and (71) we derive

$$H_2-v_2^2\left[\frac{1}{2gf_2m_2^2}+\frac{L_2}{C_2^2r_2}\right]=H_3-v_3^2\left[\frac{1}{2gf_3m_3^2}+\frac{L_3}{C_3^2r_3}\right]..(73)$$

This gives a relation between v_2 and v_3, and shows that such relation is entirely determined by the conditions affecting those two lines. Also if either v_2 or v_3 is known or given, the other may be determined from this relation.

The same relation may be generalized to apply to any of the branches of a Y connection, such as 2 and 3 of Fig. 20. To this end we may rewrite (73) as follows:

$$\left(H_2-\frac{p_c}{w}\right)-v_2^2\left[\frac{1}{2g}+\frac{L_2}{C_2^2r_2}\right]=\left(H_3-\frac{p_d}{w}\right)-v_3^2\left[\frac{1}{2g}+\frac{L_3}{C_3^2r_3}\right]\ (74)$$

Where H_2 and H_3=differences in external head along ABC and ABD respectively, p_c and p_d=pressures at C and D and f and m are both taken=1. This shows that if we know the pressure heads at the ends of the two legs of any Y branch with the other controlling conditions and the velocity in one of them, we may find that in the other.

Problem 2.—In this case we know the discharges but neither the velocities nor the sizes of the pipes. There will be, therefore, for each pipe two unknowns, a size and a velocity, and hence in all $2(2n-1)$ unknowns. We may assume that the discharge openings are fixed and the coefficients of discharge known or assumed. From the known discharges we know the product vA for each terminal element of the system. From these and equations of the form $v_1A_1=v_2A_2+v_3A_3$ we may work back and find the numerical value of vA for each element of the system. This will give us $(2n-1)$ equations.

We can then write n equations of the form (70), but shall still lack $(n-1)$, or as many equations as there are junctions. It results that the system is indeterminate. There is no unique solution and the conditions may be fulfilled in an indefinite variety of ways. A physical analysis of the conditions in a single Y branch, for example, will readily lead to the same conclusion.

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It follows that in order to make the problem definite we must fix $(n-1)$ of the variables. These must furthermore be fixed in a manner consistent with the other equations. We have already seen from (74) that the velocities in the two legs of any one Y branch are not independent, and that their relation is determined by the conditions in these legs. We are not, therefore, free to fix arbitrarily either velocity or area in both legs of any one Y branch. We may, however, fix one, and as there are $(n-1)$ such Y branches this will represent the number of variables to be arbitrarily assumed.

We therefore proceed by fixing arbitrarily the area or velocity in one leg of each Y branch (for example 7, Fig. 20). The velocity or area in the same leg will follow conversely from the relation $vA=V$, while the velocity in the other leg 4 will follow from (73) or (74). In using (74) we shall need the pressure heads at points such as B and C. These are readily found by working back from the discharge ends. Thus knowing the heads at the discharge points F, G, we readily work back, from the determined conditions in 4 and 7, to the head required at C and hence to the pressure head at C. Having found the velocity in 4 we find the area from $vA=V$, and thus by working back from the discharge points all conditions become known up to the point B. Then from the head necessary at B and the known head at A the conditions in element 1 are readily found, and thus the entire system.

As a variant on the above procedure for this problem we may, for a given leg such as 7, assume the hydraulic gradient i instead of either the area or velocity. The diameter then follows from (35) and thence the velocity. The procedure otherwise is in general the same.

It may easily result that, in working back in this way, an impossible set of conditions will develop. Thus we find that the head required at B is greater than that available at A. This will be due to the arbitrary assumption made regarding size or velocity or hydraulic gradient. In such case the assumption must be varied until the system as determined becomes feasible and consistent in its various parts.

In order to avoid this, the procedure may be advantageously varied as follows:

In Fig. 22 let ABC and ABD represent the profiles along the lines of a single Y branch. Let N be the pressure elevation at A, that is AN=pressure head supplied by pump at initial end of AB. Then from the fixed discharges, discharge openings and coefficients of discharge at D and C, the pressure heads at D and C just back of the valve are readily found (see (50)). Let these be set up as CE and DF. Then EG and FH are available for friction heads along the lines ABC and ABD respectively. We may then draw any line, straight or broken, from E to N and take such line as giving the hydraulic gradient along the pipe line ABC. The value or values of i thus determined may then be used, as before, in (35)

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to find the diameter, and thence otherwise as before. The same general method is readily applied to a more complex system. The procedure is, of course, entirely similar in case the point A of Fig. 22 is fed from an elevated reservoir. In such case, however, the point N (Fig. 22) should, strictly speaking, represent the head just inside the pipe 1 at A rather than the level of water in the reservoir. The difference is usually negligible, or otherwise we may assume the conditions in element 1, thus fixing the elevation of N and the hydraulic gradient as far as the first junction B, and then determine the remainder as above. Other variations in detail will readily occur to the engineer having to deal with such problems. The main purpose of the arbitrary assumptions which are made must be to produce, if necessary, by trial and error, a final system, harmonious and consistent in its various parts.

Problem 3.—In this case we do not have a fixed head at the initial point A. We are therefore free to either fix arbitrarily or tentatively the head at A and proceed as in the first method dis-

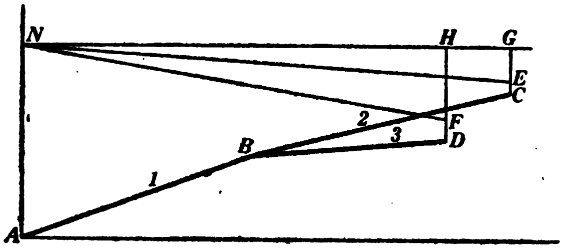

FIG. 22.—BRANCHING SYSTEM.

cussed under Problem 2, or otherwise, and preferably to fix, according to judgment, the gradients along some one line from the discharge end back to A and thus determine the head at A and the various sizes along the line. The head thus determined must then be accepted as applying to the other lines, or if not acceptable for them, another trial must be made. Thus by trial and adjustment an acceptable system may ultimately be determined.

Certain further interesting problems in connection with a ramifying system arise when a reservoir head and a pumping head are both applied at different points in the system, as in Fig. 23. This is frequently met with in connection with municipal distributing systems.

Problem 4.—Assume the system of Fig. 23 to be entirely given with all elevations of outlets, sizes of pipes, valve openings and coefficients of discharge. Assume further at any given time a given level N in the reservoir. Assume a head AM maintained by the pump. Required the flow throughout the system and the movement of the surface N.

This problem falls directly under the treatment of Problem 1.

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We have in the arrangement of the diagram seven elements in the piping, and we shall therefore require seven equations. There are three outlets, *E*, *F*, *G*, one outlet *K* under the head represented by the level *N*, and three junctions. These will furnish the needed seven equations, and we may then find the flow in all elements, including the line 5 leading to the reservoir. The flow here will, however, raise the level *N* in accordance with the dimensions of the reservoir, and as the level rises the flow in 5 will become less and less. It follows that a complete study of the case would require a series of solutions of the equations for a series of varying elevations of *N* taken at suitable intervals, and corresponding to intervals of time which will result directly from the surface area of the reservoir and the mean rate of flow along 5 for any one interval. In this way a time history of the rise of level *N* could be developed. Ultimately a point would be reached where the flow in 5 will become zero, the level *N* will remain stationary and the flow in the remainder of the system will become steady at the values determined for this condition. This terminal elevation of *N* may be directly found by assuming immediately the condition of zero velocity in 5, and hence the practical elimination of this branch with the reservoir from the problem. We then proceed with the remainder of the system exactly as in Problem 1, thus finding the flow along all elements of the system. With these known the pressure head at any given point is readily found. We therefore find the pressure head at the point *C*, and this must give therefore the elevation of the surface *N* above *C*.

In case the head *AM* maintained at *A* is, at the start, below the level required to maintain *N* stationary, then the flow in 5 will be reversed, the reservoir will become a feeder, and the level *N* will fall until it reaches a point where the conditions will be steady with *N* stationary. In any case with a fixed level *M* the level of *N* for steady conditions is readily found by assuming 5 with the reservoir removed from the system and then proceeding as above noted. Conversely, if *N* is fixed, and it is desired to know the level at which *M* must be maintained to keep *N* stationary, we should have a system with 5 pipe elements (assuming element 5 removed) and with an unknown head *AM*. This will, in the given case, make six unknowns. We have three outlets, two junctions and a given pressure head to be realized at the point *C*. These will furnish the necessary six equations.

Problem 5.—In this case we assume given the discharge at the various outlets, a given head *AM* and a level *N* to be maintained stationary. Required sizes suited to realize these ends.

The level *N* being stationary, the pipe 5 is inoperative, and we may therefore assume it removed so far as matters of velocity and flow are concerned. We shall then have as unknowns, five velocities and five areas. We shall be able to write five equations of the form $vA=V$, two for the junctions and one for the given pressure head at

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the point C. We must, therefore, arbitrarily fix two of the unknowns, as, for example, the velocities in 6 and 7, and then proceed as in Problem 2.

Or otherwise, and preferably, we may set up at the outlet, just back of the valve, vertical lines representing the pressure heads required by the discharges (see EP, FR and GQ, Fig. 23). We may then draw from P, Q and R hydraulic gradient lines back to M, which must, however, fulfil the following conditions:

1. The gradients from P and Q must have a common point on the vertical through D, as at U.
2. The gradient from U to M must pass through S.
3. The gradient from R to M must meet that from U to M on the vertical through B, as at T, and the two must be coincident from T to M.

The gradients thus determined will then serve to determine the sizes of the elements of the system, as in Problem 2.

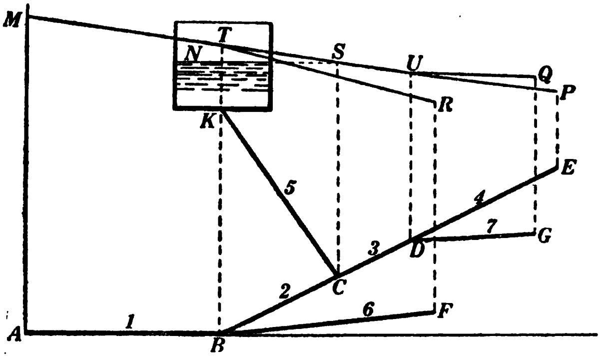

FIG. 23.—BRANCHING SYSTEM WITH DOUBLE SUPPLY.

In case the pump shuts down, the system becomes reduced to that of a single reservoir source, and may be examined as in Problems 1, 2 and 3.

Various other combinations may present themselves in connection with problems involving ramifying systems, especially such as that of Fig. 23. In all cases, however, they may be investigated by the general methods outlined in the present section.

In the case of reversed flow through such a ramifying system, a number of collectors leading ultimately to a single main, the same general principles may be applied. This, however, is a case not likely to arise.

In the case of an interconnected network with inflow at one point and discharge at another, the general method of Problem 1 will still apply. It will be found that the number of junctions plus the number of possible paths of flow will equal the number of elements in the system, and thus a complete set of simultaneous equations

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may be found which will serve to determine the velocity in each element of the system. In the case of Problems 2 and 3, the flow must be assumed or known in two out of each of the three pipes forming each junction. The flow in the third pipe will then become known from equations of the form

$$v_1A_1 = v_2A_2 + v_3A_3.$$

This will then serve to determine the entire system of flow. Then by the arbitrary fixing of velocity, size or hydraulic gradient, in the same general manner as in Problems 2 and 3 above, the remaining features of the system may be determined.

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CHAPTER II

THE PROBLEM OF THE SURGE CHAMBER

22. General Statement of Problem and Derivation of Equations

From (m) in Sec. 11, page 31, it appears that an unbalanced pressure head p/w acting over the cross sectional area of a pipe at one end will act as an accelerating force on the water in the pipe and will produce an acceleration measured by the equation:

$$\frac{L}{g}\frac{dv}{dt}=\frac{p}{w} \qquad \text{....................(1)}$$

Now suppose that we have given the arrangement of Fig. 24 comprising the following items:

1. A supply reservoir R in which the water may be supposed to remain at a constant level.
2. A main conduit AB.
3. A surge chamber or stand-pipe BG.
4. A penstock line BL terminating in a control nozzle.

Assume steady flow conditions with velocity in main conduit $AB=v_1$, and level of water in surge chamber at C. This means that of the total head BG at B, the velocity head plus the friction head has absorbed an amount measured by GC, leaving the balance as pressure head measured by BC. Now the existence of steady conditions implies a balanced system of forces on the water in AB, composed as follows:

1. Gravity with component acting A to B.
2. Pressure at A distributed over cross section of pipe and acting A to B.
3. Pressure at B distributed over cross section of pipe and acting B to A.
4. Frictional resistance along pipe and acting B to A.

Next suppose that with no change in the velocity v_1 or in the conditions generally in AB, there should develop suddenly a change in BG as a result of which the water level should drop to D. It is obvious that the force equilibrium of the water in AB would be correspondingly disturbed. The pressure head at B is now no longer BC but BD, an amount less by the head CD. The result will therefore be an unbalanced pressure head acting on the water

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in AB in the direction A to B, and measured by this distance CD. This accelerating head will immediately operate to produce an acceleration in the velocity of flow along AB measured as indicated in (1).

Similarly if with steady conditions and a velocity v_1 in AB, the level of the water in BG should suddenly rise to D_1, there would be a corresponding disturbance in force equilibrium and an unbalanced pressure head measured by CD_1 would act at B over the cross section of the line in the direction B to A, producing a retardation in the velocity of flow along AB measured likewise as in (1).

From this analysis the truth of the following general statement becomes self-evident.

At any time during the period of velocity change let the velocity

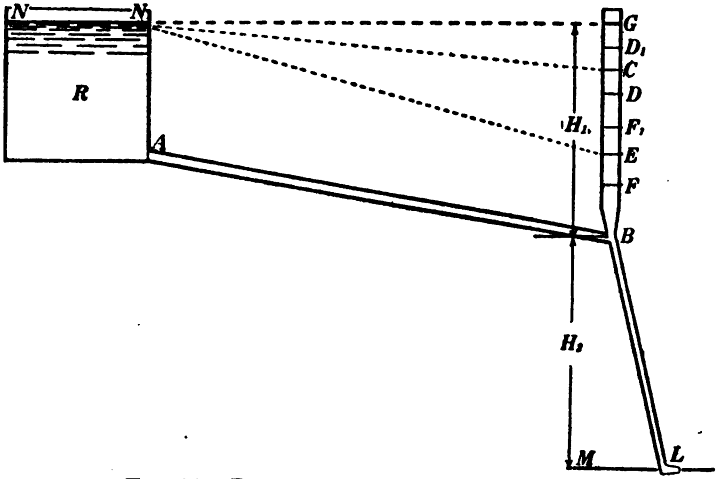

FIG. 24.—PROBLEM OF THE SURGE CHAMBER.

along $AB=v$. Corresponding to this condition the velocity and friction heads combined will have a certain value, say GC. Then if the pressure head$=BC$ (that is, if the water level is at C) the conditions for force equilibrium will momentarily obtain, and there is no accelerating or retarding head in operation on the water in AB. If, on the contrary, the pressure head differs from BC (that is, if the water level is not at C) the conditions for force equilibrium will not obtain, and there will be in operation an accelerating or a retarding head measured by the distance of the actual water level below or above the level C.

We may express this somewhat more briefly by saying that with any velocity of flow in AB, if the water level in BG is where it belongs for steady conditions, then there is no accelerating or retarding head in operation on AB, but if the water level is below

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or above the location for steady flow then there is in operation a corresponding accelerating or retarding head, measured by such difference in level. Or still otherwise; the accelerating or retarding head acting on the water in AB is measured by the difference between the level of the water as it actually is and where it belongs for steady conditions with the given instantaneous value of the velocity.

Now suppose a power house at L, the lower end of the penstock line, with fluctuating demand for power according to the accidents of the daily load curve. Suppose for the moment steady conditions with a velocity v_1 in AB and water level at C. Let there arise a sudden demand for excess power such as would require a rate of volume flow greater than that which the main conduit is furnishing. The nozzles at the power house, under governor control, will open up accordingly, the response from the adjacent penstock line BL will be prompt and we shall, after a few seconds time, have a flow through BL carrying the increased volume of water. The actual velocity in AB will, however, still remain substantially unchanged and the surge chamber BG must therefore supply the difference. As a result the level of water in the surge chamber will fall below the level for steady conditions, and as it falls will develop, as we have just seen, an accelerating head which will start in to raise the velocity of flow in AB from the initial up toward a higher final value.

In a similar manner, it is clear that if load is suddenly rejected at the power house, corresponding to a decrease in the rate of volume flow required, then the surge chamber will receive, for the time being, the excess flow coming along the line AB, and as a result the actual level in BG will rise above the level for steady flow and a retarding head will be developed; and as a result of which the velocity of flow in AB will be continuously and gradually reduced from the initial toward a lower final value.

It thus appears that any sudden change in power demand will react on the surge chamber in such manner as to disturb the level of the water from its location for steady flow conditions, and thus to develop an accelerating or a retarding head. Following these conditions in some further detail it is seen that as the velocity v gradually rises, the velocity and friction head combination will increase and the level corresponding to steady conditions will begin to fall. We shall have as time goes on, therefore, a dropping actual level of the water and a dropping level for steady flow for the momentary value of the velocity v, the difference between these two levels measuring the accelerating head in operation. Thus in Fig. 25 let OX denote a time axis, A the level of the water at the start of the change with velocity v_1 and D the level for the final steady flow velocity v_2. Then the time history of the drop in the actual level might be some curve such as $AEFD$ while that for the corresponding level for steady conditions with the momentary value of the

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velocity might be some curve such as *ABCD*. The differences between these, measured by the intercepts *BE*, *CF*, etc., give a time history of the accelerating head, as a result of the operation of which the velocity of flow will be finally raised from v_1 to v_2.

Similarly for rejected load; the rise in the water level plotted on time might give some curve such as *AEFD* (Fig. 26), while the

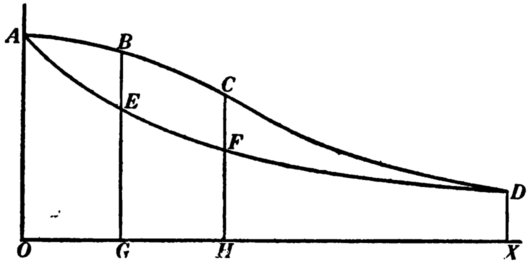

FIG. 25.—ACCELERATION HEAD.

rise in the level for steady conditions with the momentary value of the velocity might give some curve such as *ABCD*. The intercepts *BE*, *CF*, etc., furnish then a measure of the retarding head as a result of which the velocity will be finally reduced from the higher to the lower final steady flow value.

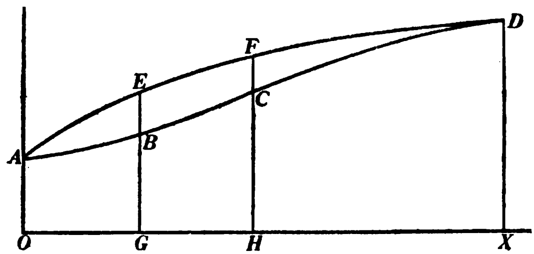

FIG. 26.—RETARDATION HEAD.

The general problem of the surge chamber operating in connection with a hydraulic power unit under governor control involves therefore the following conditions:

1. In the initial stage the power unit is assumed to be developing the power W_1 under steady flow conditions in the penstock and pipe line. While in this condition there is assumed to arise, let us say, a sudden demand for more power.

2. In the final stage and after this demand has been satisfied, the power unit is again assumed to be developing the increased power W_2 under steady flow conditions in the penstock and pipe line.

3. During the transition period, from (1) to (2) the governor is

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assumed to operate continuously to deliver to the wheel from the penstock, the flow of water required to develop the power W_2.* The rate of volume flow to the power unit will, during this period, not in general be the same as the rate of volume flow along the pipe line, and the surge chamber must therefore either supply or absorb the difference. This will occasion a change of water level in the surge chamber with a resulting accelerating or retarding head as we have seen. The change of level implies, however, a change in the pressure head at the surge chamber and hence in the net head ultimately available at the power house. But the rate of volume flow required to develop the power W_2 will depend on the net head available at the power house, and will approximately vary inversely as such head. Hence with demanded load, as the head continuously falls, the quantity of water required will correspondingly increase, while with rejected load, as the head continuously rises, the quantity of water required will correspondingly decrease.

In the detailed discussion of this problem, consideration must be given to a number of different velocities, some actual and some virtual. These may be characterized and denoted as follows:

v_1 This is the actual conduit velocity under initial steady flow conditions. It is the actual velocity with the initial head and the original load W_1.

u_1 This is a virtual velocity and is really the measure of a rate of volume flow. It is the velocity along the main conduit which would supply the quantity of water which, under the initial head, would serve to develop the final power W_2.

v_2 This is the actual conduit velocity under final steady flow conditions. It is the actual velocity with the final head and final load W_2.

u This is a virtual velocity and is really the measure of a rate of volume flow. It is the velocity along the conduit which would supply the quantity of water actually required at the wheels to develop the power W_2 under the head which prevails at any instant during the transition period. Under final steady flow conditions u becomes the actual velocity v_2.

u_m This is the maximum value of u.

w This is the actual velocity along the penstock line AB (Fig, 24), corresponding to the delivery of the quantity of water required by the wheels.

Let A_1=cross section area of main conduit (f2).
A_2=cross section area of penstock (f2).
H_1=head GB, Fig. 24 (f).
H_2=head BM, Fig. 24 (f).
$H=H_1+H_2$.

* In the actual case this condition cannot be fully realized. The divergence, however, is not of significant importance in the treatment of the problem.

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F=area of surge chamber at water level (f2).
L=length of conduit (f).
y=movement of water level from initial position (f).
z=height of actual water level above B (f).

Let b_1 denote for the main conduit line the factor L/C^2r (see Sec. 2) and b_2 the similar factor for the penstock line. We have then,

$b_1v_1^2+b_2w_1^2$=loss of head due to friction under initial steady flow conditions.

$b_1v_2^2+b_2w_2^2$=loss of head due to friction under final steady flow conditions.

$H-b_1v_1^2-b_2w_1^2$=net head under initial steady flow conditions.

$H-b_1v_2^2-b_2w_2^2$=net head under final steady flow conditions.

Referring now to Fig. 24, assume that at any instant during the transition period the actual level of the water in the chamber is at F while the level for steady flow with the existing velocity v is at E. Then the pressure head at B at this instant of time will be measured by $z=BF$ and the total head will be BF+velocity head $v^2/2g$. Hence we have $z+v^2/2g$=actual head at base of surge chamber under conditions prevailing during transition period.

Also $H_2-b_2w^2$=actual net head at power house available from H_2. Hence $z+v^2/2g+H_2-b_2w^2$=total actual head at power house under conditions prevailing during the transition period, with actual water level at F and surge or total movement of water level from C to F.

Again, assuming substantially uniform efficiency for the hydraulic unit over the range from W_1 to W_2, it results that the power developed will vary directly as the product of head by quantity per unit time or rate of volume flow. Hence we shall have the following expressions :

$v_1(H-b_1v_1^2-b_2w_1^2)$ represents load W_1 under initial head.
$u_1(H-b_1v_1^2-b_2w_1^2)$ represents load W_2 under initial head.
$v_2(H-b_1v_2^2-b_2w_2^2)$ represents load W_2 under final head.
$u(z+\frac{v^2}{2g}+H_2-b_2w^2)$ represents load W_2 under head prevailing during transition period.

We may then derive the following relations between these various velocities :

$$\frac{u}{w}=\frac{A_2}{A_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{u_1}{v_1}=\frac{W_2}{W_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$v_2=\frac{u_1(H-b_1v_1^2-b_2w_1^2)}{H-b_1v_2^2-b_2w_2^2} \quad \ldots\ldots\ldots (4)$$

$$u=\frac{u_1(H-b_1v_1^2-b_2w_1^2)}{z+\frac{v^2}{2g}+H_2-b_2w^2} \quad \ldots\ldots\ldots (5)$$

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Regarding these formulæ it may be noted that (4) is a cubic equation in v_2. This, however, is readily solved by trial and error. Also in (5), which is intended to give u for known values of z and v, the denominator contains w, which is also unknown. We may therefore proceed by assuming a value of w and then find u from equation (5) and check the result from (2). A trial and error procedure will thus readily serve to determine u for any stated condition of v and z. Or otherwise, by substituting for w in (5) from (2) the former may be reduced to a cubic equation in u and then solved by trial and error as with (4).

We may now proceed to develop the equations for the flow of the water during the transition period.

First for any velocity v in the main conduit we have:

$$\frac{v^2}{2g}=\text{velocity head.}$$

$$\frac{Lv^2}{C^2r}=\text{friction head (Sec. 2).}$$

As above let E denote the level for steady conditions with the transition velocity v, and F the actual level. Then

$$z=BF.$$

$$GE=\frac{v^2}{2g}+\frac{Lv^2}{C^2r}$$

EF=accelerating head acting at the given instant.

But $EF=GB-(GE+BF)$.

Whence from (1) we have

$$\frac{L}{g}\frac{dv}{dt}=H_1-\left(\frac{v^2}{2g}+\frac{Lv^2}{C^2r}\right)-z.$$

Also $(u-v)$=deficit of velocity in AB. That is, the actual conduit velocity is v, while the velocity corresponding to the water required is u.

Hence $A(u-v)$=deficit in rate of volume flow which must be made up by flow from the surge chamber. But

$$-F\frac{dz}{dt}=\text{rate of flow from surge chamber.}$$

Then $-F\dfrac{dz}{dt}=A(u-v)$.

Put $\left(\dfrac{1}{2g}+\dfrac{L}{C^2r}\right)=c$.

Then we have finally

$$\frac{L}{g}\frac{dv}{dt}=H_1-(cv^2+z)\;\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$-\frac{F}{A}\frac{dz}{dt}=(u-v).\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

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For certain modes of study of this problem, equations (6) and (7) may be conveniently put into the form

$$\frac{L}{g}\frac{dv}{dt}=y-c(v^2-v_1^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\frac{F}{A}\frac{dy}{dt}=(u-v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

These equations are readily seen to be the equivalent of (6) and (7). As written (8) and (9) apply to the case of demanded load and a falling level. The changes necessary for the case of rejected load and a rising level are made by changing the sign of dv/dt, by interchanging the signs of v^2 and v_1^2 in (8) and of u and v in (9).

Also from Fig. 24 the following relations are readily seen:

$$\left.\begin{array}{l} z+y=H_1-cv_1^2 \text{ (demanded load)} \\ z-y=H_1-cv_1^2 \text{ (rejected load)} \end{array}\right\} \ldots\ldots\ldots\ldots (10)$$

Equations (6), (7) or (8), (9) with (2) and (5) serve to define mathematically the conditions of the movement of the water during the transition period.

For demanded load with the actual level below the level for steady conditions, as at F, Fig. 24, (cv^2+z) will be less than H_1 and dv/dt will be positive in sign representing an acceleration of the velocity, as is required to carry the value from v_1 to the higher v_2. On the other hand, with rejected load and the actual level above the level for steady conditions, as at F_1, we shall have $GE+BF_1$ greater than H_1 and dv/dt will be negative in sign, representing a retardation of velocity, as is required to carry the value from v_1 to the lower v_2.

Likewise in (7) with demanded load $u > v$ and dz/dt is negative, implying a falling level, while with rejected load $u < v$ and dz/dt is positive, implying a rising level.

In the preceding discussion of the operation of a surge chamber and in the curves of Figs. 25 and 26 we have for simplicity assumed that the various quantities involved (water level, velocity and accelerating head) all move from the first or initial values to the final values continuously in one direction only, and hence without oscillation about such final values. Such movement is usually styled "dead beat," and is only sensibly realized with a relatively large size of chamber. In the usual case all three quantities, as above, approach and ultimately reach the final values or condition as the result of an oscillatory variation, of rapidly diminishing amplitude, about the ultimate values. Thus in the case of demanded load (see Fig. 27) the water level will rapidly drop (AB) and the level for steady conditions with the momentary value of the velocity less rapidly, ($A1$), thus indicating an accelerating head in operation during this period. In the general case the water level will reach the steady motion level for final velocity v_2 (at B) before v has reached this value. In other words during the drop

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AB, v is continuously less than u and dz/dt is negative, implying a falling level. At B this condition still continues and the level must drop still further (BC) until finally $v = u_m$ (level at C). At this point v has finally caught up with u (H) and dz/dt becomes 0 This marks the lowest level reached by the water, a level also below that for steady flow (3) with the velocity u_m. There is therefore still operative an accelerating head $3C$, which will carry the value of v beyond u_m and still further beyond v_2. This excess

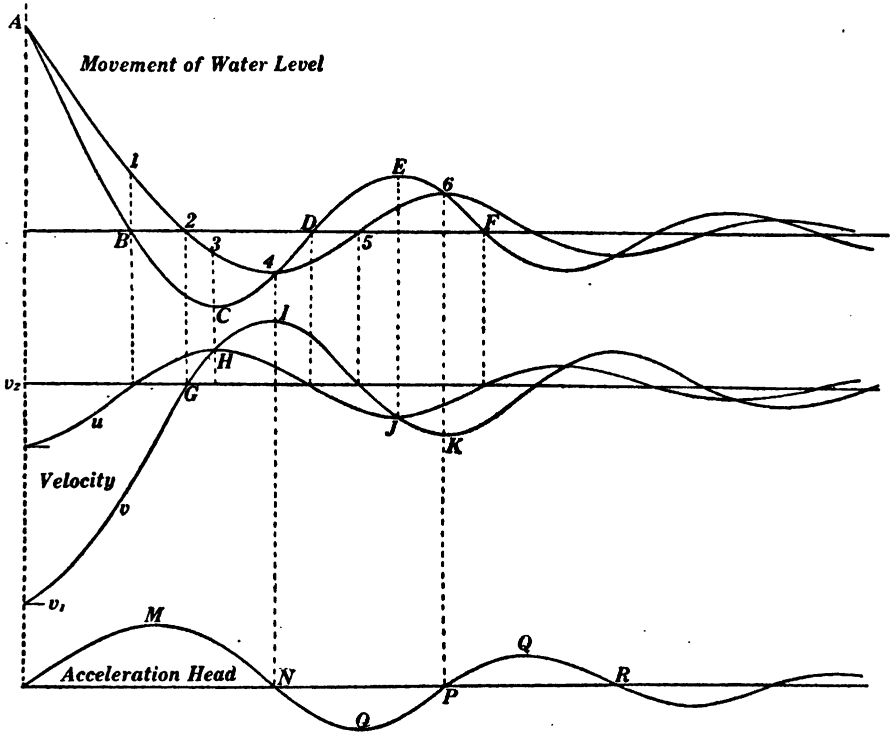

FIG. 27.—TIME HISTORY OF MOVEMENT OF WATER LEVEL, VELOCITY AND ACCELERATION HEAD

of v over u will, however, bring more water than is required by the penstock and the result will be a rising level (CD). With the rising value of v will also come a further falling level for steady conditions, (3, 4), and the rising actual level will thus soon meet and pass that for steady conditions (at 4), thus changing the acceleration head from plus to minus.

In the meantime under the influence of the accelerating head during the period $ABC4$ the velocity has been constantly rising, reaching the value v_2 at G, the value u_m at H, and the maximum

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value v_m at I, where the acceleration changes sign. The sign of dz/dt, however, still remains the same, and the water level continues to rise ($4DE$), until finally, with water level at E the velocity again becomes equal to u at a value less than v_2 (J). In the meantime the velocity decreases (IJ), while the level for steady motion rises 4, 5, 6. In this manner these various quantities see-saw back and forth, no two reaching the final steady motion values together until by a series of oscillations of rapidly diminishing amplitude, they all finally reach sensibly their ultimate values for steady motion with velocity v_2. These various oscillations back and forth may be traced readily by the curves of Fig. 27, studied in connection with the above brief analysis of the first stages of the movement. It results that in the general case the various quantities pass through a periodic or oscillatory movement, strongly dampened, or with a rapidly diminishing amplitude, until after a number of swings, greater or less according to circumstances, the accelerating head becomes sensibly zero and the water level and velocity have sensibly reached their ultimate conditions.

The extent of the first swing beyond final values (as at C, Fig. 27) will depend on the size of the chamber relative to the other characteristics of the case. The smaller the chamber the greater will be the amplitude. With a chamber of sufficient size the swing beyond final values becomes negligible, and the movement is then sensibly dead beat.

23. Treatment of Surge Chamber Equations

Differential equations of the form presented in (6), (7) or (8), (9), and representing a dampened oscillation in which the retarding force varies as the square of the velocity, do not seem to permit of direct solution by any known mathematical means. Under these conditions the following courses are open:

1. The treatment of restricted or special cases which may admit of mathematical solution. This represents a partial solution of the equations.
2. The treatment by approximate methods involving some departure from the exact relations implied by the equations, and thus making possible a solution by direct mathematical means.
3. The treatment of the equations as they stand or in simplified form by methods of approximate numerical integration. This method involves no departure in principle from the relations involved in the equations, and the only errors are those of numerical detail, and these may be made as small as desired. The direct treatment of the equations in this manner involves a trial and error process, and is necessarily tedious in numerical detail.

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4. The treatment of the equations as they stand or in simplified form by a reversed or indirect process of numerical integration which avoids the trial and error feature of (3).
5. The development, through an extension of the law of kinematic similitude, of a series of relation coefficients serving to connect the actual case with a model of workable size. Observations are then made on the model, and these are transformed, through the proper coefficients, into corresponding values for the actual case.

Space will permit of only a brief survey of these various methods of treatment.

(*a*) **Solution of Special Cases.**

Complete shut down with cylindrical surge chamber. Prof. I. P. Church* gives for the case of a complete and sudden shut down the following formulæ :

$$cv^2 = cv_1^2 - y + \frac{1}{P}(1 - e^{-Py}) \text{ .... } (11)$$

$$y_m = cv_1^2 + \frac{1}{P}(1 - e^{-Py_m}) \text{ .... } (12)$$

Where y=movement of water surface measured from initial level (f).
y_m=maximum value of y or maximum surge (f).
v=velocity corresponding to any assigned value of y (fs).
v_1=initial velocity (fs).
c=as in Sec. 22.
e=Naperian base.
$P=\dfrac{2Fcg}{AL}$ with the notation of Sec. 22.

The value of y_m is in a form which can only be solved by trial and error, but the successive approximations come very quickly and a sufficiently accurate value may soon be found. The author gives an illustrative case in which $v_1=7{\cdot}5$(fs), $cv_1^2=11$(f), $c={\cdot}1955$, $F/A=2{\cdot}25$, $L=140$(f). Whence $P={\cdot}2024.0$ as a first approximation y_m was taken=20. Then finding the value of e^{-Py_m} we have 1/57, and finding thence the value of the right-hand side of (12) we have 15·85 as the second approximation. Similarly assuming $y_m=16$ on the right-hand side we find $y_m=15{\cdot}7$, and again putting $y_m=15{\cdot}7$ on the right-hand side we find $y_m=15{\cdot}74$, which is sufficiently close check. By actual experiment in this case a value $y=16$ was observed.

Complete shut down with overflow. For this special case Prof. I. P. Church gives formulæ as follows.† (Notation transformed to

* "Cornell Civil Engineer," Dec., 1911, p. 114.
† "Cornell Civil Engineer," Jan., 1914, p. 156.

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correspond with that used in the present work. See Sec. 22 and Figs. 28, 29).

Let x=distance moved by water in main conduit after beginning of overflow (f).

Ax=volume of overflow (f3).

v_1=velocity in main conduit at instant of shut down (fs).

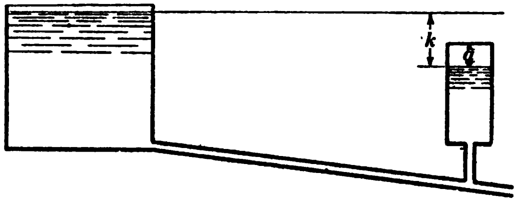

FIG. 28.—COMPLETE SHUT-DOWN WITH OVERFLOW.

v_2=velocity in main conduit at beginning of overflow (fs). Found by (11).

v=velocity in general (fs).

t=time from beginning of overflow (s).

$S=\sqrt{\frac{k-a}{c}}$ (for Fig. 28).

$S_1=\sqrt{\frac{a-k}{c}}$ (for Fig. 29).

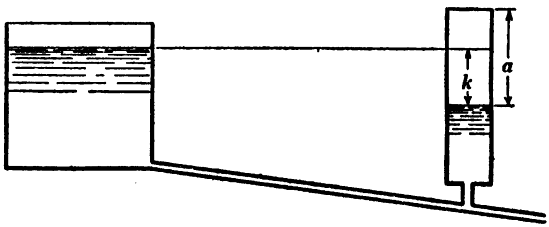

FIG. 29.—COMPLETE SHUT-DOWN WITHOUT OVERFLOW.

Then for Fig. 28:

$$x=\frac{L}{2gc}\log_e\left[\frac{k-a-cv_2^2}{k-a-cv^2}\right]\ldots\ldots\ldots\ldots\ldots(13)$$

$$t=\frac{L}{2gcS}\log_e\left[\frac{(v+S)(v_2-S)}{(v-S)(v_2+S)}\right]\ldots\ldots\ldots\ldots(14)$$

and for Fig. 29:

$$x=\frac{L}{2gc}\log_e\left[\frac{a-k+cv_2^2}{a-k+cv^2}\right]\ldots\ldots\ldots\ldots\ldots(15)$$

$$t=\frac{L}{gcS_1}\left[\tan^{-1}\frac{v_2}{S_1}-\tan^{-1}\frac{v}{S_1}\right]\ldots\ldots\ldots(16)$$

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If T=time to bring v to zero (end of first up surge) then we have

$$T=\frac{L}{gbS_1}\tan^{-1}\frac{v_2}{S_1}\qquad\qquad(17)$$

These equations give the distance x, the overflow Ax and the time t, all corresponding to any assigned value of v lying between v_2 and S for Fig. 28, or between v_2 and 0 for Fig. 29.

In the case of Fig. 28, S is the final velocity, and from (14) the time required to reach this velocity will be ∞. In the usual case, however, the approach to the final velocity is rapid, and the final condition is sensibly reached in a relatively short period of time.

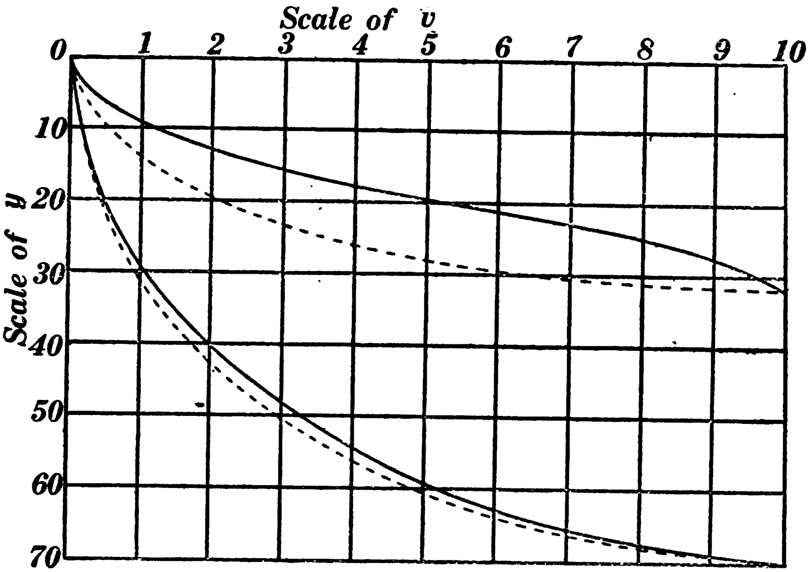

FIG. 30.—DIAGRAM OF MOVEMENT OF WATER LEVEL AND VELOCITY (OPENING).

Full opening from closure. Church's formula.* If in equations. (8) and (9) v_1 is taken as zero and neglecting governor action we put $u=v_2$ and then combine the two equations so as to eliminate dt, we find the equation:

$$ydy-cv^2dy=\frac{AL}{Fg}(v_2-v)dv.$$

Integrating between limits of 0 and y_m for y, and 0 and v_2 for v we have

$$y^2-2c\int_0^{y_m}v^2dy=\frac{AL}{Fg}v_2^2\qquad\qquad(18)$$

* "Trans. Am. Soc. C.E., 1915," Vol. LXXIX, p. 272.

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The integration indicated in the second term cannot be carried out unless the relation between v and y is known. In some cases the assumption that the relation is that of the quadrant of an ellipse, as indicated in Fig. 30, has been found to give results closely agreeing with observation.

On this assumption, the half axes being taken as v_2 and y_m equation (18) reduces to

$$y^2_m - 0{\cdot}1917cv_2^2 y_m = \frac{AL}{Fg} v_2^2 \qquad (19)$$

Whence

$$y_m = {\cdot}096cv_2^2 + v_2\sqrt{\frac{AL}{Fg} + {\cdot}0092c^2v_2^2} \qquad (20)$$

or approximately

$$y_m = \frac{h_2}{10} + v_2\sqrt{\frac{AL}{Fg}} \qquad (21)$$

Investigation through numerical integration shows that where values of AL/Fg are small, that is, where the surge chamber is large

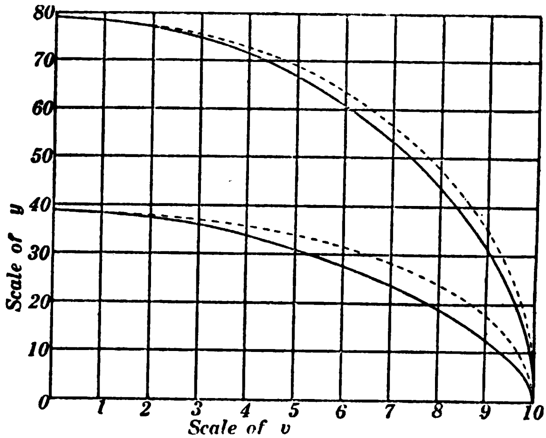

FIG. 31.—DIAGRAM OF MOVEMENT OF WATER LEVEL AND VELOCITY (CLOSURE).

and the movement of the water surface nearly dead beat, the above assumption regarding the form of the relation between v and y is not closely realized, and the application of the formula will involve a considerable error in the resultant value of y. Its use cannot, therefore, be recommended in such cases.

Full closure.—If the assumption made by Church regarding the relation between friction head and velocity be extended to the case of closure (see Fig. 31) the equation corresponding to (21) takes the form

$$y_m = \frac{h_1}{3} + \sqrt{\frac{ALv_1^2}{Fg} + \left(\frac{h_1}{3}\right)^2} \qquad (22)$$

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Numerical investigation here, likewise, shows that the formula is not applicable in cases where AL/Fg is small.

In a particular case where AL/Fg had the value 43·5 with h for full velocity 32 and a range of velocity from 0 to 10, the application of numerical integration gave the following results :

For opening, $v=0$ to $v=10$, $y_m=69{\cdot}6$.
For closure, $v=10$ to $v=0$, $y_m=78{\cdot}6$.

The application of the above approximate formulæ gives corresponding results as follows :

For opening, $y_m=69{\cdot}2$.
For closure, $y_m=77{\cdot}5$.

Again with the same maximum value of h and the range of v the same as above but with $AL/Fg=4{\cdot}35$, the application of numerical integration gives results as follows :

For opening, $v=0$ to $v=10$, $y_m=32{\cdot}0$.
For closure, $v=10$ to $v=0$, $y_m=38{\cdot}7$.

The application of the above approximate formulæ gives corresponding results as follows :

For opening, $y_m=24{\cdot}06$.
For closure, $y_m=34{\cdot}07$.

In the first case the results by formula give a close approximation to the correct values. In the second case the error is more considerable. Examination in the latter case shows that with the values as taken, the relation between v and y bears no very close relation to an ellipse and hence the results derived on such an assumption are naturally in error. In Figs. 30, 31 the full lines show the actual form of the curve between v and y for the two cases as noted, while the dotted lines show the corresponding ellipses. The degree of departure is thus plainly apparent.

For the case with full closure, Johnson* proposes the formulæ :

$$y_m=v_1\sqrt{\frac{AL}{Fg}+c^2v_1^2} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

$$t=\frac{\pi}{2}\sqrt{\frac{FL}{Ag}+\frac{c^2v_1^2F^2}{A^2}} \quad \ldots\ldots\ldots\ldots(24)$$

(t=time to reach y).

Tests of all such formulæ, through approximate integration, show that they are applicable each to a relatively narrow field of use only and that application outside of such range may lead to considerable error.

(*b*) **Approximate General Solutions.**—The following approximate equation is given by Johnson.†

$$y_m^2=\frac{AL}{Fg}(v_2'-v_1)^2+c^2(v_2'^2-v_1^2)^2. \quad \ldots\ldots\ldots(25)$$

* "Trans. Am. Soc. C.E., 1915," Vol. LXXIX, p. 265.
† "Trans. Am. Soc. Mec. Eng., 1908," p. 455.

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In this form the equation applies to the case of demanded load. For the case of rejected load the form is the same but with the interchange of v_2' and v_1. This implies equal values of y_m for the two cases. In this equation the value of v_2' is to be taken somewhat greater than v_2 for the case of demanded load and less than v_2 in the case of rejected load. The author of the formula states that for the case of demanded load the value may be taken from the equation:

$$v_2'=\frac{v_2H}{H-y}\text{.....................}(26)$$

Where H is the total head and y is a first value of y_m derived from (24) by taking $v_2'=v_2$. For rejected load the inverse ratio $(H-y)/H$ may be correspondingly inferred, although the author does not specifically refer to this case.

As a further refinement or check on these equations, with special reference to demanded load, Larner* proposes the following:

$$y_m^2=\frac{AL}{Fg}(v_2'-v_1)^2+c^2(v_2'^2-v_1^2)^2 \text{ ....}(27)$$

$$v_2'=u_1+k(u_m-u_1)\text{.......................}(28)$$

TABLE XXIII

$\frac{AL}{Fc}$	k	$\frac{AL}{Fc}$	k
2000	·50	16000	·79
4000	·62	18000	·80
6000	·67	20000	·81
8000	·70	22000	·82
10000	·73	24000	·82
12000	·75	26000	·83
14000	·77	28000	·83

To use these equations proceed as follows:

1. Use equation (25) and find a trial value of y_m; or otherwise assume a value according to judgment.
2. Substitute this value in (27) and find v_2'.
3. From the specified load change find u_1 from (3).
4. From the known characteristics of the case find the value of AL/Fc and using this in Table XXIII find the corresponding value of k.
5. Substitute the values of v_2', u_1 and k in (28) and find u_m.
6. Substitute this in (5) (see also (2)) and find z. In substituting in (5) $v^2/2g$ which is unknown may safely be approximated. This value of z with (10) will then give y_m. This is then compared with the assumed value.
7. Correct first assumption and proceed as before by trial and error.

These equations and table rest on an empirical basis derived from the examination of the results for fifteen selected cases

* "Journal Am. Soc. Mech. Eng., Jan., 1909," p. 113.

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through the method of arithmetical integration, as explained by the author in the reference cited.

(c) **Problems as Simplified by Disregarding Governor Action and by Assuming Friction Head to Vary with First Power of Velocity.**—If the volume of water delivered to the wheel during the transition period be assumed as constant at the rate $A_1 v_2$ equations (2)–(5) disappear from the problem and we have left simply (6) and (7), in the latter of which u becomes v_2. This is equivalent to a disregard of governor action during the transition period and to the assumption of a delivery of a constant volume flow of water instead of the development of constant power.

While this still leaves (6) and (7) beyond the reach of direct mathematical solution it nevertheless aids materially in treatment of the problem in various approximate or special or indirect ways. While the amount of surge as found for this simplified case is smaller than the true value, the magnitude of the error is often not serious. Especially is this the case where the friction head is a small fraction of the total head at the power house. On the other hand, where the friction head forms some considerable part of the total head the error resulting from the use of the simplified form might become significant.

In particular this simplified form of the equations is well adapted to treatment by an approximate method which has attracted the attention of a number of writers on this subject.* This method is based on the assumption that the frictional resistance in a pipe varies with the first power of the velocity instead of with the square. As a result of this assumption, equations (6), (7) becomes simplified in such manner as to admit of direct mathematical treatment.

Following are some of the more useful results.

Rejected load, partial or complete closure.—Assume in general the notation of Sec. 22. Also the following:

$\Delta v = (v_1 - v_2) =$ velocity change.

$\Delta h = c\Delta v =$ difference in friction heads for v_1 and v_2 on assumption that h varies as v.

$$a = \frac{gc}{2L}$$

$$m = \sqrt{\frac{Ag}{FL} - a^2}$$

$$B = \frac{\Delta v\left(\frac{A}{F} - ac\right)}{m}$$

$$\tan\theta = -\frac{m}{a}$$

$$t_1 = \frac{\theta}{m}$$

* Prasil, "Schweizer Bauzeitung," Band LII, No. 21–25. Warren, "Trans. Am. Soc. C.E., 1915," Vol. LXXIX.

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Then $y - \Delta h = e^{-at}(B \sin mt - \Delta h \cos mt)$..........(29)

$y_m - \Delta h = e^{-at_1}(B \sin \theta - \Delta h \cos \theta)$...........(30)

Equation (29) gives the value y for any value of t.

Equation (30) gives the value of y_m in terms of an angle θ and the corresponding time t_1, determined as above.

Numerical Example :

$L = 9000$ (f).
$F = 5000$ (f2).
$A = 75$ (f2).
$v_1 = 6{\cdot}5$ (fs).
$h_1 = 9{\cdot}5$ (f).
$v_2 = 0$ (complete shut down).

We then find by substitution of numerical values :

$\Delta v = 6{\cdot}5$.
$\Delta h = 9{\cdot}5$.
$c = 9{\cdot}5/6{\cdot}5 = 1{\cdot}4615$.
$a = {\cdot}002611$.
$m = {\cdot}00684$.
$B = 10{\cdot}63$.
$\tan \theta = -2{\cdot}620$.
$\theta = 110° - 53' = 110°{\cdot}9 = 1{\cdot}9355$.
$t_1 = 283$ sec.
$\sin \theta = {\cdot}9343$.
$\cos \theta = -{\cdot}3565$.
$at_1 = {\cdot}7388$.
$e^{-at_1} = {\cdot}4775$.
$B \sin \theta = 9{\cdot}929$.
$\Delta h \cos \theta = 3{\cdot}387$.
diff. $= 13{\cdot}316$.
$e^{-at_1} \times$ diff. $= 6{\cdot}36$.
add $9{\cdot}5$.
$y_m = 15{\cdot}86$.

While the above equations in the form given apply specifically to the case of rejected load and reducing velocity, the numerical values, on the present hypothesis regarding the relation of friction head to velocity, are the same for change in either direction between the same limits of velocity. Any problem in demanded load may therefore, for numerical solution, be converted into the corresponding problem in rejected load (change between the same velocity limits) and the solution found by the equations as above.

The error introduced by the present assumption regarding the relation of friction head to velocity is such as to give by the resulting equations a result somewhat too small for rejected load and somewhat too large for demanded load. In the case cited earlier, in comparing the results given by equations (21), (22) with those given by numerical integration, the value given by the presnet

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equations is 74 for either rejected or demanded load, lying nearly midway between the values 78·6 and 69·6 found by numerical integration.

(*d*) **Treatment of Surge Chamber Equations by Methods of Approximate Integration. Surge Chamber not Limited to Plain Cylindrical Form.**—In the methods of treatment thus far discussed, the surge chamber is assumed to be of the plain cylindrical form with uniform cross section. The assumption of a form of surge chamber with varying cross section, as for example, a tapering or conical form, would introduce further complications beyond the reach of methods of this character.

In the method of the present section, however, there are no such limitations, and the surge chamber may be of any form desired.

Writing again equations (6) and (7) we have

$$\frac{L}{g}\frac{dv}{dt}=H_1-(cv^2+z) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$-\frac{F}{A}\frac{dz}{dt}=(u-v). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

If the surge chamber is of the plain cylindrical form, the ratio F/A is constant. If the chamber is of varying cross section, then F/A will vary according to the known values of F at the water level.

Suppose now that at any given instant of time t, all values of the variables, dv/dt, v, z, u and dz/dt are known. Next take a small interval of time Δt and for the time $t+\Delta t$ assume a value of the acceleration dv/dt. We thus have two values of the acceleration, one at the beginning and one assumed at the end of the time interval Δt. If Δt is small we may take, without serious error, the mean of the two values multiplied by Δt as a measure of Δv, the change in velocity for the same time interval. This will give a value for the velocity at the end of this time interval, or for time $t+\Delta t$. Next putting in (6) the assumed value of dv/dt and the derived value of v (both for time $(t+\Delta t)$) we may solve and find the value of z for the same instant of time.

Taking now the new values of z and v with an assumed value of w in (5) we find by trial and error the correct value of u for the derived values of z and v. This value of u substituted in (7) will give the corresponding value of dz/dt. We then have two values of dz/dt (for the beginning and end of the interval Δt), and using the mean with Δt we find the resulting value of Δz, the change in z for the period Δt. Combining this with the initial value we find a value of z for the time $t+\Delta t$. We have thus found values of z for the end of the interval Δt in two different ways, one by way of (6) and one by way of (7), both starting from the assumed value of dv/dt at the time $t+\Delta t$. If this assumed value was correct, the two values z will agree. If incorrect they will disagree. The latter will,

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of course, be the result in the usual case. This operation must therefore be considered as giving a first approximation. The extent and direction of the difference in the two values of z will serve to indicate the nature of the change to be made in dv/dt by way of a second approximation. In this manner by successive steps, a set of values for all the variables is found which will satisfy the equations (6) and (7); and such are then taken as the values for the instant of time $t+\Delta t$. The operation is again repeated for another interval Δt, and so along step by step as far as it may be desired to trace the history of the movement.

As noted, the method requires a starting-point at which all values of the variables are known. This is the case at the beginning of the period of acceleration in the main conduit. At this instant, assuming the velocity in the penstocks to have acquired the value w_1 (corresponding to u_1 in the main conduit) and with the corresponding *beginning* of movement in the level of water in the surge chamber, we shall have $dv/dt=0$, $v=v_1$, $z=H-bv_1^2$ and $dz/dt=-(u_1-v_1)A/F$. These then are the initial values from which a start may be made in the manner described.

If desired other and more accurate rules for numerical integration may be employed, as for example, the following:

$$a=h/12\ (5y_3+8y_2-y_1).$$

For the area between the two ordinates y_2 and y_3 of Fig. 32. This assumes the arc of the curve to be a second degree parabola and is therefore more accurate than the use of an arithmetical mean, which assumes the arc of the curve to be a straight line and the contour to consist of broken straight lines. This rule can be used as soon as two sets of values of the various quantities are known; that is, as soon as the first point beyond the initial has been found in the manner above indicated.

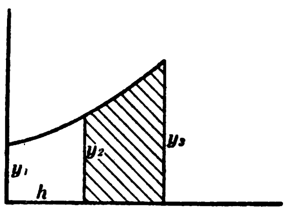

FIG. 32.
RULE FOR APPROXIMATE INTEGRATION.

In carrying out this work it is highly advantageous to carry along a series of plotted values of the different variables. In this manner by noting the trend of the curve the new value of dv/dt to be assumed in each case may be chosen very near to the correct value, and a satisfactory set of results therefore determined with the minimum number of trial and error steps.

The details of the work may be modified in various ways, as the interested reader will readily discover for himself.

By varying the time interval Δt according to the conditions of the problem any desired degree of accuracy may be realized. According to the dimensions and conditions involved and the degree of accuracy desired, the value of Δt may be taken from 5 sec. or less to 100 sec. or more.

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If instead of treating (6) and (7) in connection with (2)–(5), the simplified case is taken, assuming constant volume discharge of water to the wheels, then, as noted previously, u becomes constant $=v_2$ and the details of the process are greatly shortened.

(*e*) **Treatment of the Surge Chamber Problem through the Assumption of a Predetermined Program of Acceleration.**—The method developed in the preceding section involves necessarily a trial and error process. This results from the fact that the dimensions and proportions of the chamber are assumed as fixed, while the quantities to be determined are the consequences developing from a stated change in the load conditions.

If, on the other hand, the dimensions and proportions of the chamber are left to be determined, then we are free to fix according to choice any of the other variables entering into equations (6), (7), as, for example, the time history of the acceleration dv/dt. This is equivalent to stating the problem thus: Given a program for the water, required a surge chamber to produce it. The problem thus stated admits of solution by a straightforward process and without trial and error approaches.

For the details and possibilities of this method, reference may be made to a paper by the present author,* in which a full discussion of the subject will be found.

It may be here noted, however, that by this method there is, of course, no assurance that the form of chamber resulting from any arbitrarily assumed time history of the acceleration will be acceptable from a structural view-point. While it might produce the particular program of acceleration proposed, yet the form or dimensions might be undesirable. It results practically, however, that with a little experience the nature of the relation between the curve assumed for acceleration and the resulting form of the chamber comes to be readily appreciated, so that a form of curve suited to any generally proposed form of chamber may be assumed without difficulty.

It may be also noted that by this method, the minimum dimensions of chamber which will render the movement of the water substantially "dead beat" are readily determined.

(*f*) **Treatment of the Surge Chamber Problem by Model Experiment through the Application of the Law of Kinematic Similitude.**—Let L, d, D, A, F, v and H denote, as in Sec. 22, length of main conduit, diameter of main conduit, diameter of surge chamber, area of main conduit section, area of surge chamber section, velocity in main conduit and head in general.

Now let us assume the existence of two different cases with different values of these various characteristics, but so related as to produce, for similar load changes, similar time histories of the acceleration head in the surge chamber (*EF*, Fig. 24) and hence

* "Trans. Am. Soc. Mech. Eng.," Vol. XXXIV, p. 319.

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similar histories of the acceleration of the velocity in the main conduit.

The term similar, as here used, implies the transformation of one history into the other by a suitable change in the scales for acceleration head and for time. Assuming the time histories plotted as curves, this will imply the transformation of one curve into the other by a suitable change in the scale of the horizontal and vertical axes. Thus in Fig. 33 let the curve OA_2B_2 represent the accelerating head for case No. 2 plotted on time. Then if the vertical ordinates or dimensions of OA_2B_2 are multiplied by a factor, say ·60, and the

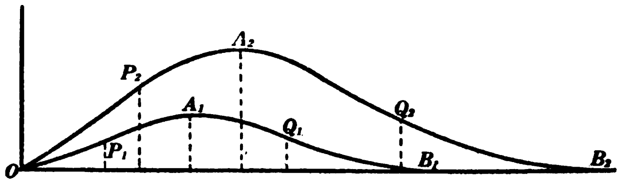

Fig. 33.—Similar Curves.

horizontal dimensions by a factor, say ·80, another similar curve will be produced as shown by OA_1B_1. Thus to every point on No. 2, as for example, P_2, A_2, Q_2, etc., there will correspond a point on No. 1, as P_1, A_1, Q_1, etc.

Writing now equations (6) and (7) we have

$$\frac{L}{g}\frac{dv}{dt}=H_1-(cv^2+z)\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$-\frac{F}{A}\frac{dz}{dt}=(u-v)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The left-hand term of (6), as we have seen in Sec. 11, is a measure of the acceleration head. Now assume this equation applied numerically first to case No. 1 and then to No. 2. At corresponding instants of time the left-hand members will be related by a fixed ratio. Hence the same must be true of the right-hand members, term by term.

Without attempting here to develop any general discussion of the principles of kinematic similitude, it may be noted that in the case of any algebraic equation which is to be applied to two different physical systems involving similar phenomena, such application being made effective through the use of a scale or transformation ratio, then each term in the equation must be subject to transformation from one system to the other through the use of the *same* scale ratio or factor. Or in other words the equation must be homogeneous in the transforming factor. The underlying reason for this may be seen by referring again to (6). The left-hand member is a quantity of the order of a vertical height; it is a linear dimension.

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Hence every member of (6) must also be a linear dimension and whatever ratio may exist for any one member as between case 1 and case 2 must also hold for all other members.

We are now ready to determine the various relation factors between the two cases in order that the assumed condition of similarity between the two acceleration curves may be fulfilled.

Let the length ratio or $L_2/L_1=p$.

Friction plus velocity head or c ratio$=c_2/c_1=q$.

Velocity ratio for any two corresponding velocities$=r$.

Time ratio for any two corresponding periods of the movement$=s$.

Then we have immediately as follows:

$$v^2 \text{ ratio}=r^2.$$
$$cv^2 \text{ ratio}=qr^2.$$

But cv^2 is a term in (6) and represents therefore a vertical dimension. Hence the same ratio qr^2 must hold between all other terms of (6) and in general between all similar vertical dimensions in the two cases. Hence we have

$$H \text{ ratio}=qr^2$$
$$z \text{ ratio}=qr^2$$
$$y \text{ ratio}=qr^2$$
$$\text{and} \quad \frac{L}{g}\frac{dv}{dt} \text{ ratio}=qr^2.$$

But the L ratio$=p$ and hence

$$\frac{dv}{dt} \text{ ratio}=\frac{qr^2}{p}$$

But the dv/dt ratio must equal the quotient of the velocity and time ratios or r/s. Hence we have

$$\frac{r}{s}=\frac{qr^2}{p}$$
$$\text{or} \quad s=\frac{p}{qr}$$

Again the dz/dt ratio must equal the quotient of the z and t ratios, or $\frac{dz}{dt}$ ratio$=\frac{qr^2}{s}$ or substituting the value of s

$$\frac{dz}{dt} \text{ ratio}=\frac{q^2r^3}{p}$$

Hence from (7) we have

$$\frac{F}{A} \text{ ratio}\times\frac{q^2r^3}{p}=v \text{ ratio}=r.$$
$$\text{or} \quad \frac{F}{A} \text{ ratio}=\frac{p}{q^2r^2}$$
$$\text{or} \quad F \text{ ratio}=\frac{p}{q^2r^2}\times A \text{ ratio}.$$

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But F ratio $=(D \text{ ratio})^2=(D_2/D_1)^2$
and A ratio $=(d \text{ ratio})^2=(d_2/d_1)^2$
so that we have finally:

$$D \text{ ratio}=\frac{\sqrt{p}}{qr}\times d \text{ ratio}.$$

$$\text{Volume ratio}=F \text{ ratio}\times\text{height ratio}=\frac{p}{q}\times(d \text{ ratio})^2.$$

Collecting for convenience we have a series of ratios as follows:

(*a*) The ratio of all similar vertical dimensions, such as maximum movement of water level, movement for final steady conditions, movement for corresponding parts of the acceleration curve or in corresponding values of the time, will equal qr^2.

(*b*) The ratio of the time intervals for the whole or for corresponding parts of the transition phenomena will equal $\frac{p}{qr}$.

(*c*) The ratio of the diameters of the chamber at corresponding levels will be $\frac{\sqrt{p}}{qr}\left(\frac{d_2}{d_1}\right)$ or $\frac{\sqrt{p}}{qr}\sqrt{\frac{A_2}{A_1}}$

(*d*) The ratio of the volumes swept through by the water surface in corresponding times will be $\frac{p}{q}\frac{d_2{}^2}{d_1}$ or $\frac{p}{q}\frac{A_2}{A_1}$.

It results that if these various relations between the dimensions and characteristics of the two cases are fulfilled, then the two acceleration curves will be similar; or otherwise, for similar velocity changes the acceleration curves will be similar and the movement of the water level in the surge chamber for one case will be in a known relation to that in the other case.

The application will be made clear by an illustrative case.

Let $L_2=30{,}000$ (f).
$d_2=8$ (f).
$a_2=50{\cdot}27$ (f).
Upper velocity $=8$ (fs).
Friction + velocity head at 8 (fs) $=66{\cdot}7$ (f).
Value of $c_2=1{\cdot}042$.

Proposed diameter of surge chamber $=45$ (f).

Suppose now that it is desired to forecast the behaviour of this case by means of a model using for the conduit say 20 feet of pipe, 1-inch internal diameter.

Then length ratio $=p=30{,}000/20=1500$.
Diam. ratio $=d_2/d_1=96/1=96$.

Suppose that it is found by experiment that an upper velocity of 5 (fs) will be most suitable for the model. Then velocity ratio $r=8/5=1{\cdot}6$. Next let the coefficient c be determined for the pipe by experiment. Suppose for a velocity of 5 (fs) the friction + velocity head is $2{\cdot}5$ (f).

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Then for the model $c_1=2{\cdot}5/25={\cdot}1$.
Then for the c ratio we have $q=1{\cdot}042/{\cdot}1=10{\cdot}42$.

Then we have as follows:

From (*b*) the time ratio $s=89{\cdot}98$
From (*a*) the vertical ratio $=26{\cdot}68$
From (*c*) the ratio $D_2/D_1=223$
Whence $D_1=(45\times12)/223=2{\cdot}42$ (i).

Suppose then that we fit up the pipe with a model surge chamber of diameter 2·42 (i) and observe the movement of the water under various conditions of load change.

Thus, for complete shut down full flow let the total rise of water be 35 inches or a 5-inch surge beyond the position of final equilibrium. Then the corresponding movement in case 2 should be $5\times26{\cdot}68=133{\cdot}4$ (i) or 11·1 (f).

Again, if the time to maximum rise of water level is 6·5 (s) in the model, the time in case 2 should be $6{\cdot}5\times89{\cdot}98=585$ (s)$=9{\cdot}75$ minutes.

Again, if it be desired to know the movement of the water level for an increase in load corresponding to a change in velocity in the main conduit of case 2 from 3 (fs) to 6 (fs) we observe the movement with the model for a change from 1·875 (fs) to 3·75 (fs), and apply the suitable ratios.

24. Differential Surge Chamber

The differential surge chamber consisting, in effect, of a small riser or stand-pipe connected to the line and to the penstock and standing within a larger chamber to which it is connected with ports, has been investigated and discussed exhaustively by Johnson.* Limitations of space forbid a detailed discussion of this combination of elements, but the interested reader should refer to the original papers, as noted, for a full consideration of characteristics offered by this device.

* "Trans. Am. Soc. Mech. Eng., 1908," p. 457. "Trans. Am. Soc. C.E.," Vol. LXXVIII.

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CHAPTER III

WATER RAM OR SHOCK IN WATER CONDUITS

ASSUME a straight inclined conduit, as in Fig. 34, leading from a reservoir at the upper end and controlled by a valve at the lower end permitting of opening or closure at any desired rate.

Suppose the water flowing steadily with a given conduit velocity v. If now the valve B is suddenly opened or closed, wholly or partially, there will be initiated at B a disturbance in the pressure condition of the water of the same nature as an acoustic wave in air, and subject to the same general laws of propagation.

As the first specific case we shall consider complete and instantaneous closure.

25. WATER RAM WITH INSTANTANEOUS COMPLETE CLOSURE: GENERAL PHYSICAL CONDITIONS

In this case we may picture the physical conditions as represented by a moving elastic column of water suddenly arrested at the lower end. We shall further, for simplicity, first assume the pipe as rigid and disregard the influence of friction on the pressure head of the water. After developing the physical conditions presented by this relatively simple and ideal case, the modifications necessary to allow for elasticity of the pipe and for friction will receive consideration.

By analogy we may picture a long spiral spring, moving endwise with a velocity v, and suddenly striking an immovable obstacle at the forward end. The result for the water column will be a compression beginning at B and propagating upward toward A, until finally the entire column will be brought to rest in a condition of compression from one end to the other. The period of time required for reaching this phase is, furthermore, just the time required for the propagation of an elastic compressive wave the length of the line, or from B to A. Physically we may picture the lower end of the column coming to rest against the face of the valve and the remainder of the column continuing on until at each point, as it reaches the same degree of compression, it also comes to rest; and thus the edge of the compressed section travels up the line with the velocity of propagation of an acoustic wave, until at the close of the period, the entire mass of the column is at rest under com-

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pression. In Fig. 35 let A_0B_0 denote the length of the column at the instant of closure, and the arrow the direction of motion. Then A_1B_1 will denote the compressed length of the column at rest, or the condition at the close of the period which we have just considered. Again, it will be clear that the kinetic energy which the

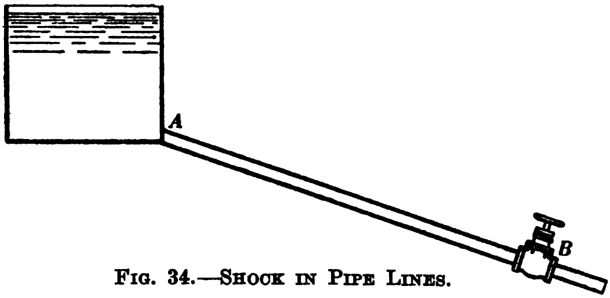

FIG. 34.—SHOCK IN PIPE LINES.

column of water possessed in virtue of its velocity v, has, at the end of this period, disappeared and become transformed into the potential energy of compression under the condition denoted by A_1B_1. In the condition A_1B_1, therefore, while the column of water is at rest momentarily, it is not in equilibrium with its surroundings, since it is under the excess pressure resulting from the compression as noted. It will, in consequence, begin to return toward normal condition, and a wave of expansion will start in at the upper end A_1 and progress downward toward B_1. The compression, in other words, will gradually yield, the particles of the column moving upward and the wave of expansion progressing downward until finally, in the phase A_2B_2, the original condition will be reached, but with the particles of water moving upward. Comparing the conditions A_0B_0 and A_2B_2, we have the volume and condition as regards pressure the same; also the energy is entirely kinetic in both cases, but reversed in direction in A_2B_2 as compared with A_0B_0.

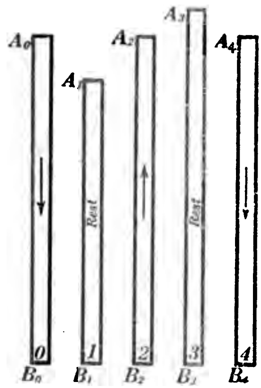

FIG. 35.
SUCCESSIVE STATES IN OSCILLATING WATER COLUMN.

Considering A_2 as virtually a free end, it is clear that the result will be the same as though the column as a whole should continue to move upward, thus relieving the pressure at B_2, and starting in a wave of further dilatation at B_2 which will progress upward toward A_2. The ultimate result will be a condition A_3B_3, in which the column will be momentarily at rest in a state of dilatation.

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The kinetic energy in the state A_2B_2 will thus be stored up against the gravity forces which serve normally to produce the condition of relative compression denoted at A_0B_0 or A_2B_2. It will be shown later that as compared with the normal pressure in A_2B_2, the drop in pressure in A_3B_3 will equal the rise in A_1B_1.

Again, while this is a condition of rest, it is not one of equilibrium, and it will be followed by a return wave of relative condensation, beginning at A_3 and progressing downward toward B_3. The result of this will be a return toward normal condition, and ultimately the column will be in the state A_4B_4 with the energy kinetic in form and the particles travelling downward, and with the original state as regards volume and pressure.

Condition A_4B_4 is thus seen to be the same as condition A_0B_0. We have thus traced the phenomena through a complete cycle, and assuming a perfectly elastic liquid, the sequence of compression

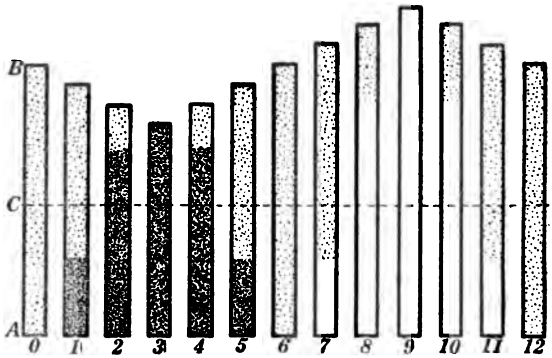

FIG. 36.—SUCCESSIVE STATES IN OSCILLATING WATER COLUMN.

and dilatation in volume with rise and drop of pressure above and below the normal, would continue indefinitely. Actually due to viscous resistance the oscillations will dampen out after a few cycles, the number and character depending on the circumstances of the case.

As a further aid in picturing the condition of the water in the various phases of this wave of compression and dilatation, reference may be made to Fig. 36, showing intermediate phases, with the parts of the column under compression or dilatation indicated by the shading.

If now we assume a pressure gauge located at the lower end of the line, it is clear that relative to normal pressure the phases from 0 to 6 (Fig. 36), will show a rise in pressure and from 6 to 12 a drop in pressure. This is shown by the line $OABCD$ (Fig. 37). This diagram, of course, assumes ideal conditions in the liquid and a gauge acting without lag or inertia. It thus appears that the

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pressure will suddenly rise to an amount OA above the normal and hold such value uniform during the phases 0 to 6 (Fig. 36), at the end of which period it will suddenly drop to the point C (Fig. 37), below normal, which value it will hold during the phases 6 to 12 (Fig. 36), following which there will be a reversal to excess pressure and a repetition of the cycle.

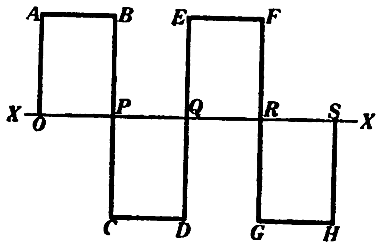

Fig. 37.—Time History of Excess Pressure at Valve. Ideal Case.

From a study of the diagram (Fig. 36), it will also be clear that at any point in the line not at the lower end, as at C, the compressive phase will not begin until the wave has travelled from A up to C, and that it will continue only during the time required for the wave to travel from C to the upper end B and return to C.

In general let

L=length of line.
x=distance to any point from lower end.
S=velocity of propagation of acoustic wave.

Then

x/S=time interval after closure to beginning of compression.
$2(L-x)/S$=duration of period of compression.
x/S=time interval from close of period of compression to end of cycle.

Similar expressions will hold for the phases of dilatation.

For a point near the lower end, or where x is small, the pressure

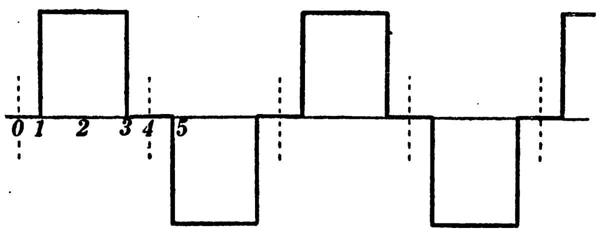

Fig. 38.—Time History of Excess Pressure at Points in the Line. Ideal Case.

diagram will therefore be similar to Fig. 38, while for a point near the upper end, or where x is large, it will be similar to 39(a), and for the upper end itself it will be similar to 39(b).

These various diagrams may be represented as a system by the solid diagram suggested in Fig. 40. $FGHJ$ represents a reference plane corresponding to normal pressure. A is a wedge lying on

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the upper side of the plane and C a similar wedge lying on the under side of the plane. The triangles D, B and E are parts of the plane. The lower edge FJ corresponds to the lower end of the pipe, the upper edge GH to the upper end, and any intermediate cutting plane 11, 22, etc., to a point distant x from the lower end. Then any such

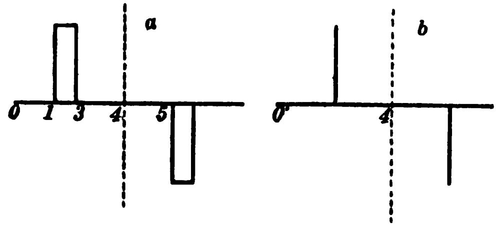

FIG. 39.—TIME HISTORY OF EXCESS PRESSURE AT POINTS IN LINE. IDEAL CASE.

plane 11, 22 or 33 will cut from such a model a section, the outline of which will give the diagram of pressure for the corresponding point in the pipe line.

We have now to show that the drop in pressure in the condition A_3B_3, (Fig. 35), will equal the rise in pressure for A_1B_1.

Consider the kinetic energy of the moving column in A_0B_0. Denote this by E. This is stored as potential energy at A_1B_1. This

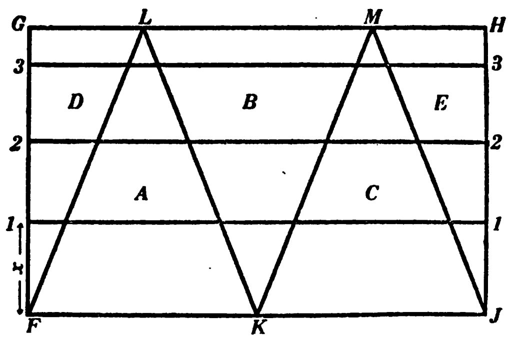

FIG. 40.—BLOCK MODEL FOR TIME HISTORY OF EXCESS PRESSURE AT ANY POINT IN LINE. IDEAL CASE.

is again transformed into kinetic energy at A_2B_2, and the latter will therefore equal E in amount though reversed in direction. This same energy E is again rendered potential at A_3B_3 against the forces of gravity, which produce the normal condition of pressure at A_0B_0 or A_2B_2. This means a reduction of the total potential energy of compression by the amount E. Relative to the total normal potential energy of compression at A_0B_0 or A_2B_2, the condition at A_1B_1 implies therefore an excess measured by E and the

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condition at A_3B_3 a decrease likewise measured by E. This will obviously imply equal changes of pressure above and below the normal.

It must be understood that this simple relation can only hold under the condition that the reduction of pressure is not greater than the original steady motion pressure. A liquid will not admit of the development of a tension, and after the pressure is reduced to 0 any further tendency in the column A_3B_3, for example, would imply a break and the entry of discontinuous conditions.

Thus near the upper end of a penstock line where, for example, the steady motion pressure is 30 lb. absolute, if a sudden shut down produces first an increment of pressure measured by 50 lb., then the pressure at the given point will rise to 80 lb. abs., but cannot drop below 0 abs. There will, however, in such case be developed a tendency for the water to separate and leave the upper end of the pipe producing discontinuity and turbulence as the manifestation of the remainder of the energy which cannot be absorbed by the development of a tensional stress.

26. Modifications in Physical Conditions Necessary to Allow for Elasticity of Pipe, for Friction and for the Head due to Velocity

So far as the more complete physical picture is concerned, we must consider that the water and the pipe form together an elastic system, and that the arrest of the former will result in a compression of the water and an extension or dilatation of the pipe. Likewise with reduction of pressure the water will expand and the pipe contract. The water and the pipe therefore enter conjointly into all interchanges and transformations of energy between the kinetic and the potential forms. With these facts in mind the changes in the physical picture necessary to allow for the elasticity of the pipe will be readily made. The mathematical development will be found in Sec. 27.

Regarding the pressure history, as affected by friction, let AB (Fig. 41) denote the line, NN the static level and KL the hydraulic grade for steady flow. Then, as we have seen in Sec. 14, the distances from AB to KL denote the values of the pressure head at points along the line.

If now any portion of the line, as a differential element in length, is brought suddenly to rest, the kinetic energy will be suddenly transformed into potential energy of compression which will manifest itself as an excess pressure, additive to the pressure already in evidence at the given point.

But the velocity is uniform throughout the length of the line (AB uniform in section) and hence the kinetic energy per element of length is uniform. Hence at each point in the line the elementary

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accession of energy in the compression form, due to sudden arrest, will be uniform and the corresponding pressure will be represented by some height such as LD.

The line CD parallel to KL will therefore mark the limits of pressure reached at successive points in the line at the instant of arrest of the corresponding elements of flow, or otherwise at the instant when the wave of compression, starting from B, reaches the point in question.

If, then, the pressure condition thus realized would remain *in statu quo* during the traverse of the compression wave front to the upper end of the line A, we should have, at the end of the compression phase, the entire line at rest, in a state of compression and with a pressure gradient CD.

This excess pressure head gradient CD, however, is not a gradient of equilibrium, and in consequence the pressure conditions realized

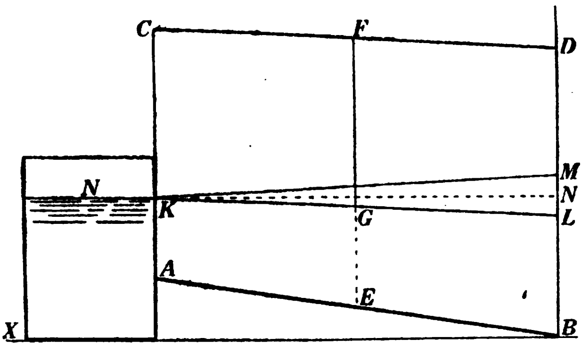

Fig. 41.—Excess Pressure as Modified by Effects due to Friction. Partial Account.

at the instant of arrest will, during the remainder of the compression phase, be subject to further change.

Thus suppose for the moment that with the wave at E and say half the line under compression, the excess pressure head gradient were FD.

Then it may be readily seen that the water in the line between E and B, at rest and under gravity, will not be in equilibrium under a total pressure head distributed according to the gradient FD. There will be a tendency for pressure energy to flow from high to low or from F toward D and thus to seek a level of uniform head.

It is of interest to note that the problem of energy flow thus arising is very similar to that presented by the flow of heat along a bar under corresponding specified temperature conditions.

The initial value of the pressure head is always on the line CD so that at the end of the compression phase the pressure gradient must end at C. The remainder of the gradient will, however, differ

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from CD as a result of energy propagation, the part toward D being raised and flattened in the approach toward a uniform level, while that nearer C will fall below CD as a result of losses of energy exceeding gains. At the instant when the compression wave front reaches A we shall have then, the original kinetic energy represented under the following items :

(*a*) Compression energy manifested as pressure and distributed according to the instantaneous condition resulting from its generation and propagation as noted above.

(*b*) A residual amount of energy in the kinetic form and involved in the movements connected with energy propagation. That is, so long as the total head is not uniform, so long will there be energy propagation involving molecular movement and hence kinetic energy.

This propagation of compression energy and approach toward uniform distribution will, of course, continue during the return of the wave up to the point when the front of the wave of recovery reaches the point in question ; that is during the entire compressive phase. The extent of the unloading or recovery will furthermore depend, at each point, entirely upon the difference between the pressure necessary for the unloaded phase and the pressure in the compression phase at that particular point.

But the pressure in the unloaded phase (6, Fig. 36) will depend in a complex manner on the influence due to friction in the establishment of the reverse flow (3 to 6, Fig. 36). Thus in Fig. 41 the pressure head at A or AK must remain unchanged whether the flow is direct or reverse. Hence with a reverse flow established we shall have a disappearance of the excess pressure head CD with a flow head represented by some gradient line such as MK.

Now the kinetic energy of reverse flow is simply the expression in kinetic form, of the excess compression energy. Hence at each point the velocity generated will be determined by the compression energy in excess and thus available for transformation into the kinetic form. But this will vary from point to point as determined by the difference between the excess pressure head at the given point and the pressure head required to maintain the flow against friction. To a first approximation, these varying amounts of compression energy would be represented by the intercepts between CD and KM. There will be, therefore, a *tendency* to develop, at each point in the line AB, a different velocity, v the initial value at A and gradually less and less as the available compression energy is less due to the growing demands for friction. But continuous flow means uniform velocity, and hence any tendency to develop varying velocities along the line will in itself tend to set up a secondary series of waves which will themselves be subject to propagation in the usual manner.

The net result of this complicated series of actions and reactions

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will be, at the end of the unloading phase (6, Fig. 36) a distribution of the total energy available under the forms :

(*a*) Kinetic energy of motion in the reverse direction or from B to A, and representing a resultant or group velocity somewhat less than the initial velocity v.

(*b*) Compression energy, due to the existing complex state of motion, distributed along the line and unavailable for expression in the kinetic form.

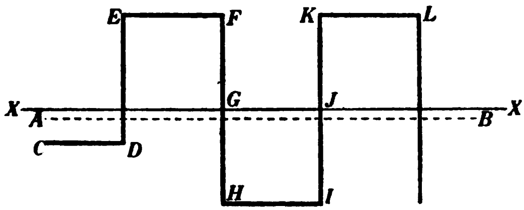

FIG. 42.—EXCESS PRESSURE AS MODIFIED BY EFFECTS DUE TO FRICTION. PARTIAL ACCOUNT.

(*c*) Compression energy as required, transformed into the work done against friction in setting up the reverse flow, and the expression of which will be a reverse flow pressure gradient, something like KM.

The time history of the pressure at any given point will evidently partake of all these complexities of pressure generation and energy

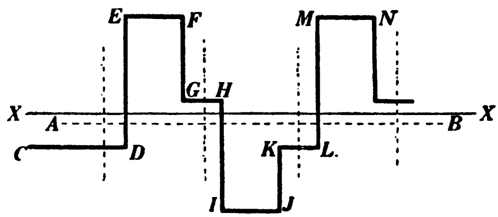

FIG. 43.—EXCESS PRESSURE AS MODIFIED BY EFFECTS DUE TO FRICTION. PARTIAL ACCOUNT.

propagation. Aside from secondary effects due to energy propagation, the time history would be somewhat as in Figs. 42, 43, 44. Using these diagrams as a first approximation we may gain some general idea of the character of history to be expected. Thus in Fig. 42 referring to conditions at the valve, let XX denote the static level, AB the level for velocity head and CD the running level, including friction head. Then pressure would start from D and on arrest of movement rise to E a distance DE measuring the excess pressure due to arrest. The further history would then consist of alternate levels EF, HI, etc., equidistant from XX and denoting the levels reached at the successive instants when the energy is all in the

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potential form (3, 9, Fig. 36). With propagation, however, the line EF will actually rise according to some complex law and the return wave will reach not quite down to H, following which will be a slight down slope resulting again from energy propagation. The history would thus show, for the extreme pressures, a series of lines slightly sloping away from XX and at distances from XX gradually decreasing with time.

Similarly, to a first approximation, at points in the line as indicated in Figs. 43, 44, the levels EF and IJ, expressing the pressure when all in the potential form (3 or 9, Fig. 36) would lie equidistant from the static level XX, while the levels GH and KL expressing the pressure when the energy is kinetic would lie at equal distances above and below the velocity head line AB.

In comparison with this incomplete ideal, the actual history would show sloping lines for EF and IJ, at gradually decreasing

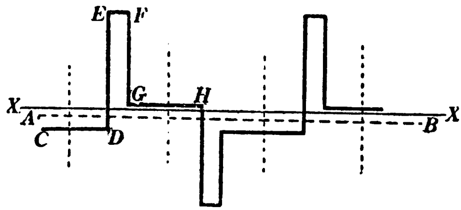

FIG. 44.—EXCESS PRESSURE AS MODIFIED BY EFFECTS DUE TO FRICTION. PARTIAL ACCOUNT.

distances from XX and lines GH and KL nearly horizontal and at nearly equal distances from a velocity head line AB which itself will gradually approach XX.

It is thus seen that as the ultimate result of the influence due to friction, an exceedingly complex condition develops and the discussion of this particular phase of the problem in further detail is quite beyond the scope of the present work. The magnitude of these secondary elements can scarcely be determined by direct mathematical means, and the results derived by such direct methods must always be understood as representing a first approximation to actual conditions, more and more nearly accurate as the friction head is less and less significant as a factor in the problem.

27. VELOCITY OF PROPAGATION OF ACOUSTIC WAVE

The well-known formulæ of physics give for the velocity of an acoustic wave in an elastic medium the equation:

$$S=\sqrt{\frac{gK}{w}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where S=velocity (fs).
g=gravity.
K=coef. of elasticity (pf2).
w=density (pf3).

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In the case of water in a pipe line, however, we do not have the simple case of a single medium. We must include the influence of the steel shell forming the envelope of the water, and the problem becomes therefore that of finding the velocity of the wave in a core of elastic water surrounded by an elastic metal shell.

Let us in the first place consider the latter. In the case of an excess internal pressure q it may be subject to excess stress in two directions, longitudinal and circumferential, measured as follows :

$$\left.\begin{aligned} T_1 &= \frac{qr}{2t} \\ T_2 &= \frac{qr}{t} \end{aligned}\right\} \quad \text{(2)}$$

where T_1=stress in longitudinal direction (pi2).
T_2=stress in circumferential direction (pi2).
q=excess pressure (pi2).
r=radius (i).
t=thickness (i).

Poisson's investigations show that in the case of a plate stressed in two directions at right angles, the relative stretches (strains) are expressed in the form :

$$\left.\begin{aligned} \frac{dx}{x} &= \frac{T_1}{E} - \frac{T_2}{aE} \\ \frac{dy}{y} &= \frac{T_2}{E} - \frac{T_1}{aE} \end{aligned}\right\} \quad \text{(3)}$$

where x and y denote the original lengths in the two directions, dx and dy the corresponding extensions, T_1 and T_2 the corresponding stresses, E the coefficient of elasticity and a a factor known as Poisson's coefficient.

Applying to the present case we may take x longitudinal and y circumferential. Then $dy/y=d(2\pi r)/2\pi r=dr/r$, and from the above formula we have

$$\left.\begin{aligned} \frac{dx}{x} &= \frac{qr}{2tE} - \frac{qr}{atE} \\ \frac{dr}{r} &= \frac{qr}{tE} - \frac{qr}{2atE} \end{aligned}\right\} \quad \text{(4)}$$

Again let K denote the cubical coefficient of elasticity for water. Then by definition of the meaning of this term we have

$$\frac{dV}{V} = \frac{q}{K} \quad \text{(5)}$$

where V=original volume.
dV=diminution of volume.
q=excess pressure.

Then $dV=\frac{q}{K}V$.

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Applying this to the original volume of water in a length of pipe x we have

$$dV \text{ (water)} = \frac{q}{K}\pi r^2 x \text{(compression)}.$$

Again for the pipe, as a result of the excess pressure q and the resulting excess stresses, longitudinal and circumferential, the new length and new radius will become $x+dx$ and $r+dr$ respectively. The new volume will then be

$$(V+dV)=(x+dx)\pi(r+dr)^2.$$

The original volume was

$$V=x\pi r^2.$$

Subtracting we find, after neglecting differential terms of the second and third orders,

$$dV \text{ (pipe)}=2\pi rx dr+\pi r^2 dx \text{ (expansion)}.$$

Substituting for the values of dr and dx from (4) we find after simple reduction of form :

$$dV \text{ (pipe)}=\pi r^2 x\left(\frac{qr}{2tE}\right)\left(5-\frac{4}{a}\right) \text{(expansion)} \quad \ldots\ldots (6)$$

The total relative change of volume between the water and the pipe will then be measured by the compression of the former plus the expansion of the latter.

Calling dV such total relative change we have

$$dV=\pi r^2 x\left[\frac{q}{K}+\frac{qr}{2tE}\left(5-\frac{4}{a}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots (7)$$

But $\pi r^2 x=V$ the original volume.

$$\text{Hence} \quad \frac{dV}{V}=\frac{q}{K}+\frac{qr}{2tE}\left(5-\frac{4}{a}\right) \quad \ldots\ldots\ldots\ldots\ldots (8)$$

We must now consider that the elastic compression of the water plus the elastic extension of the shell combine to give to the system a virtual coefficient of elasticity which we may denote by J. We may otherwise consider this change dV as an apparent or virtual change in volume, as evidenced by the shortening up of the total column of water, a part of such shortening being due to compression of the water and a part to the extension of the pipe. In any case we relate such virtual change in volume to a virtual coefficient of elasticity J for the system composed of water and pipe.

We shall then have by definition, as in (5):

$$\frac{dV}{V}=\frac{q}{J} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\text{or from (8)} \quad \frac{q}{J}=\frac{q}{K}+\frac{qr}{2tE}\left(5-\frac{4}{a}\right)$$

$$\text{or} \quad \frac{1}{J}=\frac{1}{K}+\frac{r}{2tE}\left(5-\frac{4}{a}\right) \quad \ldots\ldots\ldots\ldots\ldots (10)$$

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For steel plates we may take $a=3\cdot6$, giving:

$$\frac{1}{J}=\frac{1}{K}+\frac{1\cdot944r}{tE}\ \ldots\ldots\ldots\ldots\ldots\ldots\ (11)$$

We shall then have for the combination of water and pipe, and similar to equation (1),

$$S=\sqrt{\frac{gJ}{w}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ (12)$$

For numerical values in equation (11) we may conveniently take the pound and foot as units, and hence K and E will be measured in pounds per square foot.

This gives $K=43{,}200{,}000.$
$E=4{,}032{,}000{,}000.$

The resulting value of J substituted in (12) will then determine the velocity of propagation of the acoustic wave along the pipe line. If the pipe line were absolutely rigid, the value of J would equal K and the velocity S would be that for water alone, or about 4700 fs. Due to the influence of the elastic shell, J is always less than K, and S is less than the value for water. Values of S for various values of r/t are given in Table XXIV.

TABLE XXIV

r/t	S (fs)	q per unit v (pi2)
10	4293	57·84
15	4119	55·50
20	3964	53·41
25	3827	51·56
30	3702	49·88
35	3588	48·35
40	3485	46·96
45	3390	45·68
50	3302	44·49
55	3221	43·40
60	3146	42·39
65	3076	41·45
70	3010	40·56
75	2948	39·72
80	2890	38·94
85	2835	38·20
90	2783	37·50
95	2734	36·84
100	2688	36·22
105	2643	35·61
110	2600	35·03
115	2561	34·51
120	2522	33·98

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r/t	S (fs)	q per unit v (pi2)
125	2485	33·48
130	2450	33·01
135	2417	32·56
140	2384	32·12
145	2353	31·70
150	2323	31·20

Iron or Steel Pipe. Values of velocity of acoustic wave S and of pressure q developed per unit of velocity quenched.

(r/t=ratio of radius to thickness.)

28. Excess Pressure Developed

We may now proceed to determine the excess pressure in the pipe as a result of complete and instantaneous closure of the valve.

The kinetic energy of a cubic foot of water moving with velocity v_0 is

$$E=\frac{wv_0^2}{2g}$$

When the water comes momentarily to rest in the condition 1 (Fig. 35), this energy must exist in potential form represented by the work done in compressing the water and in extending the pipe. As we have seen above, this will be equivalent to the production of a change of volume dV in a system of virtual cubical coefficient of elasticity J.

Then as in (5):

$$\frac{dV}{V}=\frac{q}{J}$$

$$\text{or}\quad q=\frac{JdV}{V}$$

The value of q during such compression will vary from 0 to the full value, and the mean will be one-half the above or $JdV/2V$. The work done will be measured by mean pressure × change in volume. Hence

$$\text{Work}=\frac{q}{2}dV=\frac{J(dV)^2}{2V}=\frac{q^2V}{2J}\quad\text{.............(13)}$$

Hence we shall have for one cubic foot of water

$$\frac{wv_0^2}{2g}=\frac{q^2}{2J}$$

whence

$$q=v_0\sqrt{\frac{wJ}{g}}\quad\text{..................(14)}$$

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Combining (14) with (12) we find

$$q=\frac{wSv_0}{g}\text{ (pf2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

$$\text{or}\quad q=\frac{wSv_0}{144g}\text{ (pi2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Let h denote the head corresponding to pressure q. Then we have

$$h=\frac{q}{w}=\frac{Sv_0}{g}\text{ (f)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

The ratio S/g recurs so frequently in the further discussion of these problems that it will be convenient to represent it by a single symbol. To this end put

$$\frac{S}{g}=a.$$

With this notation (17) becomes

$$h=av_0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

By a different mode of combining (14) and (12) so as to eliminate w/g we also find

$$q=\frac{v_0J}{S}\text{ (pf2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$\text{or}\quad q=\frac{v_0J}{144S}\text{ (pi2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

$$\text{or}\quad h=\frac{v_0J}{wS}\text{ (f)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Since S is commonly found between 3000 and 4000, it follows that the value of q will commonly range from 40 to 54 (pi2), or the value of h from about 90 to 125 feet, per foot second of velocity arrested by instantaneous closure.

Where the pipe line is of varying diameter and of varying thickness of metal, the theoretical investigation in any precise manner becomes too complicated to serve practical purposes. Average values may, however, be usually taken in such manner as to serve practical requirements.

29. Water Ram with Rapid Complete Closure

We have thus far assumed the closure complete and instantaneous. Suppose next that it occupies a certain time interval T, and that the time history of the retardation produced at the valve is given by the curve of Fig. 45.

Any ordinate AC of this curve is then a measure of the retardation or rate of velocity change produced at the corresponding instant t. Also the area OAC will be proportional to the total change in velocity during time t, and the total area OAB will be likewise proportional to the total velocity v_0, which, at the end of time T, is reduced to 0. Under these conditions there will be

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initiated at the lower end of the pipe a continuous series of pressure waves, each representing an element of velocity change and each added to the one preceding, thus gradually building up the pressure at the valve as the sum of these elements, each of which will be proportional to an ordinate of the curve OAB. That is, the excess pressure at the valve will continuously increase by the addition of successive elements, each due to a retardation acting through an element of time and producing an element of velocity change. At any instant during the period of closure the total pressure at the valve will then be the result of a summation of all these elements, developed from the beginning of the movement up to the

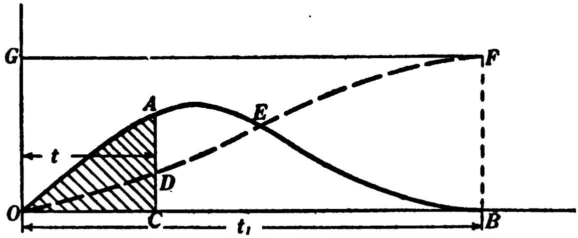

FIG. 45.—TIME HISTORY OF dv/dt AND v.

given instant of time, and this will correspond to the total change in velocity produced, that is, to an area such as OAC.

These facts stated in a physical sense may be established analytically as follows:

We have first to find the element of pressure at the valve due to a retardation dv/dt continuing through an element of time dt. The result is, of course, a reduction of the velocity v by the amount dv in time dt.

The distance covered by the wave in time dt will be Sdt. That is, a length of water column Sdt will, in time dt, be retarded by the amount of velocity change dv. Let A=cross section area and w=density. Then the mass subject to this retardation is $wASdt/g$.

The force required is measured by the product of mass by retardation. Let dQ denote such force. Then we have

$$dQ=\frac{wA}{g}\frac{dv}{dt}Sdt \qquad (22)$$

This will appear as an excess pressure at the valve, distributed over the cross section area of the column. Let dq denote the corresponding unit pressure. Then

$$dq=\frac{w}{g}\frac{dv}{dt}Sdt \qquad (23)$$

$$\text{or} \quad dq=\frac{w}{g}Sdv \qquad (24)$$

$$\text{or again,} \quad dh=\frac{dq}{w}=\frac{S}{g}dv=adv \qquad (25)$$

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In (24) or (25) the only variable term is the velocity change dv. This equation shows that the element of excess pressure dq is determined by the constant factors characteristic of the case and by the velocity change dv. More specifically it shows that it is independent of the time element dt. This somewhat surprising result is due, as is shown by the form of equation (23), to the fact that the expression for the pressure element dq contains two variable factors, one proportional to the acceleration and the other to the quantity of water involved. The first of these, expressed by dv/dt carries the time element in the denominator, while the other, represented by Sdt, carries the time element in the numerator. In the product the time element disappears, leaving the pressure element dependent solely on the velocity change dv. Or otherwise if the rate of valve closure is increased, for instance, the retardation will also be increased, but the time occupied and the quantity of water involved will be correspondingly decreased. Thus if the closure is effected in one-half the time the acceleration will be doubled and the quantity of water halved, and thus the product will remain the same.

The effect produced by each successive value of the retardation, as indicated by a curve such as OAB, will be entirely similar in form, each proportional to the element of velocity change dv, and therefore for the entire curve we shall have the sum of a series of elements, each similar in form to (24).

Summing these we have

$$q=\frac{wS\Delta v}{g}=\frac{w}{g}S(v_0-v)=aw(v_0-v) \qquad (26)$$

and
$$h=a\Delta v=a(v_0-v) \qquad (27)$$

The expression (v_0-v) or Δv for the velocity change occurs so frequently in the further discussion of these problems that we shall find it a convenience to represent it by a single term. To this end put

$$(v_0-v)=s \qquad (28)$$

In this sense s always means the aggregate change in velocity starting from the initial velocity v_0.

With this notation (27) becomes

$$h=as \qquad (29)$$

Equation (27) is general and applies either to partial or complete closure. In the latter case $s=v_0$ and $h=av_0$ as in (18).

It thus appears that the excess pressure at the valve will be dependent solely on the change of velocity produced and independent of the time required to realize such change.

It is also noted, by comparison with (18), that this value of h is the same as for instantaneous closure.

The independence of the excess pressure on the time of valve movement is, however, only realized within suitable limits as to

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length of line and time of valve movement, as will be shown at a later point.

Returning now to the various pressure waves formed at the valve, it is seen that they will all travel with the velocity S, adding themselves to the previous value of the pressure, so that when the wave corresponding to any ordinate (as AC, Fig. 45) has reached any point in the line, the pressure at such point will equal that at the valve when the given wave started.

Let the broken line $ODEF$ denote the integral curve of OAB. That is, a curve such that the ordinate at any point, as CD, is proportional to the area OAC and similarly for all other points. Then any ordinate as CD will represent the total velocity change from the origin up to time t, while the ordinate BF will represent the total velocity v_0. This curve, measuring from OB, will then give the time history of the velocity change; while measuring from GF to the curve, we shall have the time history of the velocity itself as it gradually falls from v_0 to 0.

But as we have seen, the pressure at the valve is proportional to the total change in velocity and hence to the ordinates of the curve $ODEF$. In other words, this curve will give a time history of the growth in excess pressure at the valve.

Again, due to the wave propagation, as noted above, it will be clear that at any instant t, the first wave (corresponding to $t=0$ at the valve), will have reached a distance $x=St$, and the pressure condition will correspond thereto. The wave corresponding to the ordinate AC will just be leaving the valve, and the pressure there will be represented by DC. At intermediate points along the line, between the valve and the point $x=St$, the pressure condition will be represented by the successive ordinates of the curve OD. In other words, OD is a space history of the pressure distribution along the line between the given point and the valve.

Similarly the entire curve $ODEF$ will give the time history of the excess pressure at the valve, progressing with the time from 0 at the beginning of the movement to BF at the close, and covering the total time of valve movement T. Likewise the same curve will give for this instant (at the end of T) the space distribution or history of the pressure along the line, represented by BO, the valve being at B with the pressure BF, and the farthest point reached, distant $x=ST$, being at 0 with pressure zero.

It follows further that at any point $x=St$ the time history of the pressure will be the same as at the valve, but retarded by a time interval t.

These various conclusions assume, of course, no disturbance in the conditions, due to reflection from the upper end of the line.

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30. Water Ram with Rapid Partial Closure

For simplicity of treatment, the discussion thus far has assumed complete closure.

The treatment of Sec. 29 embodies, however, the more general case as shown by equations (26), (27).

In the various formulæ for complete closure we have in any case only to substitute for v_0, the velocity of flow, the change in velocity expressed by $(v_0-v)=\Delta v=s$.

Thus for any change from v_0 to v, whether instantaneous or gradual (so long as the conditions of Sec. 31 are fulfilled) the values of q and h are as given in (26), (27).

The generality of these results traces back to (24), (25) which will have the same form whether the closure is partial or complete. In any case the value of the integral of dv is $\Delta v=s$, the velocity arrested, complete or partial as the case may be.

Hence in one case this integral will have the value v_0 and in the other $(v_0-v)=\Delta v=s$.

31. Conditions for Realization of Assumptions of Secs. 25–30

Reference has been made to the conditions under which the results of Secs. 25–30 may be realized. From the preceding discussion it will be clear that at the valve the pressure will continue to increase according to a time history as given by some curve such as *ODEF* (Fig. 45), unless interfered with by reflection from the upper end of the line as discussed in Sec. 25. Let L=length of line and T=time of valve movement. Then if $L=ST$ it is clear that the first impulse to leave the valve will just reach the upper end of the line at the end of the period, and the entire history of the growth in pressure at the valve will be spread out along the length of the pipe. If the length L is less than ST the first impulses to leave the valve will have started back, as a reverse or partial unloading of the pressure, before the close of the period T. If L is only slightly less than ST the upper end of the line only will be affected by this unloading and the lower end, and especially at the valve, will show the same condition as in the case when $L=ST$. If $L=ST/2$ or $T=2L/S$ the return from the upper end will just reach the valve at the close of the period. For a shorter value of L or a longer value of T there will be a certain amount of unloading and reflection at the valve, depending on the circumstances of the case.

Broadly speaking, then, the values:

$$L=ST/2$$
$$\text{or}\quad T=2L/S$$

furnish the critical conditions regarding the pressure at the valve. For convenience of notation let us represent the time $2L/S$ by the

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symbol z. Then if T is less than z the conditions assumed in Secs. 29, 30 will be realized at the valve and the pressure will be the same as for instantaneous valve movement. If T is greater than z there will be a certain amount of unloading and reflection at the valve and formulæ (18), (29) no longer apply.

If T lies between $z/2$ and z there will be a length of the line measured from the upper end, which will undergo partial unloading before the close of the valve movement—a length less and less as T approaches $z/2$.

The condition of pressure resulting from values of T greater than $z/2$ and involving reflection back and forth from the ends of the line will be considered in later paragraphs.

32. Derivation by Different Methods of Certain of the Preceding Formulæ for Excess Pressure

The subject of shock or water hammer in pipe lines is of so great general importance that it may be well to derive certain of the preceding results in a somewhat different manner.

Consider first instantaneous full closure. Then holding in mind the total column of water of length L we find, after it has been brought to rest in the compressed state 1 (Fig. 35), that it has shortened up a certain amount. This is due, as we have seen, partly to the actual compression of the water and partly to the extension of the pipe.

The shortening up of the column is measured by the distance traversed in time $t=L/S$ by what may be termed the upper end of the water column with cross section A moving with velocity v_0. The distance moved is v_0L/S and the total apparent change in volume is Av_0L/S.

If then J represents the virtual cubical coefficient of elasticity as defined in (9) we have as before:

$$\frac{\Delta V}{V}=\frac{q}{J}$$

$$\text{or} \quad q=\frac{\Delta V}{V}J \qquad \text{(30)}$$

But ΔV is the total apparent decrease in volume noted above. Hence,

$$\Delta V=\frac{Av_0L}{S}$$

But AL=total volume$=V$ and hence

$$\frac{\Delta V}{V}=\frac{v_0}{S}$$

and from (30) $q=\dfrac{v_0J}{S}$ as in (19).

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But from the law of propagation for an acoustic wave we have from (12)

$$J=\frac{wS^2}{g}$$

Whence we find $q=\dfrac{wSv_0}{g}=awv_0$ as in (15).

Again assume partial instantaneous closure reducing the velocity from v_0 to v_1. Then during the time t required for the compressive wave to travel the length of the line L, we shall have a discharge at the lower end under velocity v_1 and a volume discharged Av_1t. Likewise at the upper end we must consider the water flowing in under a velocity v_0 until the lapse of time $t=L/S$ when the compressive wave will reach the upper end and the entire column for the moment will be in a state of compression and moving with velocity v_1. The total volume of inflow will then be measured by Av_0t. The difference between the inflow at the upper end and outflow at the lower will measure the apparent change in volume due to water compression and pipe extension. Hence we shall have

$$\Delta V=\frac{A(v_0-v_1)L}{S} \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

We have again the fundamental relation:

$$\frac{\Delta V}{V}=\frac{q}{J}$$

$$\text{or} \quad q=\frac{\Delta V}{V}J.$$

But $AL=V$ and from (31) we have

$\dfrac{\Delta V}{V}=\dfrac{(v_0-v_1)}{S}$, and hence from (9)

$$q=\frac{v_0-v_1}{S}J$$

which is the generalized form of (19).

Again substituting for J as in (12) we find

$q=\dfrac{wS\,(v_0-v_1)}{g}=aws$, as in (26).

Again consider the work energy relation. The volume of the pipe will be AL and the original energy $wALv_0^2/2g$. After time L/S this energy will be represented by the following items:

1. The kinetic energy of the water in the pipe moving with velocity v_1 and measured by $wALv_1^2/2g$.
2. The work done in forcing out at the lower end the volume of water Av_1L/S under the excess pressure q. This will be measured by qAv_1L/S.

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3. The work done in producing the apparent change of volume $\Delta V = A(v_0 - v_1)L/S$. This will be measured by $qA(v_0 - v_1)L/2S$. See equation (13).

We may then write a work energy equation:

$$\frac{wALv_0^2}{2g} = (1) + (2) + (3).$$

Combining and reducing we find

$$q = \frac{wS(v_0 - v_1)}{g} \text{ as in (26).}$$

33. Water Ram in Pipe Lines When Lower End of Pipe is Held Rigid

The formulæ and methods of Secs. 25–30 assume that the lower end of the pipe is free to move longitudinally. Only on this assumption can longitudinal stress be developed in accordance with equation (2) and only on this assumption will the virtual coefficient of elasticity J have the value as given in equation (10).

If we now assume the lower end of the pipe rigidly fixed, then $dx = 0$ and we have as the only change in the dimensions of the pipe

$$\frac{dr}{r} = \frac{pr}{tE}$$

Following this value through with exactly the same method as in Sec. 27, we find

$$\frac{dV}{V} = \frac{p}{J} = \frac{p}{K} + \frac{2pr}{tE}$$

$$\text{or} \quad \frac{1}{J} = \frac{1}{K} + \frac{2r}{tE}$$

Comparing this with (11) we find it the same except for the coefficient 2 as compared with 1·944.

Referring to equation (3), it is seen that the presence of a stress (and its resulting strain) at right angles to a given stress will reduce the strain or extension which the latter stress would by itself produce. The circumferential stretch due to a given internal pressure will then be less than if the circumferential stress existed alone and not in conjunction with the longitudinal stress. With the latter eliminated, as in the case of a pipe with the end rigid, the circumferential stretch will therefore be larger than with the two stresses coexisting and, as the equation shows, the total extension in volume is practically the same in the one case as in the other. In fact so far as the equations apply and with the value of Poisson's modulus assumed for steel, it appears that the volume expansion of the pipe is slightly greater with the lower end rigid than when free. Practically the difference is not significant.

We may therefore conclude that whether the lower end of the

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pipe is rigid or free the total volume expansion under an excess pressure p will be substantially the same, and therefore the equations of Secs. 27–30 for J and for the excess pressure resulting from a sudden quenching of velocity may be practically employed independent of the degree of constraint of the lower end of the pipe.

34. Rapid Opening from Complete Closure

In the case of a rapid or practically instantaneous opening of the valve from complete closure, the case comes under the general method of treatment of Secs. 25–30. A similar system of acoustic waves will be formed, beginning with a wave of expansion or drop of pressure, and followed by excess and defect in alternation, as discussed in Sec. 25. The amplitude of these pressure waves will, however, not be the same as for the case of sudden closure. If the opening is instantaneous, the pressure at the valve will be reduced from the total static head behind the valve to the pressure on the discharge side.

Actually the valve cannot be opened instantaneously and the drop in pressure will be somewhat less than this amount. Let q denote the pressure drop and v the velocity of the water movement in the wave toward the valve. Then there will subsist between q and v for pressure drop and velocity generated the same relation as in Sec. 28 for pressure rise and velocity quenched, and we shall have for the velocity of the water forming the wave toward the valve the value,

$$v=q\sqrt{\frac{g}{Jw}}=\frac{h}{a}; \text{ see (14) (18) } \ldots\ldots\ldots\ldots (32)$$

Where h now denotes the head due to pressure drop.

This wave will be propagated toward the upper end of the line with a velocity S as in Sec. 25. In case the gradient of the pipe is such that at all points the absolute statical pressure is greater than q, then during the passage of the wave the pressure will at all points remain positive and the wave will reach the upper end with its amplitude or pressure drop practically unchanged. At this instant the entire column of water is moving toward the valve with the velocity v and with a pressure q below the normal value. At the inlet, however, the pressure must remain normal and reflection at this point will result in the propagation toward the valve of a wave of normal pressure and with a water velocity v relative to the expanded part or $2v$ relative to the pipe. This will in turn be reflected at the valve under conditions representing a change in velocity between $2v$ and the existing velocity at the valve. If there are no disturbing conditions, and especially if effective reflection from the partly open valve could be realized, then the result would be an excess pressure wave q propagated up the line; and thus the series of pressure drop and pressure excess would alternate, forming

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a general pressure history similar to the case for closure as discussed in Sec. 25.

In the actual case, however, disturbing conditions may enter, and in any event the reflection from a partly open valve is imperfect. The conditions contemplated in the physical picture are not therefore completely realized, and the excess pressure is usually considerably less than the pressure drop and the series of alternations of pressure above and below normal rapidly damps out to a negligible amount.

Again, if due to the gradient of the line there should be a point where the pressure drop would render the pressure zero or negative, then there will result a discontinuity in the physical conditions of the problem and the wave will travel on to the inlet with a greatly reduced amplitude. The reflected wave will then represent a much reduced velocity and the series of pressure fluctuations will dampen out very rapidly.

Under certain conditions of discontinuity with a rapidly opened valve, the return pressure may not even pass the normal static value, or indeed it may not even reach such value, the return from the initial pressure drop gradually flattening out to the pressure value with friction head under final steady conditions.

Broadly speaking, the initial pressure drop will vary directly with the degree of initial opening. With full opening from closure the drop will reach nearly down to the pressure on the discharge side of the valve. With only partial opening, the initial drop will be reduced.

The further analytical development with discussion of this case will be found in Secs. 41, 42, as a special case of the general problem of valve opening.

35. Rapid Opening from Partial Initial Opening

The general phenomena attendant on such cases are broadly similar to the case of opening from initial closure, with a closer and closer approach as the initial opening is less. With increase in the initial opening there is a rapid decrease in the initial pressure drop and a decrease in the value of the return excess pressure wave, and a rapid approach toward the condition of dead beat return from the initial drop to the final steady flow pressure value.

The analytical treatment of this case will be found in Sec. 41 as a special case of the general problem of valve opening.

36. Law of Increase of Pressure with Time, Valve Closure

The equations developed in Secs. 28–30 give the maximum or ultimate value of the pressure reached, but do not furnish any indication of the time history of the growth of such pressure.

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These equations apply at the valve so long as the time of valve movement T is not longer than $z=2L/S$, and throughout the pipe so long as T is not greater than $z/2=L/S$.

The excess pressure will clearly depend, among other things, on the rate of closure of the valve. The usual assumption in this connection is of uniform closure; that is, of a uniform rate of decrease of valve opening.

Independent of any such assumption regarding the rate of valve closure, however, but assuming T not greater than $2L/S$ we may investigate the law of pressure change as follows:

Let A = cross section area of pipe.
a = area of valve opening.
S = velocity of acoustic wave.
u = velocity through valve.
v_0 = original value of v.
f = coefficient of efflux through valve.
$h=q/w$ = excess pressure head.
Put $m=a/A$.

We have then three equations as follows:

$$v=mu \quad \text{(33)}$$

This expresses the continuity of flow along the pipe and through the valve.

$$h=a(v_0-v)=as \quad \text{(34)}$$

This expresses, in accordance with (27), the excess head developed at the valve corresponding to any reduction of velocity (v_0-v), and hence the excess head at the valve at the instant when the pipe line velocity is v.

$$\frac{u^2}{2g}=f\left(H+h-\frac{Lv^2}{C^2r}\right)^{*} \quad \text{(35)}$$

This expresses the value of the head on the discharge side of the valve, $u^2/2g$, transformed under efficiency f from the total net head on the upper side of the valve, made up of the original head H plus the excess pressure head h, minus the friction head Lv^2/C^2r (see Chap. I (44)).

Putting (35) all in terms of u and transforming we have

$$Mu^2=H+h \quad \text{(36)}$$

$$\text{Where } M=\left(\frac{1}{2gf}+\frac{Lm^2}{C^2r}\right)$$

* In this expression, the term Δy (see Chap. I (44)), representing the difference in the external pressure at the two ends of the line, is omitted. This does not involve any lack of generality in the present treatment. In any of the subsequent equations of the present chapter, wherever H occurs, $H+\Delta y$ may be substituted for it, if Δy has a value other than 0, thus giving full generality of treatment in this respect.

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If then between (33), (34) and (36) we eliminate u and v and reduce the equation in h we have

$$h^2-2\left(av_0+\frac{(am)^2}{2M}\right)h+(av_0)^2-2H\frac{(am)^2}{2M}=0 \quad \ldots\ldots(37)$$

Put $E=av_0$

$$F=\frac{(am)^2}{2M}$$

Then (37) takes the form

$$h^2-2(E+F)h+E^2-2FH=0 \quad \ldots\ldots\ldots\ldots(38)$$

Solving this as a quadratic in h we have

$$h=(E+F)-\sqrt{F^2+2F(H+E)} \quad \ldots\ldots\ldots\ldots(39)$$

When $m=m_0$ we find $E^2=2FH$, and h in (39) reduces to $h=0$ as it should.

When $m=0$, $F=0$ and h reduces to av_0 as it should.

If then we take a series of values of m from m_0 to 0 and corresponding to valve movement from start to full closure, we may, with the known hydraulic characteristics of the case, find the values of E and F and hence of h.

We may then find s from (34), thence v and u if desired from (33).

The time history of the pressure head h and the velocity v is shown for four typical cases in Fig. 48.

In a the value of h remains negligibly small during the early part of the movement, only beginning to rise at the very last, and then jumping with great rapidity to its maximum value at the instant of complete closure.

In b the rise is slow at first and then more rapid, but in less pronounced degree than in the case of a.

In c the curve is of the same general character, but more nearly approaches a uniform rate of pressure rise.

In d the curve is only slightly convex to the axis of time, showing a nearly uniform rate of increase.

The history of s will be the same as that of h and the history of v will hence be similar, but in the inverse direction, as noted on the diagrams.

In a the reduction in velocity is negligible during the valve movement up to the last tenth and then the velocity is rapidly reduced to zero, accompanied by the rapid upshoot in the value of h as noted above. In b, c, the rate of reduction of v changes progressively toward the condition indicated in d, where a nearly uniform rate of reduction is realized.

The characteristics of these various cases, as noted, lead to the following general conclusions :

The controlling condition giving rise to a pressure history such as that of a is a large value of m_0, that is a valve or nozzle opening nearly or quite the full size of pipe. In the case of a, $m_0=1{\cdot}00$. This implies a pipe with gravity flow and hence the entire head H

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used up in friction and velocity, or since the latter head is small, it follows that substantially the entire head H is used up in friction.

Other things equal, the head h will be small with large values of F, and will increase more and more slowly with decrease in m in accordance as the value of L is greater and greater.

Going to the other extreme as represented by the case of d the initial value of m is very small (·02) and the head is very high (2700 (f)). At full pipe line velocity of 8·04 (fs) the velocity head is about 1 foot and the friction head is 85·5. Hence but a small fraction (3·2%) of the total head is used up in friction and velocity, the remainder (96·8%) being available under steady conditions as pressure head.

The cases represented in b and c are intermediate between those of a and d. The fraction of head used in friction in the four cases is progressively ·968, ·65, ·17, ·032. While other factors will modify the form of the curve to some extent, the general progression from that of a to that of d will correspond to a lesser and lesser fraction of the total head absorbed in friction.

37. Law of Decrease of Pressure with Time, Valve Opening

The treatment of the case of valve opening is in effect contained within that of closure, as in Sec. 36. It is only necessary to remember that for valve opening, h is negative and to modify the resulting equations accordingly.

The treatment of this case in further detail will, however, be found in Sec. 41, as a part of the more general treatment of the case of valve opening.

38. Gradual Closure: Time Long Relative to Time $2L/S$ for Double Traverse of Acoustic Wave*

In this case there will be a series of reflections back and forth from the two ends of the line, somewhat after the manner assumed in Sec. 25. The total effect, however, may be considered as made up as a summation of successive effects due to successive movements of the valve and to the consequent successive elements of velocity reduction and the resulting elements of pressure change. Thus as in Sec. 29 an elementary or differential change in velocity

* *Notation:* In this and following sections of the present chapter it will be found of special convenience to denote the value of the reduction in velocity $(v_s - v)$ by the single symbol s and also a series of values of h, s and other quantities belonging to a series of time instants t, $t-z$, $t-2z$, $t-3z$, etc., by h, h_1, h_2, h_3, etc. That is, no subscript implies a value relating to the instant of time t, a subscript 1, to an instant earlier by z, a subscript 2, to an instant earlier by $2z$, etc.

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dv taking place in an elementary or differential time dt will produce at the valve an elementary or differential pressure change measured by

$$dh = a dv$$

and this element will move with velocity S as an elementary pressure wave along the pipe and suffer reflection back and forth somewhat as indicated in Sec. 25.

At any point in the line therefore, and at any subsequent time, the total net excess pressure head h will be the algebraic summation of all such elements, both direct and reflected, as have, during such time, reached or affected such point.

With reference to such conclusion, however, one important reservation must be made.

The reflection of pressure waves in a liquid back and forth from the two ends of the line, as assumed in Sec. 25, assumes complete or perfect reflection from each end of the line. Also in the ideal case, the damping effects due to viscosity are neglected.

With regard to reflection from the upper end of the line, such reflection is based on the condition of a constant pressure at this point, and such condition must obviously be fulfilled, since just beyond the upper and open end of the pipe we can only have the reservoir pressure, which is assumed constant in value. Hence we may with propriety assume that, at this end of the line, reflection will be realized with a close approach in manner and degree as assumed.

On the other hand, the reflection from the valve end assumes the valve closed before the reflected wave returns to this end of the line. A dead end, and at which the velocity must become zero, is therefore the implied condition for the complete reflection assumed in these earlier sections. With the conditions of the present section, however, the reflected wave returns to the valve end before closure and while water is still issuing. The reflection cannot therefore be complete.

The degree to which reflection is realized will presumably depend on the closeness to which the condition of a dead end at the valve with zero pipe line velocity is approached.

Thus with an area of valve opening at the start the full size of the pipe and in the early stages of the movement, the area will be but slightly reduced, the excess pressure developed will tend to increase the issuing velocity u, and there will be but slight reduction in the main pipe velocity v, and in consequence the reflection must be quite incomplete. As the valve approaches the closed position, however, the area through the valve will become much reduced, the pipe velocity v will become small and more complete reflection should be realized. Otherwise, considering the valve as equivalent to a diaphragm moved across the opening, we may say that in the early stages of the movement there is but a small area of diaphragm available against which a reflected wave can form, while near the

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close of the movement such area will be larger, and more complete reflection will be realized.

Again, in the case of a pipe line under high head and with a nozzle area a, even when wide open, small compared with the pipe line area A, it seems probable that, at all stages of the valve movement, relatively more complete reflection should be realized. The closure of a nozzle from area a to zero may, in effect, be considered as the last stages of a closure of the complete area from A down through a and to zero. From another viewpoint we may consider that the end area available for the support of a reflected pressure wave will be $(A-a)$ even when the valve is wide open and will gradually increase to A as the valve is closed.

These points still remain in uncertainty, however, and there is much need for further experimental study of the general problem in order to determine more definitely the extent to which reflection of pressure waves can be realized from the delivery end of a pipe discharging water through a nozzle or opening of varying area in relation to the c.s. area of the pipe.

We must, however, in general conclude that the reflection of pressure waves from the valve end of a pipe under discharge will be more or less imperfect or incomplete, approaching complete reflection as the flow of water becomes less and less, and realizing substantially complete reflection from and after the moment of valve closure.

In Sec. 40 will be found some further discussion of the subject of partial reflection at the valve.

The condition of complete reflection may in general be considered as a limiting case to which actual cases will approach as the attendant circumstances may determine. It becomes therefore a matter of interest to develop, at least in general outline, the results which may be expected in such limiting case.

To this end and holding in mind the principles of Sec. 25 we have

$$dh=a(dv-2dv_1+2dv_2-2dv_3+\text{etc.})\ldots\ldots\ldots(40)$$

whence

$$h]_0^t=a(v]_0^t-2v]_0^{t_1}+2v]_0^{t_2}-2v]_0^{t_3}+\text{etc.})\ldots\ldots\ldots(41)$$

Or with the special notation of the present section,

$$h=a(s-2s_1+2s_2-2s_3+\text{etc.})\ldots\ldots\ldots\ldots\ldots(42)$$

Thus in (40) the first term represents the element generated at the valve at the instant t, the second term the element generated at the valve at the instant $(t-z)$ or t_1 and which has, in the meantime, travelled to the upper end of the line and back again, arriving at the instant t and, by complete reflection as discussed in Sec. 25, operates to reduce the pressure condition at that instant by $2adv_{t-z}$, or by twice the amount of the element generated at the instant $(t-z)$. Similarly the third term represents the element generated at the valve at the instant $(t-2z)$ and which has in the meantime completed one full cycle of four traverses of the length L, and thus

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returns to the valve at the instant t as a wave of positive pressure and by reflection then gives a positive element of $2adv_{t-2z}$ or twice the element generated at the instant $(t-2z)$.

The remaining terms, however numerous, indicate successive elements generated at the valve at successive time intervals of z counting backward from the instant under consideration. The number of terms, therefore, will be given by the whole number next below t/z. These terms, as readily seen, will have alternately minus and plus signs according as they have made an odd or an even number of double traverses of the length L.

In equation (41) the successive terms indicate each the summation of a series of elements adv, all of which are similar in sense and time history (number of double traverses of pipe line length). Thus for the first term the time period is 0 to t, giving the summed effect (all in the positive sense) of all elements as formed and previous to propagation or reflection. For the second term the time interval is 0 to $(t-z)$ or t_1, giving the summed effect (all in the negative sense) of all elements which have had time to make the double traverse $2L$ with return to the valve and reflection at that point. For the third term the time interval is 0 to $(t-2z)$ or t_2, giving the summed effect (all in the positive sense) of all elements which have had time to make two double traverses, or one complete cycle, with return to the valve in the positive sense and reflection at that point.

In this manner the various terms are made up, each representing the summation of elements which have made at least a given whole number of round trips from the valve to the upper end and return, successively 0, 1, 2, 3, etc.

It should be noted that this entire development of an expression for the resultant h in the case of a closure extending over a time t larger than z is only an extension, by the process of summation, of the principles and methods developed in Secs. 25 and 29.

Re-writing (42) we have

$$h=a(s-2s_1+2s_2-2s_3+\text{etc.})\quad\ldots\ldots\ldots\ldots(42)$$

We shall have similarly

$$h_1=a(s_1-2s_2+2s_3-\text{etc.})\quad\ldots\ldots\ldots\ldots(43)$$

It is readily seen that these two expressions after the terms in s_1 have the same terms with opposite signs. Hence

$$h+h_1=a(s-s_1)\quad\ldots\ldots\ldots\ldots(44)$$

Again, in equation (42) put

$$B=2s_1-2s_2+2s_3-\text{etc.}\quad\ldots\ldots\ldots\ldots(45)$$

We have then

$$h=a(s-B)\quad\ldots\ldots\ldots\ldots(46)$$

Noting the make-up of h_1, as in (43), we also readily see that

$$h_1=a(B-s_1)$$

or

$$aB=h_1+as_1\quad\ldots\ldots\ldots\ldots(47)$$

Thus from either equations (44) or (46), (47) it appears that the value of h for a given time t can be immediately determined if we

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can find s for the same time and knowing also h and s for a time $(t-z)$. Or otherwise if we know h and s for a time t and can find s for a time $t+z$ we can find h for $t+z$ and so on for a series of values of t separated by the interval z.

Those results are of remarkable simplicity, connecting, as they do, successive values of h separated by the time interval z.

The determination of the values of h during any period of time in general, involves three distinct phases or time periods:

1. t between o and z.
2. t between z and T.
3. t beyond T.

With proper interpretation (44), (46) and (47) apply generally to all three periods. Thus for the first period the subscript 1 implies a time $(t-z)$ negative and in such case the term is to be omitted, giving in (44):

$$h=as \qquad (48)$$

For the second period the equations apply as written.

For the third period, for $t=T$ and beyond, s becomes v_0. Hence for values of t between T and $T+z$ we shall have from (44):

$$h+h_1=a(v_0-s_1) \qquad (49)$$

while for $(t-z)>T$ or $t>(T+z)$, both s and s_1 become v_0, and we have

$$h=-h_1 \qquad (50)$$

It now remains to provide means for the determination of the value of s corresponding to any given time t.

In Sec. 36 equations have been developed for the determination of the time history of each of the various quantities h or q, u, v, s, and on the assumption of a time of closure T less than z. These equations will therefore apply directly and without change to the present problem for the first time period from $t=0$ to $t=z$.

For the second time period from $t=z$ to closure, or $t=T$, we proceed as follows:

Equations (33) and (36) apply to this time period without change. Instead of (34) we have (46), and this becomes

$$h=a(v_0-v-B) \qquad (51)$$

Combining these so as to eliminate u and v, as in Sec. 36, and reducing the equation in h we have similar to (39):

$$h=(E+F)-\sqrt{F^2+2F(H+E)} \qquad (52)$$

Where $E=av_0-aB$

$F=\dfrac{(am)^2}{2M}$ as before,*

and $aB=h_1+as_1$

When $m=0$, $F=0$ and $h=E=av_0-aB$ as it should from (51) with $v=0$.

* For an expression for F not explicitly involving the quantities m and f (as in notation of equation 36), see Appendix II.

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It will be noted that (52) differs from (39) only in the term E which here appears as $av_0 - aB$ instead of av_0.

Repeating for convenience (33) and (36) we have

$$v = mu \qquad (53)$$

$$Mu^2 = H + h \qquad (54)$$

$$\text{Where } M = \frac{1}{2gf} + \frac{m^2L}{C^2r}$$

and $m = a/A$ and where a may vary in any specified manner with the time.

The condition of uniform rate of valve area closure is often assumed in dealing with problems of shock. In such case if a_0 is the original or full opening we shall have

$$m = \frac{a_0}{A}\left(1 - \frac{t}{T}\right) = m_0\left(1 - \frac{t}{T}\right) \qquad (55)$$

It should be especially noted, however, that in this general method of treatment it is not necessary that the valve follow any particular law in closing and in particular that the treatment is in no wise limited to the case of linear closure. The various equations of the present section expressing h as the sum of a series of terms are entirely independent of any term expressing the time rate of valve movement. The valve opening is represented solely by the factor m which appears in equations (36) and (38), and no matter what the character of the valve movement may be, so long as it is known, it will be possible to assign to a series of values of t the corresponding series of values of m. This insures, therefore, the solution of the problem for any assigned rate or character of valve movement.

In any case, therefore, the program for finding a series of values of h extending over any period of time from 0 to t is as follows:

1. The conditions of the valve movement determine the values of m. If the ratio of closure is assumed uniform then m for any value of t is given by (55).

2. From the known hydraulic and other characteristics of the case find the values of the various terms in equation (52), omitting B.

3. With a time interval Δt as selected and the various numerical values of the terms in (52), putting $B=0$, find the corresponding series of values of h up to the value of t next smaller than z. Then find the resulting values of s, v and u, if desired, using (48) and (53). This will give a series of values covering the period up to z or up to the nearest point below z. A further point may then be found in the same manner for $t=z$. If the time interval is exactly contained in z, the regular series will contain the point for $t=z$ as its last member. A value of the time interval Δt which is either equal to z or an even submultiple is to be advised in a program of computation of this character.

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4. Then taking for t a value $z+\Delta t$, find from (47) the value of $aB=h_1+as_1=2s_1$ or $2s$ at time Δt, and with this value of B and the appropriate values of the other terms find from (52) the value of h, and thence s from (46) and thence v and u if desired.

5. Continue the process by taking next $t=z+2\Delta t$ and find from (47) the value of $B=2s_1$ or $2s$ at time $2\Delta t$, and similarly for successive steps. Beyond $t=2z$ the value of B will no longer equal $2s_1$ but will always be correctly given by (47).

6. Beyond $t=T$ the procedure is simplified through (49), (50) and, as will be seen, the time history of h then becomes a periodic curve with alternate positive and negative maxima equal to the value realized at $t=T$, and with a complete period of $2z$.

It thus appears, in the development of such a series of values of h, that each set of terms found for a time t serves to determine through (47) and (52) the h and thence the s, v and u for the time $t+z$.

It further results that a value of h at any single value of t cannot in general be found except as a member of the series extending at least from $t=z$. The complex condition of multiple reflection, back and forth, does not seem to permit of representation in a single equation with known terms, unless, indeed, such equation were made of such complexity as to represent in effect the series of operations required to determine the series of values as above indicated.

Sample Computation.—In order to indicate the character of the numerical operations in connection with the solution of equation (52) the following sample computation is given.*

The basic data taken are as follows:

$$
\begin{array}{lrl}
 & L= & 4000\cdot \quad \text{(f)}. \\
 & D= & 1\cdot \quad \text{(f)}. \\
 & H= & 500\cdot \quad \text{(f)}. \\
\text{Chézy Coef.} & C= & 110\cdot \\
 & f= & \cdot 96. \\
 & S= & 4000\cdot \quad \text{(fs)}. \\
 & m_0= & \cdot 05.
\end{array}
$$

Arrest of valve at half closure or $m_1=\cdot 025$.

Rate of valve closure $\cdot 10m_0=\cdot 005$ in time $z=2$ (s).

We then find $L/C^2r=1\cdot 322$ and $1/2gf=\cdot 01618$.

The initial conditions are found in the column under $t=\cdot 00$. To this end only the quantities necessary are entered. $M=1/2gf+Lm^2/C^2r$ results as shown, and then $u^2=H\div M$ and $v=mu$.

For $t=z$, $B=O$ and hence $E=av_0$.

M is next found from the value of $m=\cdot 045$ and similarly F.

In this form of computation equation (52) is put in the form (56), and the solution proceeds as indicated, giving a value $h=44\cdot 8$. The remainder of the column then furnishes a check on the numerical accuracy of the work, and also serves to determine E for the next step with $t=2z$.

* See also Sec. 39.

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Thus we find u from the relation $u^2=(H+h)\div M$ (see equation (54)), and v from $v=mu$, s from $s=(v_0-v)$ and aB_n from $aB_n=(h+as)$ (see equation (47)).

In this notation B_n denotes the B for the next step ahead.

Then with these values we find E_n from $E_n=(av_0-B_n)$, where likewise E_n denotes the value of E for the next step ahead. This gives $E_n=905{\cdot}5$, which is then entered as the E for the next step $t=2z$, and thus the computation proceeds.

The intermediate steps for $t=3z$ and $4z$ are omitted and the results then given for $t=5z$ carried through in the same manner.

Then for $t=6z$, m and hence M and F remain the same as for $t=5z$, and thus the computation follows through giving a value of $h=-16{\cdot}67$.

The numerical check referred to is found in the relation $h=a(s-B)$ (see equation (46)). Thus the difference between the as in one column and the aB_n found in the preceding column should equal the value of h. It will be noted that this relation checks out within the nearest tenth, which is as close as the number of places included in the computation will secure.

The full results for the case up to $t=10z$ are shown graphically in Fig. 56*b*.

SAMPLE COMPUTATION

1.	t	·00	z	$2z$	$5z$	$6z$
2.	m	·050	·045	·040	·025	·025
3.	E		995·	905·5	612·9	507·
4.	m^2	·0025	·0020	·0016	·00063	
5.	$1{\cdot}322m^2$	·0033	·00268	·00211	·00082	
6.	·01618	·01618	·01618	·01618	·01618	
7.	M	·01948	·01886	·01829	·01700	·01700
8.	am		5·59	4·969	3·106	
9.	$(am)^2$		31·25	24·69	9·647	
10.	$2M$		·03772	·03658		·03400
11.	F		828·6	675·	283·7	283·7
12.	$H+E$		1495·	1405·5	1112·9	1007·
13.	$H+E+F$		2323·6	2080·5	1396·6	1290·7
14.	$(H+E)^2$		2235025·	1975400·	1238600·	1014050
15.	$(H+E+F)^2$		5399100·	4328400·	1950500·	1665900·
16.	Diff.		3164075·	2353000·	711900·	651850·
17.	$\sqrt{}$		1778·8	1534·	843·75	807·37
18.	$(E+F)$		1823·6	1580·5	896·60	790·70
19.	h	·00	44·8	46·5	52·85	(−)16·67
20.	$H+h$	500·	544·8	546·5	552·85	483·33
21.	$\div M$	25665·	28890·	29880·	32520·	28430·
22.	$u=\sqrt{}$	160·2	170·	172·86	180·33	168·61
23.	v	8·01	7·65	6·914	4·508	4·215
24.	v_0	8·01	8·01	8·01	8·01	8·01
25.	s	·00	·36	1·096	3·502	3·795
26.	as	·00	44·7	136·1	435·1	471·4
27.	aB_n	·00	89·5	182·6	487·95	454·73
28.	av_0	995·	995·	995·	995·	995·
29.	E_n	995·	905·5	812·4	507·	540·27

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Graphical Treatment of Equation (52).—It is seen that equation (52) may be readily put into the form

$$h=(E+F)-\sqrt{(H+E+F)^2-(H+E)^2}\,\ldots\ldots\ldots(56)$$

This form readily lends itself to graphical solution as in Fig. 46. Lay off AB, BC, CD representing respectively H, E and F. Then $AD=H+E+F$. With AC as radius draw the arc CQ. To this draw the tangent DP and then the radius AP. Then $AP=H+E$ and PD=the radical in (56) and swinging over the arc BR from D as centre we have $h=PR$.

Pressure at any Point in the Line.—The discussion thus far has related solely to the pressure at the valve. The same principles

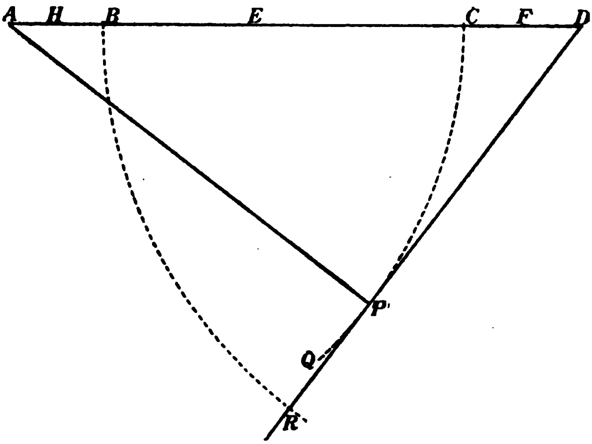

FIG. 46.—GRAPHICAL SOLUTION OF EQUATION GIVING VALUE OF PRESSURE HEAD h.

as developed in Sec. 38 and generalized for any point P at a distance x from the valve, enable us to write down a general equation similar to (42) as follows:

$$h=a(s_{t-i}-s_{t-(z-i)}-s_{t-(z+i)}+s_{t-(2z-i)}+s_{t-(2z+i)}-\text{etc.})^{*}\ldots(57)$$

As in (43) each of these terms represents the summation of those parts of the final result which have had, so to speak, a common life and which thus admit of summation. The first term is the summation of the elements generated at the valve, and which have at least traversed the distance x between the valve and the point P, distant from the valve by the time interval i. The second term is the summation of the elements generated at the valve, and which have at least traversed the distance L to the upper end and back again to the point P, a distance $2L-x$ and requiring a time $(z-i)$. These come back as an unloading of pressure and hence

* In this and subsequent equations, i=time x/S.

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appear with the negative sign. The third term is similarly the summation of the elements which have at least traversed the distance L to the upper end and back again to the valve and back by reflection there to the point P, a distance $2L+x$ and requiring a time $(z+i)$. This likewise operates as an unloading term and hence appears with the negative sign. Similarly the other terms represent the summation of elements corresponding each to a common distance of propagation and approaching P alternately from the upper end and from the valve.

It is of interest to note that when the point P is at the valve and $x=0$, the two unloadings represented by the second and third terms occur simultaneously at the valve, and become by their sum $2s_{t-z}$ or $2s_1$ as in equation (42). In this manner by the assumption of $x=0$ or $i=0$, equation (57) becomes reduced to (42), and the latter is thus seen to be only a special case of the former when $x=0$.

Writing equation (57) for time $(t-z)$ and adding the two values thus found, we have by the cancellation of all terms after the second,

$$h+h_1=a(s_{t-i}-s_{t-(z-i)}) \dots\dots\dots\dots (58)$$

It will be noted that at any given instant of time, s is the same throughout the length of the pipe. Hence the investigation of the pressure at the valve end will give a series of values of s which may be applied throughout the length of the line as indicated in (57) or (58).

To this end we need a general history of s on time, and this may be most conveniently laid down graphically from the results derived for the valve end.

With such a graphical history of s, the determination of h for any point in the line and at any time becomes, through (57) and (58) a matter of simple routine.

Equations (57) and (58) are entirely general and may be thus applied over any time period, with the single interpretation that when any subscript is zero or negative, such term is to be omitted. In this manner (57) could be employed as far as might be desired. If the number of terms becomes large, however, the operation grows in numerical complexity, and in this case the relation expressed in (58) may be employed with advantage.

It is also of interest to note in (57) that if $x=L$, $i=z/2$ and the successive terms become equal in pairs with opposite signs and thus $h=0$ as it should.

Assumption of Uniform Value of dv/dt.—In discussing the problem of shock the assumption is sometimes made of a uniform value of dv/dt, that is, of a uniform retardation in the velocity v. In such case each of the elements dv will be equal, and the value of s up to $t=T$ will always be given by

$$s=nt \dots\dots\dots\dots\dots\dots\dots (59)$$

Where n is the constant value of $-dv/dt$

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Equation (44) for this case becomes

$$h+h_1=anz=\text{constant} \ldots\ldots\ldots\ldots\ldots \quad (60)$$

An examination of (42), remembering that subscripts when 0 or negative imply the omission of the term, shows readily that h starting from 0 increases uniformly up to $t=z$, where it reaches the value anz. It then begins to decrease at the same rate, as is also implied in (60). Consideration of (42) and (60) shows that in such case the graphical history of h would consist of a series of straight line slopes running from 0 to a maximum of anz and then down to 0 again, and with a complete time period of $2z$. This will hold as long as the valve is in movement. After closure and beyond $t=T+z$, we shall have, as before noted, $h=-h_1$. That is, any two values with a time interval z will be equal and apposite in sign. Examination of this part of the history in detail will show that the graph will be as indicated in Fig. 47, where the full lines show the up and down slopes during the period of valve movement, and the dotted lines show the various forms which the graph would take

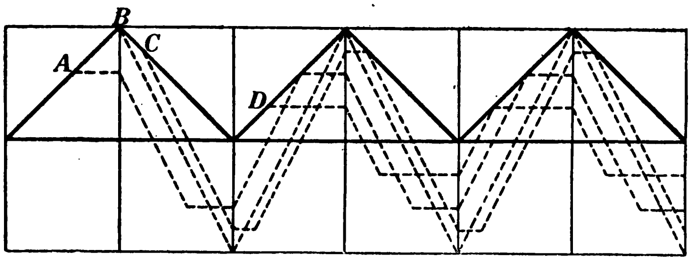

Fig. 47.—History of Pressure Head h assuming Uniform Rate of Velocity Change (Closure).

after closure, dependant on the value at $t=T$. Thus if $T<z$ the graph will follow the dotted line as indicated from A on. If the terminal point when $t=T$ lies on a down slope as at C the graph will follow the dotted line as indicated and similarly for other cases.

These graphs are readily seen to fulfil the following laws:

1. Values separated by a time interval z are equal and apposite in sign.
2. The complete period is $2z$.
3. The graphs will in general have flat tops and sloping sides, the slope of the sides being twice that of the full line part belonging to the period of valve movement.
4. The flat tops mark maximum and minimum values, which are alternately positive and negative in sign and equal in numerical value.
5. The numerical value of the maximum (or minimum) equals the value reached at the instant of valve closure.

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6. If the instant of valve closure is on an up slope, as at A or D, the graph starts off on a flat top at this value of h and continues for the rest of a period z. It then falls until $t=T+z$, and reaches a numerically equal negative value, and so on as shown.
7. If the instant of valve closure is on a down slope as at C, the graph falls during the remainder of a period z, reaching a negative value equal numerically to the value at valve closure, and then starts off with a constant value of h, which it holds until $t=T+z$, when it rises and follows its course as shown.

It is of interest to note that if the instant of valve closure occurs at the end of an odd number of periods z, bringing the point to the top of one of the slopes, the graph will continue as a series of right line slopes implying a series of positive and negative extreme values equal to anz.

If, on the other hand, the instant of valve closure occurs at the end of an even number of periods z, bringing the point to the bottom of one of the slopes where $h=0$, the value of h will continue to hold zero as a constant value, and the excess pressure head in this case disappears with the closure of the valve and remains 0 indefinitely thereafter.

While this analysis of the conditions resulting from an assumed uniform value of dv/dt has an interest as a part of the general problem, and especially as illustrating the effective manner in which the equations above deduced serve to determine a wide diversity of results according to changing conditions, it must be recognized that the fundamental condition of uniform retardation is one not likely to be met with in actual practice.

39. Gradual Partial Closure, Time as in Section 38

The discussion of Sec. 38 has assumed full closure as the final condition. If instead the valve remains partly open, we have in effect an arrest of valve movement at a time T followed by a constant value of m and such values of h, s, v, u as may develop. We have here to distinguish two periods:

1. The period from $t=0$ to $t=T$, when the valve movement is arrested.
2. The period subsequent to $t=T$.

Obviously the condition for the first period is the same as for the corresponding part of a case of assumed full closure with the same rate of valve movement up to $t=T$, and as treated in Sec. 38.

For the second period we shall have relations between h, s, v, u

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expressed by the same general equations as before but now with a constant value of m.

The general procedure for finding the time history of h during this period is therefore the same as described above for the case of full closure, but with a constant value of m. This will give an entirely different course to the sequence of values. Often the graph of h during this period will show a series of excursions positive and negative of decreasing amplitude and gradually approaching 0, while the values of v and u will approach those for steady conditions under the given fixed value of m. The general character of such graphs is given in Figs. 54–56. In some cases, as determined by the characteristics, the return of h to the zero value may be by gradual decrements without oscillation through positive and negative values (see Fig. 54). While the equations would, furthermore, indicate an indefinitely long period of time for the attainment of final steady conditions, the influence of viscosity and the failure to realize reflection as assumed, will rapidly obliterate the oscillations of pressure and reduce the values to those for final steady flow conditions. The rapidity of return to zero, or to a negligible value, also varies in marked degree with the characteristics of the case (see Figs. 55, 56).

Naturally, in dealing with partial closure, any law of reflection at the valve end may be assumed, either full reflection or partial, according to the various methods of estimate as discussed in Sec. 40.

40. Partial Reflection at Valve (Closure)

The treatment thus far has assumed full reflection of the pressure wave at the valve, a condition which, as we have already seen, will be only imperfectly realized in practice. It becomes, therefore, of interest to examine the results of an assumption of partial rather than full reflection at the valve.

There are at least four assumptions which may be taken as a basis for the specification of a partial reflection, as follows:

1. A constant fraction of full reflection.
2. A fraction of full reflection measured by the ratio $(m_0-m)/m_0$. That is, reflection in proportion as the valve opening is reduced in area from full opening to closure.
3. A fraction of full reflection measured by the ratio $(A-a)/A$. That is, reflection in proportion as the cross section area of the pipe A is closed over at the valve end.
4. A fraction of full reflection measured by the ratio $\frac{s}{v_0}$. That is, reflection in proportion to the decrease in the velocity of the water.

We shall briefly indicate the results as developed from these various assumptions regarding the degree of reflection realized.

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Case 1.—Let f denote the constant fraction. Then equation (42) will become

$$h=a(s-(1+f)s_1+f(1+f)s_2-f^2(1+f)s_3+\text{etc.})\ldots\ldots(61)$$

The second term represents at time t an unloading s_1 with reflection fs_1. The third term develops first at time $(t-z)$ as an unloading s_2 with reflection fs_2 and then at time t as a second unloading fs_2 with second reflection f^2s_2, giving by the sum of the two latter the term as written ; and similarly for subsequent terms.

Denote all terms in the parenthesis after the first by B we then have

$$h=a(s-B)\ldots\ldots\ldots\ldots\ldots\ldots(62)$$

With the make-up of (61) we then readily find, in a manner similar to that followed with full reflection the following relations :

$$h+fh_1=a(s-s_1)\ldots\ldots\ldots\ldots\ldots\ldots(63)$$

$$aB=fh_1+as_1\ldots\ldots\ldots\ldots\ldots\ldots(64)$$

It will be noted that these equations all reduce to the forms for full reflection, as in (42), (44), (47) if we put $f=1$.

We may then proceed, exactly as indicated for full reflection, using equations (52), (53), (54) but with the value of B as in (64).

Case 2.—We shall in this case have for f a varying value given by

$$f=\frac{m_0-m}{m_0}$$

There will be, therefore, a value of f for each instant of time during the closure, and in particular a series of values for the instants t, $t-z$, $t-2z$, etc. These we may denote by f, f_1, f_2, etc., the same as for the series of values of h, s, etc.

We shall have, then, instead of (61) the equation :

$$h=a[s-(1+f)s_1+f_1(1+f)s_2-f_1f_2(1+f)s_3+\text{etc.}]\ldots(65)$$

The second term represents at time t an unloading s_1 plus a reflection fs_1. The third term develops first at time $(t-z)$ as an unloading s_2 with reflection f_1s_2 and then at time t as a second unloading f_1s_2 with second reflection ff_1s_2, giving by the sum of the two latter the term as written ; and similarly for the other terms.

Denote, as before, everything within the brackets after the first term by B. Put also

$$P=s_1-f_1s_2+f_1f_2s_3-\text{etc.}\ldots\ldots\ldots\ldots(66)$$

Then $P_1=s_2-f_2s_3+\text{etc.}\ldots\ldots\ldots\ldots\ldots\ldots(67)$

Whence we derive

$$P+f_1P_1=s_1$$

or $P=s_1-f_1P_1\ldots\ldots\ldots\ldots\ldots\ldots(68)$

We have then for (65) $h=a(s-B)\ldots\ldots\ldots\ldots\ldots\ldots(69)$

and from the composition of (65), (66) it is seen that

$$B=(1+f)P\ldots\ldots\ldots\ldots\ldots\ldots(70)$$

We also readily derive the relation

$$h+h_1=a(s-fs_1-P_1(1-ff_1))\ldots\ldots\ldots\ldots(71)$$

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It will be noted that (65) and (71) reduce to the forms for full reflection if we put $f=1$.

We may now proceed as described for the case with full reflection finding h from (52) with the value of B as given by (70), (68) and finding s and v from (69) or (71).

Case 3.—In this case the resulting fundamental equation is the same as (65) for Case 2, but, with different values of f, f_1, etc., we shall have here:

$$f=\frac{A-a}{A}=1-\frac{a}{A}=(1-m) \quad \text{.............}(72)$$

With a prescribed program of movement for the valve, therefore, f becomes known for any instant of time and we may then use the equations of Case 2 but with the appropriate values of f.

Case 4.—In this case we have for valve closure

$$f=\frac{s}{v_0} \quad \text{......................}(73)$$

and hence for the series f, f_1, f_2, etc., we shall have s/v_0, s_1/v_0, s_2/v_0, etc.

We may, therefore, use the same general equations as for Case 2 except that in this case we do not know, for any given time t, the value of f, since this depends on v and this in turn on u or h. This implicit relation does not, however, introduce any new variable into the equations and we proceed by taking equations (33), (36), (69) with the special values of B and f as given in (70) and (73). From these equations, and substituting for s in terms of v, we eliminate v, u and s, and derive, as in Sec. 36, the value of h,

$$h=(E+F)-\sqrt{F^2+2F(H+E)} \quad \text{...........}(74)$$

where $E=a(v_0-2P)$

and $F=\frac{(am)^2}{2M}\left(1-\frac{P}{v_0}\right)^2$

the same in form as in Sec. 36 and in the previous cases, and differing only in the values of E and F.

We may, therefore, follow through step by step first using (39) for $t<z$, and to determine the initial values of P, and then using these results in (74) for the determination of h, u, v and s for times between z and $2z$, and then using the latter values to determine those for the next interval z, and so on as previously indicated for the case of full reflection.

As between these four bases for estimating the degree of reflection realized, we have little experimental evidence as a guide. There is no reason for assuming a constant value of f, but a constant value somewhat less than 1 would presumably give more accurate results than to assume it constant at 1, as with the assumption of complete reflection.

For a case where the open valve area is the full size of the pipe,

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assumptions 2 and 3 become the same, and in such case assumption 4 will differ widely from these in the values of f through the period of closure.

For a case where the open valve area is small relative to the cross section area of pipe, assumption 3 will imply practically complete reflection, while in such case assumptions 2 and 4 will more nearly agree.

41. Gradual Opening: Time Long Relative to Time $2L/S$ for Double Traverse of Acoustic Wave

The treatment for the gradual opening of a valve is, in principle, contained in that of Sec. 38 for closure. We shall have the same relations as expressed in the fundamental equations of that section but with the following interpretations:

The change of head h is now a decrease instead of an increase and is essentially subtractive instead of additive.

The change of velocity s will be reversed in direction; that is we shall have

$$s=v-v_0$$

instead of (28) and if the opening starts from complete closure we shall have

$$s=v.$$

With this understanding we may, parallel to (36), (51), (33), write the fundamental equations as follows:

$$Mu^2=H-h$$
$$h=a(v-v_0-B)$$
$$v=mu$$

Combining these as for the case of closure, we derive the equation for h in the same general form

$$h^2+2h(E+F)+E^2-2HF=0$$

$$\text{or}\quad h=-(E+F)+\sqrt{F^2+2F(H+E)} \qquad (75)$$

$$\text{Where}\quad E=av_0+aB$$

$$F=\frac{(am)^2}{2M}$$

and $aB=h_1+as_1$ as before.

If the valve movement starts from full closure, we shall have $v_0=0$ and $E=aB$.

It follows naturally that this general method of treatment may be applied to the determination of the value of h at any point in the line and assuming any law of reflection at the valve end, the same as for closure and as developed in detail in Sec. 38.

With the proper definition of the quantities involved and with the proper interpretation of the results, therefore, the entire treatment developed in Sec. 38 may be directly applied to the case of opening as well as closure.

It must, however, be remembered that the total pressure head at any point in the line cannot become negative and hence, any

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result implying a final absolute pressure negative in value will imply rather a break in the water column, turbulent conditions and a state of affairs generally where the physical basis assumed for the treatment of Sec. 38 no longer exists.

In (75), if we put $h=0$ and reduce, we find

$$E^2=2FH.$$

This is then the condition that $h=0$. If $E^2>2FH$, h is negative, in this case implying an excess pressure. If $E^2<2FH$, h is positive, implying here a lowering or decrease of pressure.

In some cases F may be large compared with $(H+E)$ and hence $(H+E+F)$ large compared with $(H+E)$. In such case an approximate value of the radical (see equation (56)) will be $(H+E+F)-(H+E)^2/2(H+E+F)$. This will give

$$h=H-\frac{(H+E)^2}{2(H+E+F)}$$

With F large and E zero or small (opening from complete closure or near closure) the term following H will be small and hence the drop in head h at the time of arrest of valve movement will be nearly the entire head H, leaving the net pressure head represented only by the small term $(H+E)^2/2(H+E+F)$, or in other words, reducing the absolute pressure head nearly to that due to the atmosphere.

This condition will be approximately realized for all values of F equal to or greater than eight to ten times $(H+E)$ or for

$$F \gtreqless (8 \text{ to } 10)\,(H+E)$$

with a closer and closer approach to the limit $h=H$, as the ratio of F to $(H+E)$ exceeds these lower values.

The same as in the case of closure the general problem of opening includes three time periods:

1. t between 0 and z
2. t between z and T
3. t beyond T.

First Time Period. $t=0$ to $t=z$.

For this period, $B=0$ and the value of E reduces to av_0. Further, if the movement is from complete closure, $v_0=0$, and we have

$$h=-F+\sqrt{F^2+2FH}\ldots\ldots\ldots\ldots\ldots(76)$$

This gives, then, the value of h where the valve movement is completed within the time z.

It is of interest to compare this with the value of h for closure through the same range of valve movement and within the same time.

Let m and v denote the final steady motion values at the end of opening or at the beginning of closure. Then substituting for F its value $(am)^2/2M$ and noting that H/M=steady motion u^2 and that m^2u^2=steady motion v^2, it is readily shown that $2FH=(av)^2=E^2$.

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Then designating the two values of h by subscripts c and o for closure and opening we have:

$$h_c = av \qquad (77)$$

$$h_o = \sqrt{F^2 + (av)^2} - F \qquad (78)$$

and from these we readily derive the relation

$$h_c^2 = h_o^2 + 2h_oF \qquad (79)$$

In the general problem of valve opening we may have, for the initial phenomena, two cases according as T is greater or less than z.

(*a*) T greater than z.

In this case, from what precedes, it is clear that we shall normally expect a rapid drop at the start; either reaching $h=H$ practically, and holding such value up to $t=z$ or approaching such value more or less closely as determined by the characteristics of the case and in particular by the relative values of F and $(H+E)$.

(*b*) T less than z.

In this case we shall have the same general trend of values as in (*a*) except that whatever value is realized at $t=T$ will he held uniform until $t=z$.

Second Time Period. $t=z$ to $t=T$.

For this period the general equation (75) must be employed. The course of the values of the net pressure head will show a return upward from the low value reached at $t=z$ followed by a course which will be determined by the circumstances of the case and which will be illustrated at a later point.

Third Time Period. $t=T$ onward.

For this period the fundamental relations expressed in equation (75) still hold, but with the special condition m=const. This will simplify somewhat the operations involved, but in general this equation must be used in order to determine the course of the pressure from $t=T$ until substantially steady conditions are realized. It may be noted, of course, that in case T should be less than z the value of h will remain constant at its value for $t=T$ until $t=z$, after which equation (75) with m=const. will begin to apply.

The general course of h during this third period will show, normally, a series of fluctuations gradually bringing the net pressure head to the value for final steady flow conditions corresponding to the given amount of valve opening.

Illustrative cases will be noted at a later point.

Partial Reflection at Valve (Opening).—The discussion of Sec. 40, with the equations developed, applies without change to the case of valve opening, except that in Case 2 the value of f will be $(m_1-m)/m_1$ where m_1 is the ultimate value of m, while in Case 4 we shall have

$$f = \frac{v_1 - v}{v_1}$$

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Where v_1=ultimate steady motion velocity with $m=m_1$

$$E=a(v_0+2P)$$

$$F=\frac{(am)^2}{2M}\left(1+\frac{P}{v_1}\right)^2$$

42. Discussion of Formulæ of Sections 38–41 with Numerical Cases*

Closure. Time of Valve Movement T Equal to or Less than Time z for Double Traverse of Acoustic Wave.—This case is treated in Sec. 36. Its treatment is also, of course, contained within that of Sec. 38.

The maximum value of h is always found at the instant of complete closure and is measured by

$$h_m=av_0.$$

If the closure is only partial the maximum value of h is always found at the end of valve movement and is measured by

$$h_m=as=a(v_0-v).$$

It will, however, require a solution of the equation (52) in order to determine the value of h or v.

The course of the pressure rise in various cases has been discussed in Sec. 36 and in connection with the diagrams of Fig. 48.

Closure. Time of Valve Movement T Greater than Time z for Double Traverse of Acoustic Wave.—This case is the one most likely to arise in practice. A large number of numerical cases have been worked through and from which the diagrams of Figs. 49–56 have been prepared.

During the first period from $t=0$ to $t=z$, the course of the curve is rising, the same as for the corresponding part of one of the curves of Fig. 48.

* In Figs. 48–86 showing pressure and velocity change for various cases of valve closure or opening, the following characteristics have been assumed, for convenience, uniform in value:

$D=1$(f).

Chézy coefficient $C=110$, and hence

$C^2r=3025$.

Coefficient efflux $f=\cdot 96$, and hence

$1/2gf=\cdot 016177$.

Velocity $S=4000$ (fs).

Note should also be made that in the formulæ relating to closure (equation (52), etc.) h positive implies *excess* of pressure, while in the formulæ relating to opening (equation (75), etc.) h positive implies *defect* of pressure or depression. Also in all diagrams relating to closure, h positive is laid off upward while in those relating to opening, h positive is laid off downward.

On certain diagrams, the heavy dot calls attention to the point where $t=z$ and the circle to the point of arrest of valve movement.

Full lines refer to pressure head h.

Dotted lines refer to velocity v.

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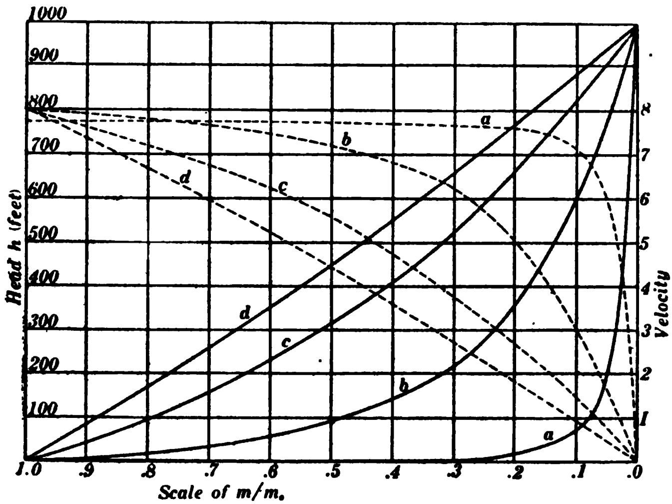

Fig. 48.—History of Pressure Head h and Velocity v (Closure).

	L	H	m_0	m_1	z	T
Case a	4000	80	1·00	·00	2·0	z or less
,, b	,,	130	·15	·00	,,	z ,,
,, c	,,	1500	·05	·00	,,	z ,,
,, d	,,	2700	·02	·00	,,	z ,,

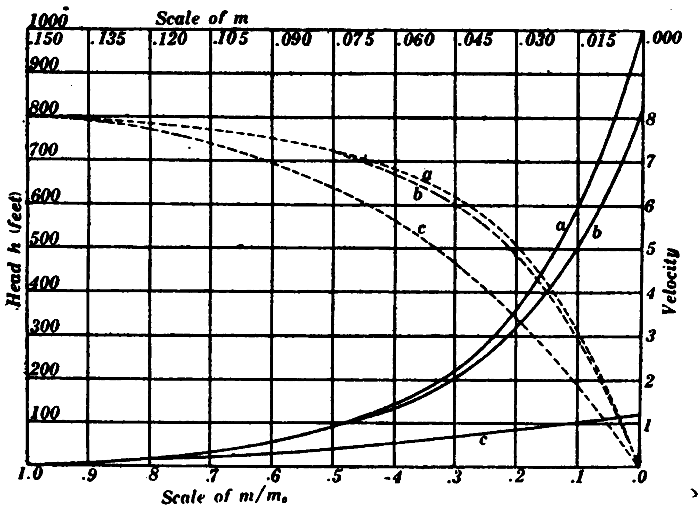

Fig. 49.—History of Pressure Head h and Velocity v (Closure).

	L	H	m_0	m_1	z	T
Case a	4000	130	·15	0	2·0	z
,, b	,,	,,	·15	0	,,	$2z$
,, c	,,	,,	·15	0	,,	$10z$

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From this point on the course will depend on the time T in relation to z and on the hydraulic characteristics of the case.

The more typical cases are as follows:

(*a*) The curve may continue on reproducing very nearly the form of Fig. 48*a*, but reaching a value of h somewhat lower at the point of complete closure.

Fig. 49*b* represents a case of this type.

(*b*) The curve may continue on rising gradually, but with a more decided break at the point where $t=z$ and reaching a maximum value at complete closure considerably less than in the case where $T<z$.

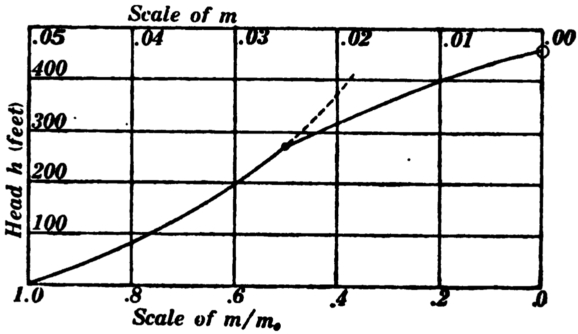

FIG. 50.—HISTORY OF PRESSURE HEAD h (CLOSURE).

L	H	m_0	m_1	z	T
20,000	850	·05	0	10	$2z$

Dotted line shows continuation for Case $T=z$

Figs. 49*c*, 50 represent cases of this type.

(*c*) The curve may continue nearly horizontal, or with a nearly constant value of h, until the closure is complete. In some cases there may be a slight gradual rise, or again a gentle rise to a maximum followed by a slight decline, or again an immediate sharp decline.

Figs. 51*b*, *c*, 52*b* represent cases of this type.

(*d*) The curve may break into a series of slopes up and down, giving alternate maxima and minima, and implying a periodic fluctuation in the pressure with a total period of $2z$. Again, the general trend of such a history may be gradually up or down or practically horizontal.

Fig. 52*c* represents a case of this type.

Where the head H is mostly used up in friction, as in the case of Figs. 48*a*, 49*a*, a moderate lengthening of the time T will give a result similar to that of Fig. 49*b*. That is, the same general character will be retained with a sharp increase in h during the last moments

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of closure, but reaching a final value less and less as T is longer and longer. The important point here is that the maximum value is not reached until complete closure and determination through any form of computation on the basis of the theory of Sec. 38 must necessarily proceed according to the methods outlined in that section, and no approximate formula based on an assumed form of time history entirely different in character can be expected to give results having any rational relation with those furnished by the more complete theory.

Passing to the other extreme in a case where only a very small part of the head H is used in friction, as in Fig. 52, and where the

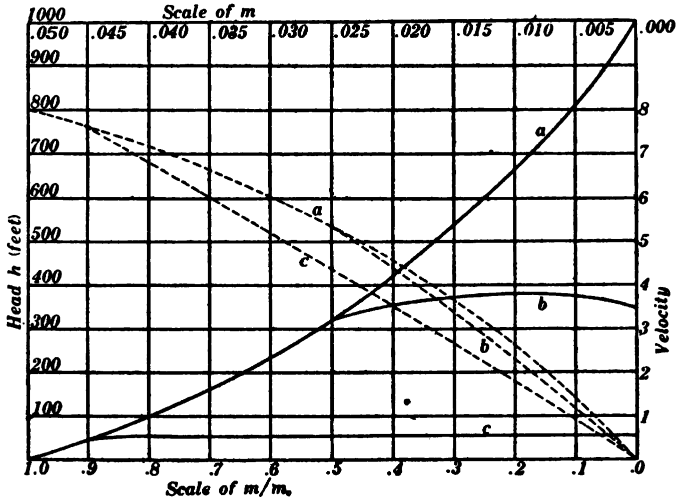

FIG. 51.—HISTORY OF PRESSURE HEAD h AND VELOCITY v (CLOSURE).

	L	H	m_0	m_1	z	T
Case a	800	430	·05	·00	·4	z
,, b	,,	,,	·05	·00	,,	$2z$
,, c	,,	,,	·05	·00	,,	$10z$

curve for $T \gtrless z$ is nearly linear, the result of increasing T will be to give a periodic or fluctuating form of curve as in c. For intermediate conditions the curve will assume intermediate forms, as in Figs. 50, 51.

In most cases of a break-up of the curve into periodic fluctuations with the period $2z$, the maximum value of h will not greatly exceed that for the instant when $t=z$. Hence in such cases a good approximation to the maximum value will usually be given by solving equation (52) with the value of m when $t=z$.

The same will be true for cases where the curve continues practically at a constant value of h, or with only a slight rise as in Fig. 51b, c.

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For cases such as those of Figs. 49, 50, however, the value of h continues to rise to the end, and the value for $t=z$ will not give a proper indication of the final or maximum value reached.

A detailed analysis of equation (52) would serve to give certain indications regarding the course of the curve beyond the point where $t=z$, and therefore as to whether the value of h for this point might be taken as a reasonable maximum. Such analysis, however, is complex and hardly more useful than the direct trial of one or two points. Such a trial will usually serve to give some

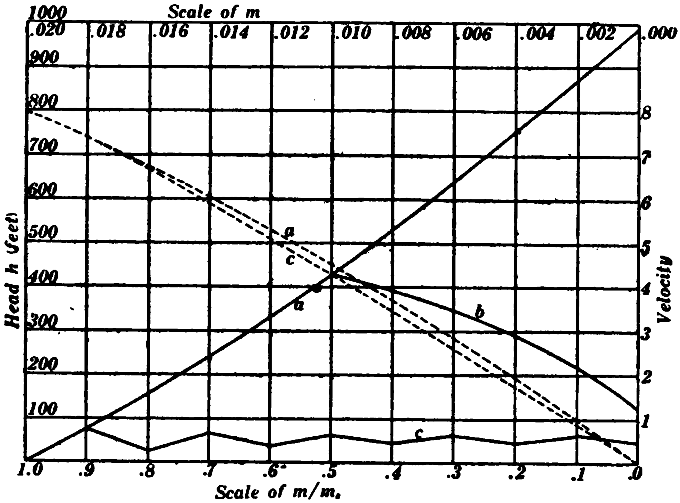

FIG. 52.—HISTORY OF PRESSURE HEAD h AND VELOCITY v (CLOSURE).

	L	H	m_0	m_1	z	T
Case a	20,000	3000	·02	·00	10	z
„ b	„	„	·02	·00	„	$2z$
„ c	„	„	·02	·00	„	$10z$

The velocity curve for Case b differs so slightly from that for Case a that no attempt is made to show the two lines separately.

indication of the future course of the curve and hence of the probable location of the maximum value of h.

It will also be found, if the curve has a flat top or nearly uniform value of h from $t=z$ onward, that the Allievi formula (81) or the mass-acceleration formula (95) will give a good approximation to the value.

The main question remains as to the form of the curve, and whether any such approximate formulæ will apply with reasonable accuracy. The simplest way of answering this question will usually be through the actual examination of the curve itself by equation (52), and this will in itself give directly the values sought, if carried to the point of maximum value of h.

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The time required for a computation of this character will not usually exceed a period reasonably permissible in such a study. After becoming accustomed to the work it will be found that with the aid of slide rule and table of squares and square roots a single point can be comfortably determined in about ten minutes, and thus a curve for $T=10z$, or say twenty seconds for a line 4000 feet long, in a couple of hours or less. If the solution is carried out graphically through the method of Fig. 46 the time will be reduced, though naturally with some loss of accuracy.

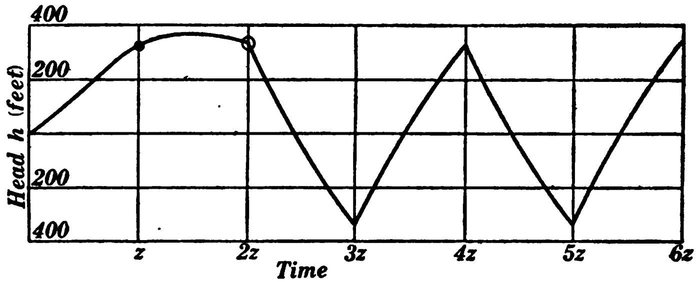

FIG. 53.—HISTORY OF PRESSURE HEAD h SHOWING COURSE AFTER FULL CLOSURE.

L	H	m_0	m_1	z	T
800	430	·05	·00	·40	$2z$

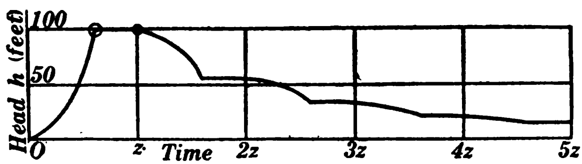

FIG. 54.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE MOVEMENT AT $\cdot 6z$ (CLOSURE).

Rate of valve closure same as for complete closure in time z.

L	H	m_0	m_1	z	T
800	42	·20	·08	·40	$\cdot 6z$

Course of History of h after Arrest of Valve Movement.—After arrest of the valve at full closure, the curve of h, as indicated in equation (50), will fluctuate between plus and minus values equal to the value reached at the instant of closure. Such a course is indicated in Fig. 53. As previously noted, secondary disturbances not included in the theory will ultimately reduce the amplitude to a negligible amount.

In the case of arrest of valve movement at partial opening, the value of h will return to zero, either by a series of stepwise slopes or by alternate fluctuations, as indicated in Figs. 54, 55, 56.

Valve Opening from Complete Closure. Time T Equal to or Less than Time z for Double Traverse of Acoustic Wave.—In this case

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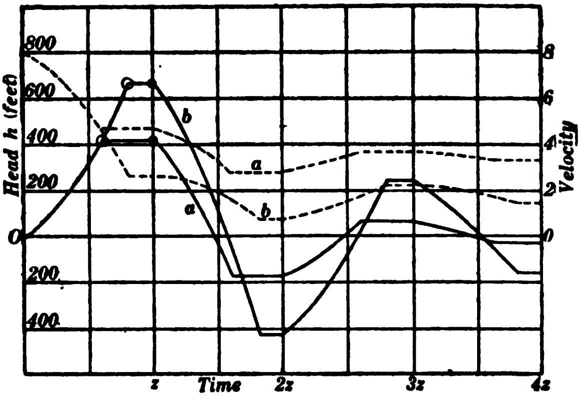

FIG. 55.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE MOVEMENT (CLOSURE).

Rate of valve closure same as for complete closure in time z.

	L	H	m_0	m_1	z	T
Case a	4000	500	·05	·02	2·0	·6z
,, b	,,	,,	·05	·01	,,	·8z

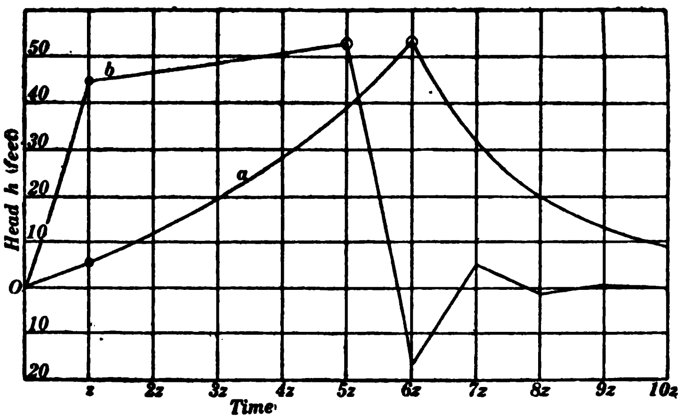

FIG. 56.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE MOVEMENT (CLOSURE).

Rate of valve closure same as for complete closure in time 10z.

	L	H	m_0	m_1	z	T
Case a	800	42	·20	·08	·40	6z
,, b	4000	500	·05	·025	2·00	5z

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there will be a continuous drop in pressure (increase in value of h), reaching a minimum at the close of the movement when $t=T$.

Where the ultimate opening is nearly or quite the full size of the pipe and hence the friction head a large part of the total head H and with F large relative to H, the drop will be abrupt and will reach down nearly to atmospheric pressure. Where the ultimate

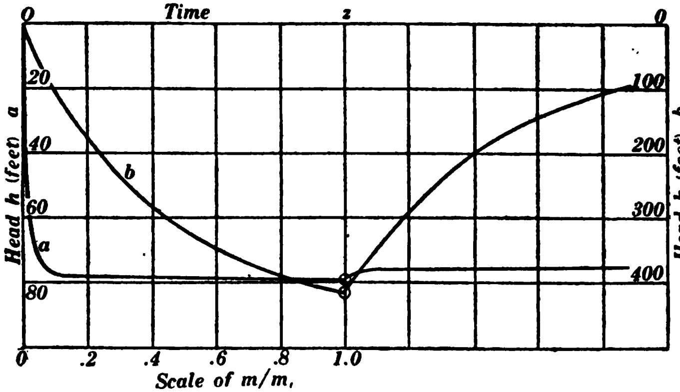

FIG. 57.—HISTORY OF PRESSURE HEAD h (OPENING).

	L	H	m_0	m_1	z	T
Case a	4000	80	·00	1·00	2·00	z
„ b	„	500	·00	·05	„	z

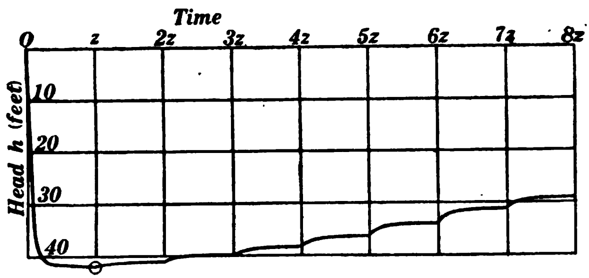

FIG. 58.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

L	H	m_0	m_1	z	T
800	42	·00	·20	·40	z

opening is small relative to the full size of pipe the drop will be more gradual and the minimum will be higher. That is, the ultimate values of h will approach more and more nearly to H as the drop is more abrupt, in accordance with the conditions noted above, and contrariwise in opposite cases. These characteristics are shown in Fig. 57.

Following the minimum value reached when $t=T$, the pressure

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head h will return toward zero, the course of the history depending on the circumstances of the case. Three typical courses may be noted.

(*a*) The head h will return toward and ultimately to zero by way of a series of gentle stepwise lifts, as shown in the case of Fig. 58. In an actual case, due to the damping-out effects of friction and other secondary causes, such a case would develop as a practically smooth gentle up-slope gradually approaching the axis of zero pressure.

(*b*) The head h will return toward and ultimately to zero by way of a series of more definitely marked stepwise lifts, as shown in the cases of Figs. 57*b*, 59, 60*a*.

(*c*) The head h will show a series of alternations on either side of the axis, as in the case of Figs. 60*b*, 61, 62. This means an alternation of value, plus and minus, with diminishing amplitude and gradual approach to ultimate zero value.

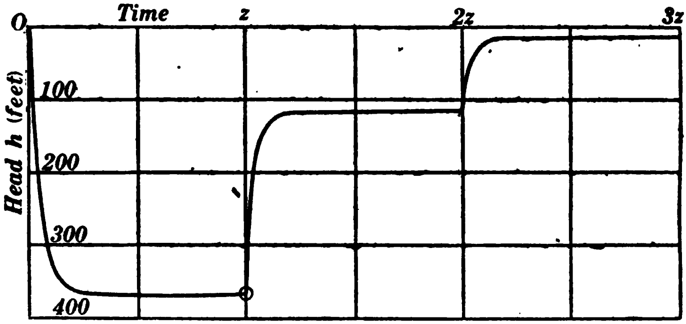

FIG. 59.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

L	H	m_0	m_1	z	T
20,000	425	·00	1·00	10	z

In cases where the ultimate opening is nearly or quite the full size of pipe and the friction head forms the larger part of the total head H, while at the same time H itself is moderate or small, the form of the return will be similar to that of Fig. 58. In similar cases, but where H itself is large (implying L large), the form will be similar rather to those of Figs. 57*b*, 59.

In cases where the ultimate opening is small compared with the size of pipe and where the friction head forms but a small part of the total head H, but where H itself is large (implying L large), the return will be by way of alternating plus and minus values (type c), as in Fig. 60*b*. In similar cases, but where H is moderate or small, the return may be by way of type (*a*) or (*b*), as the values involved may determine.

Intermediate cases will depend on the values involved. Generally let m_1 be the final value m for full or ultimate opening. Then the

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smaller the value of H/m_1 the more definitely will the return be by way of a curve of type (*a*), while the larger the value of H/m_1 the more definitely will the return be by way of a curve of type (*c*), while type (*b*) will be found for intermediate values.

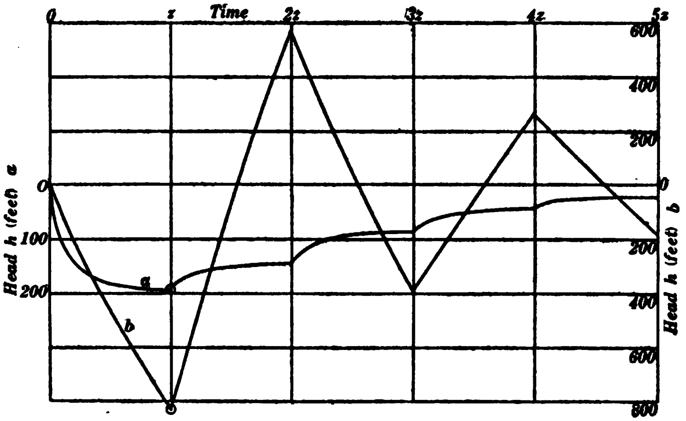

FIG. 60.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

	L	H	m_0	m_1	z	T
Case a	4000	200	·00	·10	2·0	z
„ b	„	2700	·00	·02	„	z

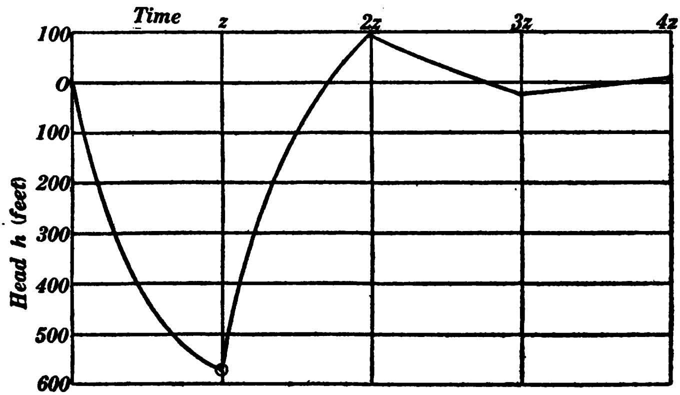

FIG. 61.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

L	H	m_0	m_1	z	T
20,000	850	·00	0·5	10	z

Actually there is no sharp line of demarcation between these various cases, and, geometrically, one type of curve shades into another by insensible gradations.

A detailed and somewhat complex analysis of the equations

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involved would make possible the laying down of more definite criteria regarding the type of curve to be expected in any particular case. This, however, does not seem justifiable in the present work.

Maximum value of h. In the present case, as noted above, the maximum value of h is found at the end of the valve movement or when $t=T$. It may therefore be computed from equation (75) by substitution of the proper values.

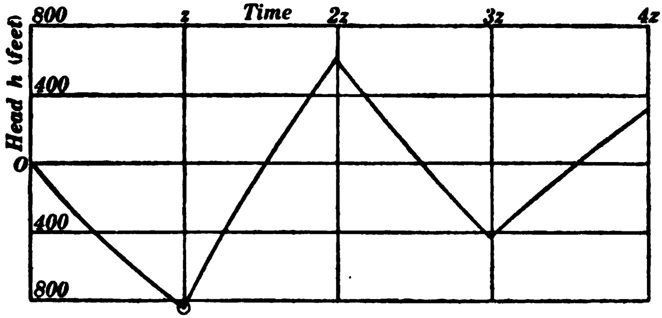

FIG. 62.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

L	H	m_0	m_1	z	T
20,000	3000	·00	·02	10	z

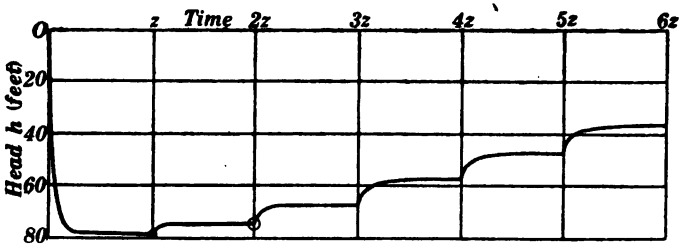

FIG. 63.—HISTORY OF PRESSURE HEAD h AFTER ARREST OF VALVE (OPENING).

L	H	m_0	m_1	z	T
4000	80	·00	1·00	2·0	$2z$

Valve Opening from Complete Closure. Time T Greater than Time z for Double Traverse of Acoustic Wave.—In this case we may note the three periods as before :

1. From $t=0$ to $t=z$ the pressure will drop continuously following, for that part of the opening up to $t=z$, the same law as in the case of $T=z$. At the instant $t=z$ the maximum value of h or greatest drop in pressure will be reached.
2. From $t=z$ to $t=T$ the curve will rise either in periodic lifts or following a series of alternations about a mean value

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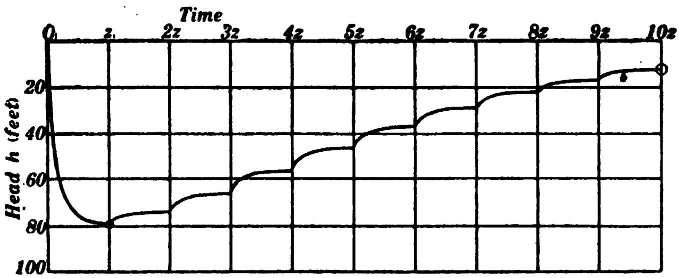

FIG. 64.—HISTORY OF PRESSURE HEAD h WITH INCREASE IN VALUE OF T (OPENING).

L	H	m_0	m_1	z	T
4000	80	·00	1·00	2·0	10z

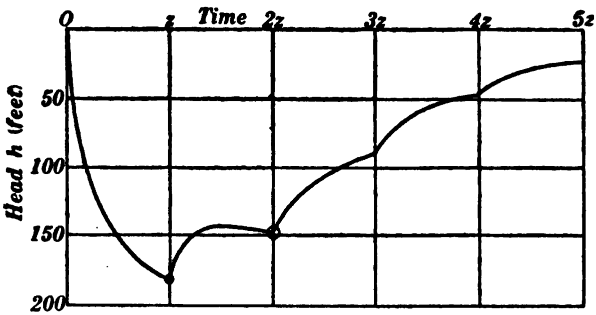

FIG. 65.—HISTORY OF PRESSURE HEAD h (OPENING) $T > z$.

L	H	m_0	m_1	z	T
4000	200	·00	·10	2·0	2z

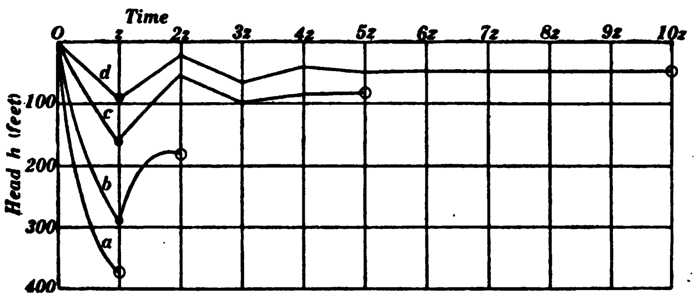

FIG. 66.—HISTORY OF PRESSURE HEAD h (OPENING). INFLUENCE OF INCREASING T.

	L	H	m_0	m_1	z	T
Case a	800	430	·00	·05	·40	z
,, b	,,	,,	·00	·05	,,	2z
,, c	,,	,,	·00	·05	,,	5z
,, d	,,	,,	·00	·05	,,	10z

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holding a general trend nearly horizontal or slightly upward. Here again a small value of H/m_1 will normally determine a form similar to that of Fig. 58 or in Fig. 63 for the time period z to $2z$, or again in Fig. 64. On the other hand a large value will, for the same time of opening,

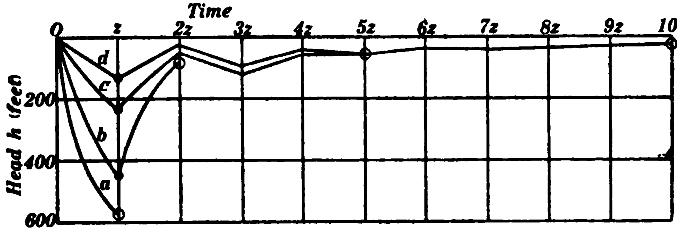

FIG. 67.—HISTORY OF PRESSURE HEAD h (OPENING). INFLUENCE OF INCREASING T.

	L	H	m_0	m_1	z	T
Case a	20,000	850	·00	·05	10·	z
,, b	,,	,,	·00	·05	,,	$2z$
,, c	,,	,,	·00	·05	,,	$3z$
,, d	,,	,,	·00	·05	,,	$4z$

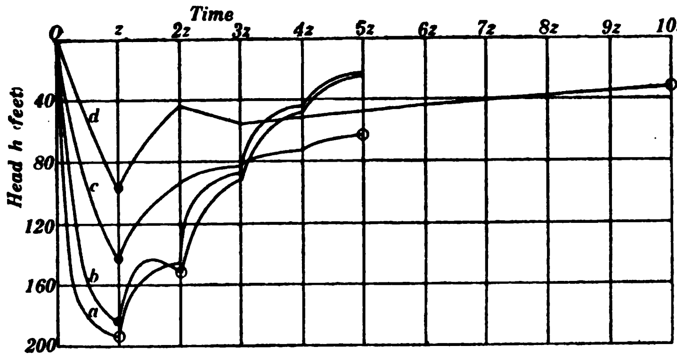

FIG. 68.—HISTORY OF PRESSURE HEAD h (OPENING). INFLUENCE OF INCREASING T.

	L	H	m_0	m_1	z	T
Case a	4000	200	·00	·10	2·0	z
,, b	,,	,,	·00	·10	,,	$2z$
,, c	,,	,,	·00	·10	,,	$5z$
,, d	,,	,,	·00	·10	,,	$10z$

determine forms showing marked periodic lifts or alternations about a mean value, either nearly uniform or slightly rising (see Figs. 65–70). As shown by the diagrams these cases shade insensibly the one into another.

3. For t beyond T, the value of h will return to 0 either by periodic lifts or by alternate plus and minus values similar to those

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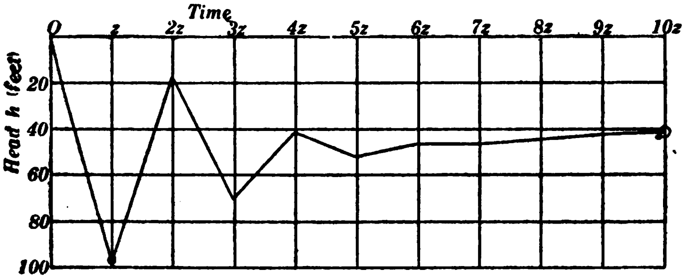

FIG. 69.—HISTORY OF PRESSURE HEAD *h* (OPENING).

L	H	m_0	m_1	z	T
4000	500	·00	·05	2·0	10z

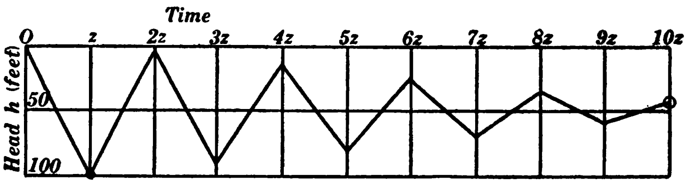

FIG. 70.—HISTORY OF PRESSURE HEAD *h* (OPENING).

L	H	m_0	m_1	z	T
4000	2700	·00	·02	2·0	10z

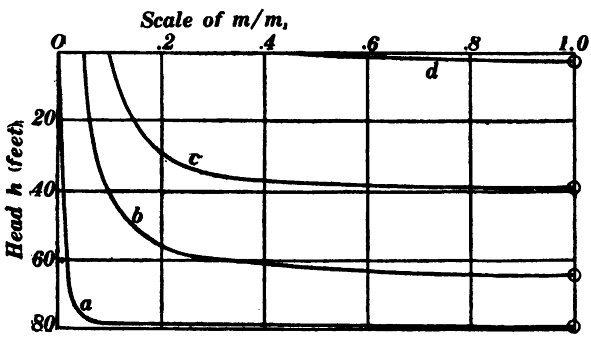

FIG. 71.—HISTORY OF PRESSURE HEAD *h*, STARTING FROM PARTIAL OPENING.

Rate of Valve Opening the same in all cases.

	L	H	m_0	m_1	z	T
Case a	4000	80	·00	1·00	2·0	z
,, b	,,	,,	·05	1·00	,,	·95z
,, c	,,	,,	·10	1·00	,,	·90z
,, d	,,	,	·50	1·00	,,	·50z

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of Figs. 57–62, for return after $T=z$. In general the return is without alternation of sign except for large values of the ratio H/m combined with values of T only moderately greater than z (see also Figs. 63, 65).

Maximum value of h. In this case the maximum value of h

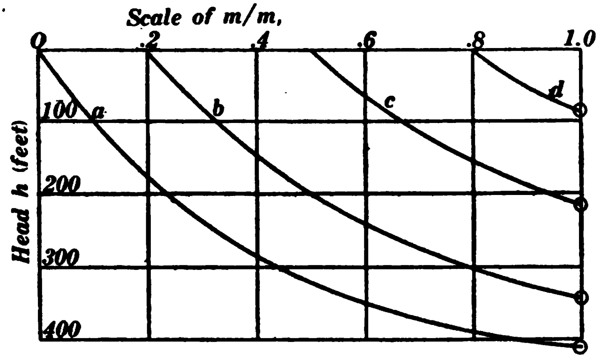

FIG. 72.—HISTORY OF PRESSURE HEAD h, STARTING FROM PARTIAL OPENING.

Rate of Valve Opening the same in all cases.

	L	H	m_0	m_1	z	T
Case a	4000	500	·00	·05	2·0	z
„ b	„	„	·01	·05	„	·8z
„ c	„	„	·025	·05	„	·5z
„ d	„	„	·04	·05	„	·2z

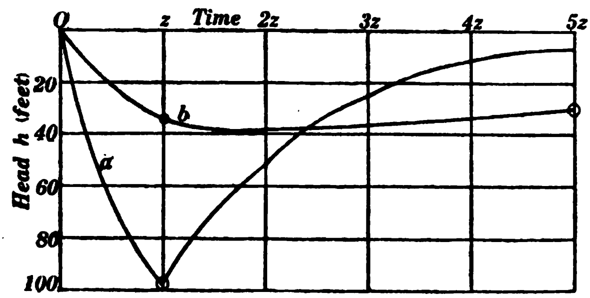

FIG. 73.—HISTORY OF PRESSURE HEAD h, STARTING FROM HALF OPENING.

	L	H	m_0	m_1	z	T
Case a	4000	200	·05	·10	2·0	z
„ b	„	„	·05	·10	„	5z

(drop in pressure) is found when $t=z$. It may therefore be computed from equation (75) by substitution of the proper values.

Valve Opening from Initial Partial Opening. Time T Equal to or Less than Time z for Double Traverse of Acoustic Wave.—In cases where the ultimate valve opening is nearly or quite the full size

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of pipe and the initial opening one-half the ultimate or greater, the initial velocity v_0 will be but slightly less than the final velocity v_1. In such case the increase in velocity from v_0 to v_1 will be small, and the whole program of values for h will be correspondingly reduced.

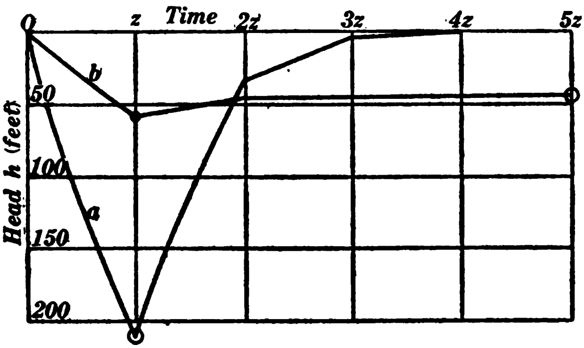

FIG. 74—HISTORY OF PRESSURE HEAD h, STARTING FROM HALF OPENING.

	L	H	m_0	m_1	z	T
Case a	800	430	·025	·05	·40	z
,, b	,,	,,	·025	·05	,,	$5z$

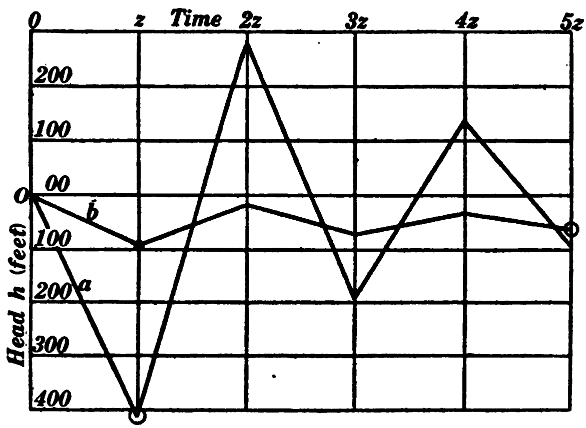

FIG. 75.—HISTORY OF PRESSURE HEAD h, STARTING FROM HALF OPENING.

	L	H	m_0	m_1	z	T
Case a	4000	2700	·01	·02	2·0	z
,, b	,,	,,	·01	·02	,,	$5z$

Thus with the data of Fig. 48a and with $m_0 = ·5$ the value of $v_0 =$ 7·594, while $v_1 = 7·731$, leaving only a residual $v_1 - v_0 = ·137$ to be realized by the further opening of the valve. In this case the values of h are negligible, only reaching a maximum of 2·4 feet.

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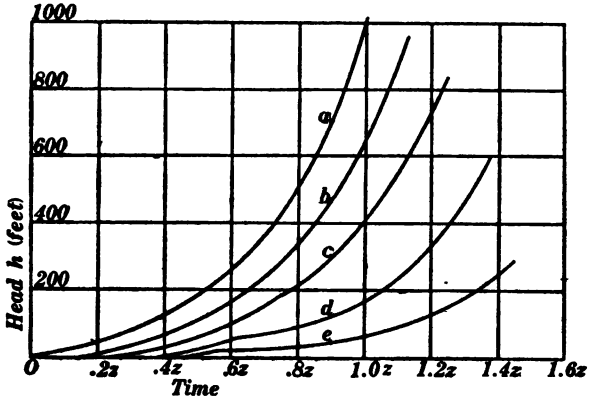

FIG. 76.—HISTORY OF PRESSURE HEAD h FOR VARIOUS POINTS IN PIPE LINE (CLOSURE).

L	H	m_0	m_1	z	T
4000	200	·10	·00	2·0	z

Case a at ....valve
,, b ,, 1000 feet from ,,
,, c ,, 2000 ,, ,, ,,
,, d ,, 3000 ,, ,, ,,
,, e ,, 3600 ,, ,, ,,

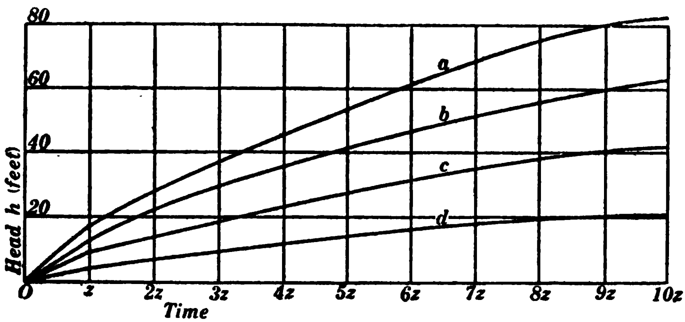

FIG. 77.—HISTORY OF PRESSURE HEAD h FOR VARIOUS POINTS IN PIPE LINE (CLOSURE).

L	H	m_0	m_1	z	T
4000	200	·10	·00	2·0	10z

Case a at ....valve. Origin 0 at $t=0$
,, b ,, 1000 feet from ,, ,, 0 ,, $t=·25$ sec.
,, c ,, 2000 ,, ,, ,, ,, 0 ,, $t=·50$,,
,, d ,, 3000 ,, ,, ,, ,, 0 ,, $t=·75$,,

For better comparison the four cases are brought to a common origin of time. Actually these origins are displaced relatively by quarter seconds, as noted above.

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On the other hand, with the same data and with $m = \cdot 10$ and $\cdot 05$, the initial velocities are 5·216 and 3·13, and the values of h at full opening are 39·2 and 64·0 respectively (see Fig. 71).

In cases where the ultimate valve opening is small compared with the size of pipe, the initial velocity will vary nearly with the initial opening, and in such cases the ultimate drop in pressure at

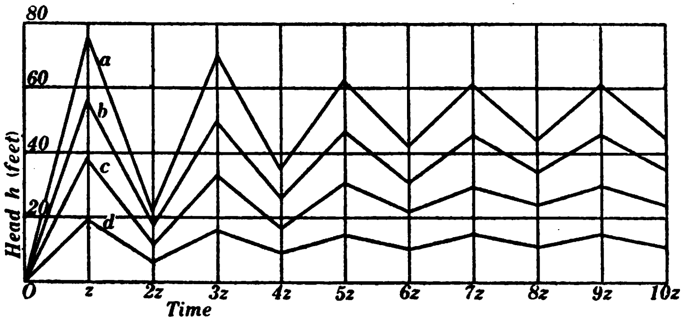

Fig. 78.—History of Pressure Head h for Various Points in Pipe Line (Closure).

L	H	m_0	m_1	z	T
20,000	3000	·02	·00	10	$10z$

Case a at ....valve. Origin 0 at $t=0$
„ b „ 5000 feet from „ „ 0 „ $t=1\cdot 25$
„ c „ 10,000 „ „ „ „ 0 „ $t=2\cdot 50$
„ d „ 15,000 „ „ „ „ 0 „ $t=3\cdot 75$

For better comparison the four cases are brought to a common origin of time. Actually these origins are displaced relatively by time intervals, as noted above.

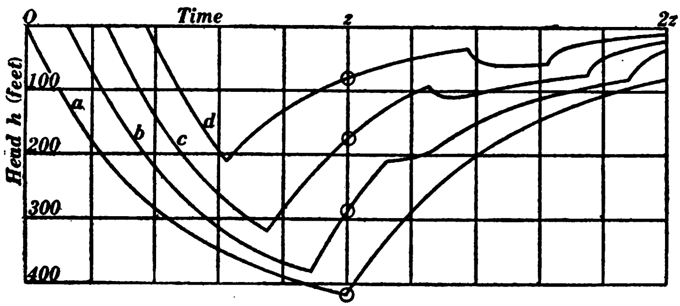

Fig. 79.—History of Pressure Head h for Various Points in Pipe Line (Opening).

L	H	m_0	m_1	z	T
4000	500	·00	·05	2·0	z

Case a at ....valve
„ b „ 1000 feet from „
„ c „ 2000 „ „ „
„ d „ 3000 „ „ „

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full opening within the time $t=z$, will vary approximately with the amount of valve movement or nearly with the velocity (v_1-v_0) to be acquired (see Fig. 72).

With cases intermediate between these extremes, the results will be of the same general character, but varying in less direct

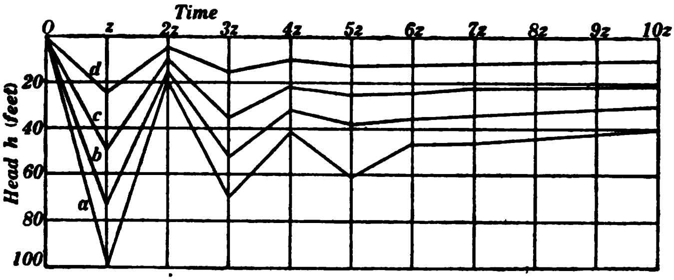

FIG. 80.—HISTORY OF PRESSURE HEAD h FOR VARIOUS POINTS IN PIPE LINE (OPENING).

L	H	m_0	m_1	z	T
4000	500	·00	·05	2·0	10z

Case a at ....valve. Origin 0 at $t=0$
„ b „ 1000 feet from „ „ 0 „ $t=·25$
„ c „ 2000 „ „ „ „ 0 „ $t=·50$
„ d „ 3000 „ „ „ „ 0 „ $t=·75$

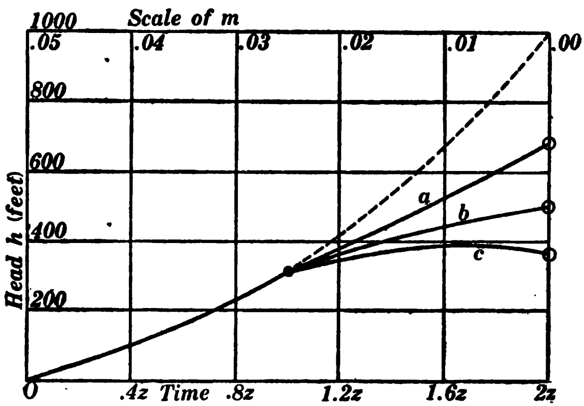

FIG. 81.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (CLOSURE).

L	H	m_0	m_1	z	T
4000	500	·05	·00	2·0	2z

Case a Percentage of reflection at valve ·00
„ b „ „ „ „ „ ·60
„ c „ „ „ „ „ 1·00

The dotted line shows the course of the curve with full reflection for full closure in time z.

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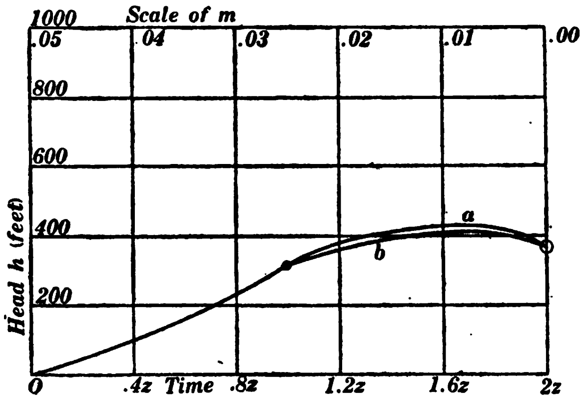

FIG. 82.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (CLOSURE).

L	H	m_0	m_1	z	T
4000	500	·05	·00	2·0	$2z$

Case a Percentage of reflection at valve $(m_0-m)/m_0$

" b " " " " " s/v_0

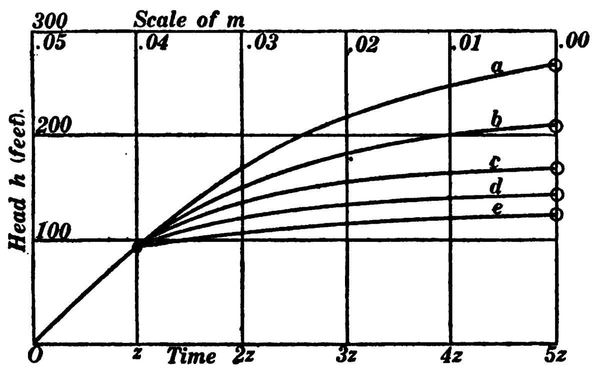

FIG. 83.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (CLOSURE).

L	H	m_0	m_1	z	T
4000	500	·05	·00	2·0	$5z$

Case a	Percentage of reflection at valve	·00
" b	" " " " "	·25
" c	" " " " "	·50
" d	" " " " "	·75
" e	" " " " "	1·00

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ratio with valve movement as the final opening is large in comparison with the area of pipe. In all cases, however, the maximum drop will vary roughly with the velocity $(v_1 - v_0)$ to be acquired.

Subsequent to the arrest of valve movement at full opening, the head h will return to zero either by periodic lifts or through

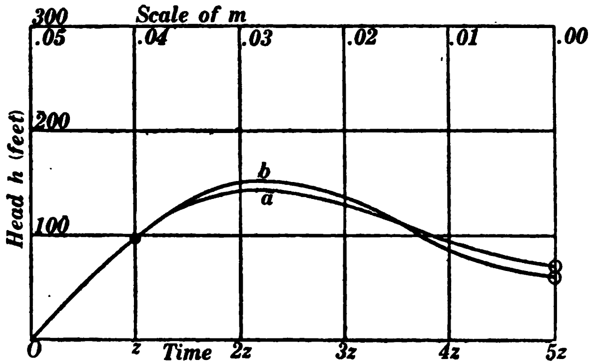

FIG. 84.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (CLOSURE).

L	H	m_0	m_1	z	T
4000	500	·05	·00	2·0	5z

Case a Percentage of reflection at valve $(m_0 - m)/m_0$
" b " " " " " s/v_0

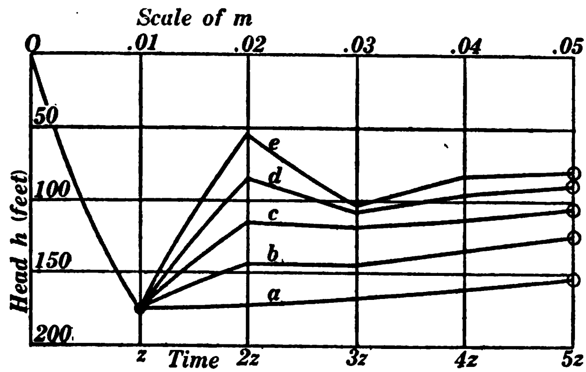

FIG. 85.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (OPENING).

L	H	m_0	m_1	z	T
4000	500	·00	·05	2·0	5z

Case a Percentage of reflection at valve ·00
" b " " " " " ·25
" c " " " " " ·50
" d " " " " " ·75
" e " " " " " 1·00

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alternating plus and minus values, as the characteristics may determine, and in accordance with the same general relations as outlined for the case of opening from full closure.

Maximum value of h. In this case the maximum value of h (drop in pressure) is found at the end of the valve movement or when $t=T$. It may therefore be computed from equation (75) by substitution of the proper values.

Valve Opening from Initial Partial Opening. Time T Greater than Time z for Double Traverse of Acoustic Wave.—The effect of extending the time in these cases is similar to that in opening from full closure, as discussed above. Such increase of time reduces

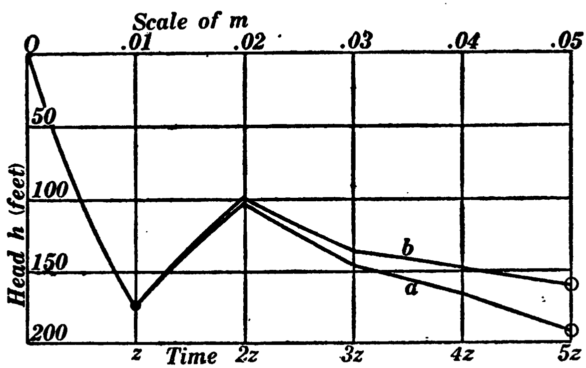

FIG. 86.—HISTORY OF PRESSURE HEAD h WITH ASSUMED PARTIAL REFLECTION AT VALVE (OPENING).

L	H	m_0	m_1	z	T
4000	500	·00	·05	2·0	$5z$

Case a Percentage of reflection at valve $(m_1-m)/m_1$
,, b ,, ,, ,, ,, ,, $(v_1-v)/v_1$

the ultimate or extreme drop in pressure and modifies the general character of the curve beyond the point where $t=z$, as indicated in Figs. 73, 74, 75.

After the close of the valve movement, the value of h will return to zero in the same general manner as for other cases and as previously noted.

Maximum value of h. In this case the maximum value of h (drop in pressure) is sometimes found when $t=z$, or again when $t=T$, or in some cases between $t=z$ and $t=T$. It will therefore result from an application of (75), as may be required.

Pressure History at any Point in the Line.—Following the method outlined in Sec. 38, Figs. 76–80 show the history of h at various points in the line for selected cases with hydraulic characteristics as noted.

Partial Reflection at Valve.—Due to the absence of any definitely assured basis for estimating the proportion of reflection at

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the valve end of the line, no attempt has been made to work out any considerable number of numerical cases. In Figs. 81–86 are shown a few cases in illustration of the application of the method to a numerical problem. Figs. 81–84 show the form of the curves in the case of valve closure, and Figs. 85, 86 correspondingly for valve opening. In Figs. 83, 85, for the interest attaching to extreme conditions, curves have been worked out on the assumption of a constant value of $f=0$ and $f=\cdot 25$, though doubtless in most cases the actual value is greater than the larger of these. The value $f=0$ implies reflection from the upper end of the line, but no reflection from the valve end. These various curves will repay careful study in connection with those for $f=1\cdot 00$ or full reflection at the valve, and which are given on the same diagrams for convenience of comparison.

43. Approximate Formulæ

Due to the inherent complexity of the relations involved in the general problem of shock or water hammer, there have been many attempts to find approximate or working formulæ which might be sufficiently accurate for most practical cases. There is perhaps no problem connected with practical hydraulics for the treatment of which a greater number of approximate or working formulæ have been proposed. The difficulty of developing any such formula which shall be fairly general in its application is, however, shown by the extreme divergence among the results given by the application of these various formulæ to the same problem. It is true that by introducing special limitations or conditions a formula may be developed which shall be relatively simple, and at the same time fairly accurate for the special conditions implied. All such formulæ lack generality, and it is just here that danger enters in connection with their use. The author of a proposed method or formula may well understand its limitations and the conditions under which it may safely be used; but once the formula has found a place in engineering literature, these limitations are apt to be forgotten and such formulæ are often proposed or used for cases to which they are entirely inapplicable. Under such circumstances the results obtained are wholly misleading, and their use implies a confidence which is quite without foundation in actual fact.

Due to these considerations it has seemed desirable to note, in the present section, some of the better-known formulæ which have been proposed and which are in current use in varying degrees.

In this discussion there is implied no criticism of these formulæ as such or of their authors, but rather an attempt to show the relation between such formulæ and the more complete theory developed in the preceding sections, and thus to indicate the general conditions under which such formulæ may be safely or appropriately employed.

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The Allievi Formula.—The general equations developed by Allievi* rest upon the same fundamental assumptions regarding the physical phenomena involved as the equations and methods of Secs. 36 and 38. Allievi's development is, however, based on certain restrictions or limitations as noted below.

Applied to the same problem and with the same interpretations and conditions, the equations of Secs. 38, 41 give identically the same results as those of Allievi. A somewhat different method of development has, however, been here preferred by reason of the better picture which it enables us to form, of the physical state of the column of water within the pipe and along which an acoustic wave is travelling back and forth with reflection at the ends, all as developed in detail in preceding sections.

The restrictive conditions which are assumed in the development of the Allievi equations are as follows:

1. The omission of the influence due to friction.
2. The omission of the head $v^2/2g$ corresponding to the main pipe line velocity v.

The latter is usually small compared with the other quantities involved, and its omission is a matter of no serious importance.

The former is of serious or minor importance according to circumstances. For closure the influence of the friction head is relatively large at the start and decreases as the pipe line velocity v decreases, vanishing with $v=0$ at complete closure. The omission of the influence due to friction will affect the course of the history of h during the closure and the final value to some extent, but in many cases not seriously.

In general the error due to the omission of friction will be more and more important as the friction head is a larger and larger fraction of the total head H.

In the case of opening, the influence of friction is minimum at the start, when, as a rule, the greatest drop in pressure occurs, and increases, reaching a maximum with the final steady motion velocity. If the opening is from complete closure, the influence of friction on the maximum drop is relatively small; if from part opening, the error due to its omission will be more significant, increasing in importance as the friction head is a larger and larger fraction of the total head H.

These general equations of Allievi have, however, for the most part, been put aside in favour of an approximate formula proposed by him and used currently by many engineers without a proper understanding of its derivation or limitations. These are in brief as follows:

We may write equation (42) showing the composition of h in terms of the successive values of s as follows:

$$h=a\begin{pmatrix} s-s_1+s_2-s_3+\text{etc.} \\ -s_1+s_2-s_3+\text{etc.} \end{pmatrix} \quad \ldots\ldots\ldots\ldots (80)$$

* "Annali della Societá degli Ingegneri," Rome, Vol. XVII, 1902.

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Put $F=s-s_1+s_2-s_3+\text{etc.}$
Then remembering the special notation of Sec. 38 we shall have

$$F_1=s_1-s_2+s_3-\text{etc.}$$

Hence (80) becomes

$$h=a(F-F_1).$$

It is also evident by inspection that

$$s=F+F_1.$$

Now it is evident that if F increases uniformly with the time so must also F_1, and so must also their sum or s. Likewise it is evident that $F-F_1$ then becomes a constant quantity, or h=constant.

It was then assumed that a uniform rate of decrease of valve opening, that is a uniform rate of decrease in m, as used in the present system of notation, will during the period from $t=z$ to $t=T$ result in a uniform rate of increase in F, and hence in a uniform rate of decrease in v and in a constant value of h.

While it was recognized that these conditions will not be realized with mathematical precision, it was assumed nevertheless that, for practical purposes, the time histories of v and h during the time period from $t=z$ to $t=T$, may, with uniform rate of valve closure, be taken as showing a substantially uniform rate of decrease for v and a substantially constant value for h.

Similarly for opening, it was assumed that with a uniform rate of valve area increase, the history of h after the first drop, will show a partial return followed by a nearly uniform value for the remainder of the time period up to $t=T$.

A formula is then developed for the determination of this assumed uniform value of h during the time period from $t=z$ to $t=T$.

The formula itself is developed in terms of the ratio between the total pressure head, which we may here denote by y, and the original head H. In terms of the present notation we have then

$$y=H\pm h\left(\text{omitting } \frac{v^2}{2g}\right)$$

Next let $x=\dfrac{y}{H}$

Then the following equation in x is deduced

$$x^2-x(2+n^2)+1=0$$

and of this equation we have the two roots

$$x=1+\frac{n}{2}(n\pm\sqrt{n^2+4}) \qquad \text{(81)}$$

where $n=\dfrac{Lv_0}{gTH}$

Of the two values in (81) the + sign applies to the case of closure and the − sign to that of opening.

Regarding the two roots given by the + and − signs of the radical, it may be noted that their product is 1, so that if either is known the other may be immediately found.

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The limitations in the proper use of this formula arise from the fundamental assumption that a uniform rate of valve movement will, for this period, determine a uniform rate of decrease in v or a uniform value of h. In some cases this will hold, at least approximately. In others, as for example, in Fig. 49 the rate of decrease of v is widely divergent from uniformity, and correspondingly the history of h shows no approach to a flat top or nearly uniform value during the time period $t=z$ to $t=T$. Generally speaking, if the friction head is a large part of the total head, the history of h shows no approach to the form assumed for the Allievi formula, and the application of the formula to such a case will give a result widely divergent from that indicated by the more complete theory.

In the case of valve opening, furthermore, the formula does not profess to give the value of the first extreme drop, but rather the mean of the subsequent history between $t=z$ and $t=T$. Here again, however, where the final value of m is large, the history of y will show small similarity to that assumed in the formula, and in such case likewise the use of the formula will lead to a result having but remote relation to that indicated by the more complete theory.

Warren's Formula.*—*Complete Closure.* If we assume that the conditions of closure are such that the excess pressure rises from 0 according to a linear law during the time $z=2L/S$ and then remains constant, we have the same general conditions as assumed in the Allievi formula. Warren, however, does not assume necessarily a linear rate of valve area closure, and rests these general assumptions regarding pressure change rather on a certain amount of observation of actual cases with valve areas controlled by governing devices in normal power plant practice, and in which cases the change in pressure closely fulfilled a program such as assumed.

With this assumption the principle of impulse and momentum may be invoked to obtain a simple approximate value of the assumed constant excess head reached during the time period from $t=z$ to $t=T$.

Considering the mass of water in the pipe and neglecting the relatively small amount which flows out during closure we have as follows:

The average force acting over the cross section of pipe during time from $t=0$ to $t=z$ is $Awh/2$.

The uniform force acting during time from $t=z$ to $t=T$ is Awh.

The total change of momentum produced is $LAwv_0/g$.

Then remembering that the sum of the products of force by time during which it is in operation will equal the change in momentum produced, we have:

$$\frac{Awhz}{2}+Awh(T-z)=\frac{LAwv_0}{g}$$

* "Trans, Am, Soc. C.E., 1915," p. 238.

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Whence $h\left(T-\frac{z}{2}\right)=\frac{Lv_0}{g}$

$$\text{or}\quad h=\frac{Lv_0}{g\left(T-\frac{z}{2}\right)} \qquad (82)$$

This is Warren's formula.

Obviously the limitations in the use of this formula are substantially the same as for the Allievi formula. Both assume (however realized) substantially the same form of time history for h and where the circumstances are such as to realize approximately such a form of time history, the formula should apply with close approximation.

In other cases the results will be in error more and more widely as the form of history for h departs more and more widely from that assumed.

If again, in this formula, we should assume T very long relative to z then the formula will reduce approximately to

$$h=\frac{Lv_0}{gT} \qquad (83)$$

which has been proposed as a formula for water hammer conditions.

This is equivalent to assuming the value of h uniform throughout the entire period of closure, and on this assumption the formula may be directly deduced on the principle of impulse and momentum.

It will be noted that this value is one-half that given by the Joukovsky formula. Obviously both cannot be correct, at least for the same case. As noted under the next heading the Joukovsky formula applies to one assumed history of h and that of (83) to an entirely different assumed history, while that assumed by Warren and Allievi is a combination of the two.

Any one of these histories might by chance be approximated to in actual experience, but a study of the diagrams of h as developed by the use of the more complete theory indicates how small the probability of any one case conforming to any one of these assumed histories, and in consequence the remote chance that any one of the formulæ based thereon could be safely employed in any given case.

Joukovsky's Formula.*—It is assumed (however realized) that the rate of decrease of velocity is uniform, or in other words that the rate of increase of s is uniform. In such a case, as we have seen in Sec. 38, the history of h will show a series of slopes up and down from a minimum of 0 to a maximum determined by the value reached at $t=z$. We have then simply to find the value at this instant $t=z$ as the maximum value reached during the movement.

The rate of increase of s will be v_0/T per unit time.

The rate of increase of h will be, then, av_0/T per unit time.

* "Memoirs Imperial Academy of Sciences, St. Petersburg, 1897," Vol. IX.

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This rate is uniform from $t=0$ to $t=z$ when the maximum value is reached. Hence for such maximum value we shall have

$$h=\frac{zav_0}{T}=\frac{2Lv_0}{gT} \qquad (84)$$

which is Joukovsky's formula.

Vensano* has extended this to a pipe line of varying diameter in the form

$$h=\frac{2(L_1v_1+L_2v_2+L_3v_3+\text{etc.})}{gT} \qquad (85)$$

Where L_1, L_2, etc., denote the lengths of the various sections and v_1, v_2, etc., denote each the initial velocity of flow in these various sections.

The limitations on the use of this formula spring from the special assumption made regarding the form of the time history of v. In Allievi's formula the assumption is made that the change of velocity is uniform during the period $t=z$ to $t=T$. In Warren's formula the assumption relates to the form of the time history of h, but agrees in form for the time period z to T, with that assumed by Allievi. In the present case the assumption is made of uniform rate of decrease of v from the start of the movement. It is only this assumption which can justify the form of time history for h which furnishes the basis for the formula, and it is of special importance that such conditions should be fulfilled for the time period 0 to z.

It may be shown, however, that at the most the condition assumed can only be approximately realized, and in no case will a uniform rate of valve closure determine a uniform rate of velocity decrease, especially in the early part of the movement and hence in the time period 0 to z.

At the best, therefore, the special condition assumed as a basis for the development of this formula can be only imperfectly realized and in many cases the departure will be extreme.

Generally speaking the greater H relative to L and the smaller the area of the valve opening compared with pipe cross section, the more nearly will the conditions here contemplated be realized. In inverse cases the departure from the conditions assumed may become so wide as to vitiate the formula for practical use.

Formulæ Based on Principle of Mass and Acceleration.—Several writers† have proposed formulæ for change of pressure in pipe lines, based on the principle of mass and acceleration and considering the body of water within the pipe as forming a single mass subject to an accelerating head measured by h.

In the development of such formulæ we may conveniently start with the fundamental equations (33), (36) as follows:

$$v=mu \qquad (86)$$

$$Mu^2=H+h \qquad (87)$$

* "Trans. Am. Soc. C.E., 1918," p. 185.

† See particularly A. H. Gibson, "Water Hammer in Hydraulic Pipe Lines." New York: D. Van Nostrand.

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and to which instead of (34) we add the equation:

$$\frac{dv}{dt}=\frac{gh}{L} \text{ (see Sec. 11 } (n)) \quad \dots\dots\dots\dots (88)$$

Assuming also uniform valve area closure, we have

$$m=(m_0-kt) \quad \dots\dots\dots\dots (89)$$

The combination of these equations should serve to give a complete solution of the problem, but, unfortunately, the resulting equation does not seem to admit of integration and reduction in analytical terms.

If, however, the influence due to friction is omitted, the equations become amenable to treatment. In such case M becomes a constant and equal to $1/2gf$ and for (87) we have

$$u^2=2gf(H+h) \quad \dots\dots\dots\dots (90)$$

If then we combine (86), (88), (89), (90), eliminating v and h, we shall have the equation:

$$u^2-2fkLu-2gfH=-2fL\,(m_0-kt)\frac{du}{dt} \quad \dots\dots (91)$$

For $2gfH$ put its value u_0^2.
Also put $fkL=P$
and $\sqrt{P^2+u_0^2}=R$.
We may then reduce (91) to the form

$$\frac{du}{(R+P-u)\,(R-P+u)}=\frac{dt}{2fL\,(m_0-kt)} \quad \dots\dots (92)$$

Reducing the left-hand member into partial fractions and integrating, we find

$$\log\frac{R+P-u}{R-P+u}\Bigg]_{u_0}^{u}=\frac{R}{P}\log\frac{m_0-kt}{m_0}$$

Evaluating the above equation between u and u_0 and reducing, we have

$$\log\left(\frac{R+P-u}{R-P+u}\right)=\log\left(\frac{R+P-u_0}{R-P+u_0}\right)+\frac{R}{P}\log\left(\frac{m_0-kt}{m_0}\right) \quad ..(93)$$

$$\text{or} \quad \frac{R+P-u}{R-P+u}=\frac{R+P-u_0}{R-P+u_0}\left(\frac{m_0-kt}{m_0}\right)^{\frac{R}{P}} \quad \dots\dots (94)$$

From (93) or (94) it follows that when $t=0$, $u=u_0$. Again at the close of the full movement when $t=m_0/k$ we shall have

$$u=R+P=\sqrt{P^2+u_0^2}+P \quad \dots\dots\dots\dots (95)$$

From the form of (94) it is seen that u steadily increases during the movement of the valve, from its value $u_0=2gfH$ at $t=0$ to its maximum $(R+P)$ just at the instant of complete closure.

The value of h follows directly from (90):

$$h=\frac{u^2}{2gf}-H \quad \dots\dots\dots\dots (96)$$

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and hence h will increase continuously with u and will reach its maximum value with u at the instant of complete closure, or in the case of partial closure at the final instant of valve movement.

In the case of complete closure $k=m_0/T$ and with $u=(R+P)$ we readily reduce (96) by substitution of values, to the form

$$h=\frac{m_0L}{gT}(R+P) \quad \text{.................(97)}$$

For the case of opening, the analytical treatment is in effect contained within that for closure as above. It is only necessary to introduce a change of sign in dv/dt and in k, and to suitably interpret the terms of the preceding equations.

We shall have then for opening,

$$\log\left(\frac{R-P-u}{R+P+u}\right)=\log\left(\frac{R-P-u_0}{R+P+u_0}\right)+\frac{R}{P}\log\left(\frac{m_0}{m_0+kt}\right) \quad \text{..(98)}$$

$$\frac{R-P-u}{R+P+u}=\frac{R-P-u_0}{R+P+u_0}\left(\frac{m_0}{m_0+kt}\right)^{\frac{R}{P}} \quad \text{........(99)}$$

$$\text{and} \quad h=H-\frac{u^2}{2gf} \quad \text{..................(100)}$$

If the valve starts from a nearly closed position then m_0 is nearly 0, and toward the close of the valve movement $m_0/(m_0+kt)$ will become nearly 0. Hence in equation (99) the last term approaches 0 as a limit as t increases, while correspondingly u will approach the limit $(R-P)$, as shown by the other members of the equation. This value is readily seen to be less than u_0. From the form of (99) it thus follows that u steadily decreases during the movement of the valve and with small m_0 approaches the value $(R-P)$ as a limit when t is large. Hence, as shown by (100), there will be a continuous increase in the value of h (drop in head), and the maximum value will be reached at the close of the time T when the movement under consideration is completed.

From (100) we derive in the same manner as for (96) the maximum value of h, corresponding to $u=(R-P)$ in the form

$$h=\frac{m_0L}{gT}(R-P) \quad \text{.................(101)}$$

For two cases, one of closure and one of opening, with the same values of P and R, and denoting opening and closure by subscripts 1 and 2, we have for the limit values of u and h:

$$u_1u_2=R^2-P^2=u_0^2=2gfH \quad \text{............(102)}$$

We also readily derive the relation:

$$h_1h_2=\left(\frac{kLu_0}{g}\right)^2=\frac{2fH}{g}(kL)^2 \quad \text{...........(103)}$$

It must be remembered, however, that these limit values of h imply closure either complete or nearly so, and opening from full closure or nearly so.

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The rapidity of approach of u and h to these limit values will depend on the circumstances of the case. Generally u and h will approach more quickly and more nearly to their limit values with

Head H or u_0	. large
T . .	. long
Valve area	. small relative to pipe cross section.

In the case of valve opening, as shown by (98), with small values of m_0, the value of h rapidly and closely approaches its limit value, and as m_0 becomes smaller and smaller the rapidity of approach becomes rapidly greater. When $m_0=0$, that is when the valve starts from the shut position, the approach to the limit value is instantaneous, at least so far as indicated by these equations. In such case, therefore, the value of h would hold substantially uniform during the period of valve movement at the limit value, and the velocity in the pipe line would be given substantially by the equation:

$$v=mu=kt(R-P) \qquad (104)$$

It should be noted that this value applies only in the case of the valve starting from the closed position.

As the value of m_0 is relatively larger; that is, as the valve starts from a position more and more open, the growth of h (drop in pressure head) is relatively less abrupt and the actual value realized at time T is a smaller fraction of the limit value, as in (101).

A comparison of (97), (101) with the Allievi formula, equation (81) leads to the surprising result that when expressed in terms of the same quantities the two formulæ are identical. Equations (97), (101) express the maximum or limit values of h based on principles of mass and acceleration alone and without reference to the formation or propagation of acoustic waves. Allievi's equation starts with the same fundamental theory as developed in Secs. 38, 41, but as the result of the process of development with the introduction of special or limiting assumptions, the same ultimate expression as in equations (97), (101) is reached.

Obviously the same limitations regarding the use of (97), (101) apply, as in the case of the Allievi formula, and as noted in connection with that formula.

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CHAPTER IV

STRESSES IN PIPE LINES

44. INTRODUCTION

IN approaching this subject we must first inquire as to the sources of load capable of producing stress.

These are as follows:

1. Internal pressure.
    (α) Balanced.
    (β) Unbalanced.
2. Weight of pipe line or element under consideration.
3. Weight of water contained in pipe line or element under consideration.

Balanced internal pressure implies uniform pressure in opposite directions over equal projected areas, as in the case of a completely closed chamber or on opposite sides of a pipe. It is well known that such a load, in an element with circular cross section, produces tension alone. Where the cross section is non-circular, bending moments involving tension, compression and shear will be combined with the tension arising directly.

Unbalanced internal pressure implies a condition of unbalanced force so far as the element under consideration is concerned, such unbalanced force tending to displace or separate the element from the remainder of the system or from its environment or attachments. The stress developed by such unbalanced pressure may be tension, bending or cross breaking, shear, or compression and in all combinations according to the details of the case.

In discussing stresses developing in these various ways in pipe lines we find it convenient to consider different combinations of conditions as follows:

Two conditions regarding distribution of pressure.

(*a*) Static or no flow.

(*b*) Steady flow.

We may properly assume that in all cases the transverse dimensions (diameter usually) will be so small compared with the head involved as to permit of assuming the pressure uniform over any given cross section of the pipe or pipe line element. Regarding vertical extension, however, we may have the two cases:

(*c*) Element or system under consideration substantially in one horizontal plane.

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(*d*) Element or system under consideration extending over considerable differences of horizontal level.

It results in case (*ac*) that the pressure may be taken as uniform over any section involved and in general throughout the system.

In case (*bc*) the differences in pressure at different sections will be due to changes of area and hence of velocity, and not to changes of level. Also we may take the pressure as uniform over any given section.

In case (*ad*), depending on differences in level, there will be differences in static pressure throughout the system.

In case (*bd*), likewise, there will be differences in pressure due to changes in level as well as changes in velocity.

45. Ring Tension in a Cylindrical Pipe or Element with Circular Cross Section

We assume static conditions (*a*) above. The pressure under conditions of flow will be less than under conditions of rest. In this case, furthermore, the question of vertical extension enters only as a factor in the intensity of the pressure.

Let D=diameter (i).
t=thickness (i).
p=internal pressure (pi2).
T=actual working stress in longitudinal joint (pi2).
e=efficiency of longitudinal joint.

Then the familiar formulæ of mechanics give us the following:

$$\left.\begin{aligned} T&=\frac{pD}{2et} \\ p&=\frac{2teT}{D} \\ t&=\frac{pD}{2eT} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It should be noted that whenever a pipe or pipe line is under pressure, the ring tension stress is always operative.

46. Longitudinal Stress

We assume conditions (*a*) as above, and for the reasons cited in connection with ring tension. The full longitudinal stress in a pipe is only developed when there is a cap or closure across the section or in the case of bends, angles or turns; and in all cases the pipe must be sufficiently free from external constraint to permit longitudinal movement under the end pressure developed. A typical case of longitudinal tension is furnished by a pipe closed at the end with the end free to move.

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Under these conditions the formulæ of mechanics give us

$$\left.\begin{aligned} T&=\frac{pD}{4et}\ \text{(pi2)}\\ p&=\frac{4teT}{D}\ \text{(pi2)}\\ t&=\frac{pD}{4eT}\ \text{(i)} \end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots(2)$$

The notation in these equations is the same as for ring tension, except that T and e must be taken as relating to the circumferential joint.

These latter equations show the well-known fact that, other things equal, the longitudinal stress on a circumferential section is just one-half the circumferential stress on a longitudinal section, or that the ring tension is just twice the longitudinal stress.

47. Stresses Due to Angles, Bends and Fittings

In approaching this subject it is first necessary to consider the character and measure of the load which may be thrown on a pipe line as a result of the internal pressure on such elements of the line. The load will be represented by certain unbalanced forces acting between the angles, bends or fittings and the remainder of the line, and as a result of which, stress will be developed either in the line itself or in some form of anchor or tie which is intended to carry such load and thus relieve the line. It will be necessary here to consider the four combinations of conditions, (*ac*), (*ad*), (*bc*), (*bd*), as noted at the opening of the chapter.

Case (*ac*). **Static Conditions with Influence Due to Differences of Level Insignificant.**—It is well known from mechanics that a complete enclosure under uniform internal fluid pressure is in complete equilibrium under the forces developed by such pressure. That is, no condition of internal fluid pressure can develop any force tending to move the enclosure as a whole. Parts or elements of pipe lines, however, are not completely closed. Such a part or element of a pipe line system presents therefore an incomplete enclosure under internal fluid pressure, and as a corollary from the equilibrium of a complete enclosure it follows that an incomplete enclosure will not necessarily be in equilibrium. It follows, further, that the forces required to maintain the part or element in equilibrium will be represented exactly, both in magnitude and direction, by the fluid pressures over the areas which would be required to completely close such part or element. This conclusion applies to any part or element of a pipe line such as an individual pipe or part thereof, an angle, Y branch or valve body. Hence we deduce the following broad proposition.

Assume any part of any pipe line system under uniform pressure.

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Then the resultant forces, due to internal fluid pressure, will be represented in magnitude and *reversed* direction by the fluid pressure over the areas which would be required to make of such element or part a complete enclosure. Conversely the forces or system of forces required to maintain such an element or unit in equilibrium will be represented in magnitude and direction by the fluid pressures over the areas required to produce complete closure and without reversal.

This may be illustrated as in Fig. 87. Let AC denote a straight uniform length of pipe. Then the areas required to close the system would be represented by AB and CD. Hence the force required to maintain equilibrium would be represented by the fluid pressures on the sections AB and CD. But these are equal and opposite. Hence the force is zero and the section of pipe is in equilibrium, a well-known fact which common sense readily tells us.

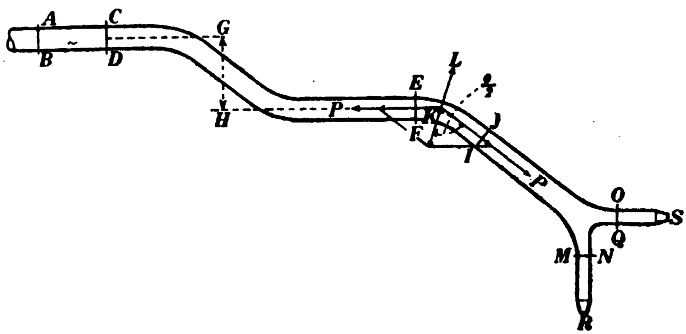

FIG. 87.—RESULTANT FORCE ON PIPE LINE ELEMENTS—STATIC CONDITIONS.

Again, take the part between C and E. Here the closing areas will be sections CD and EF. The pressures are equal and opposite in direction, but do not act along the same line. Hence to maintain equilibrium there will be required a couple with GH as arm. The resultant force on the section itself is, of course, measured by a couple equal in amount and opposite in direction. The resultant force on the section is therefore a couple measured by $p\pi D^2/4 \times GH$, and tending to turn the element in a clockwise direction.

If instead of a pipe of uniform section we have a difference between the two areas as at AB and CD (Fig. 88), we shall have two forces opposite in direction, but unequal in value, and the resultant will be a force and a couple.

Again, take the element of line between EF and IJ (Fig. 87). Denote for convenience the total pressure over a cross section of the pipe by P. Then the balancing system will be represented by two forces P and P applied at the centres of the sections EF and

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IJ. These may, of course, be readily combined into a single resultant measured by $2P \sin \theta/2$. This reversed will then be a measure of the resultant acting on the element and along the line KL.

Again, with an element containing a Y branch such as $IJMO$, we shall have three sections IJ, MN, OQ and three forces, and

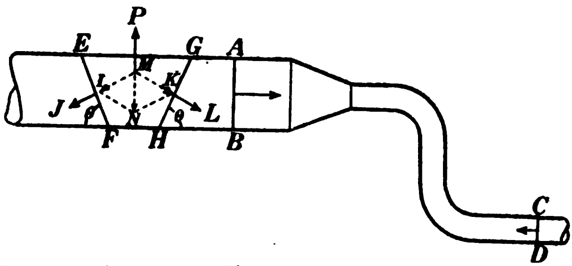

Fig. 88.—Resultant Force on Pipe Line Elements—Static Conditions.

a combined resultant according to the methods of elementary mechanics.

Again, suppose we should consider a part of a straight length contained between two oblique imaginary planes such as EF, GH (Fig. 88). Then the closing areas will be two ellipses of area $(\pi D^2 \operatorname{cosec} \theta)/4$, and the forces required for equilibrium will be

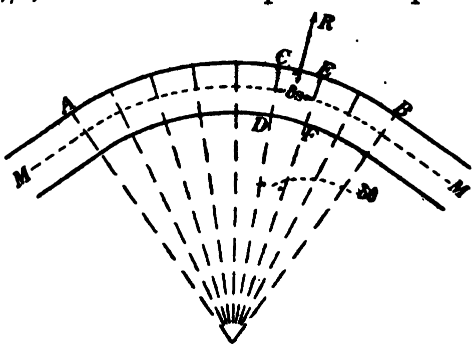

Fig. 89.—Distribution of Resultant Load along Pipe Line Bend.

measured by the pressure p acting over this area. This will give the forces IJ, KL, or by transfer and composition, a single resultant MN. The force exerted on such an element will then be equal and opposite, or MP.

Distribution of Unbalanced Pressure along a Pipe Bend.—The analysis developed above serves to determine the unbalanced load on the bend as a whole. It becomes of interest, however, to determine

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likewise the law of distribution of this unbalanced load along the line of the bend. Thus referring to Fig. 89, let AB denote the bend with short straight lengths at each end. Let $CDEF$ be a small element of the bend and $\delta\theta$ the corresponding angle. Then from the principles above developed we shall have for the resultant unbalanced pressure on this element a force δF measured by

$$\delta Q = 2pA \sin\frac{\delta\theta}{2}$$

where p=pressure in pipe and A=cross sectional area. Now if $\delta\theta$ is small, sin $\delta\theta/2$ approaches $\delta\theta/2$, and hence for a small element we shall have

$$\delta Q = pA\,\delta\theta.$$

Now let δs=length of arc of mid curvature for the element and ρ the radius of curvature. Then $\delta\theta = \delta s/\rho$. Hence we have

$$\delta Q = \frac{pA\,\delta s}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

At the limit when $\delta\theta$ and δs become differentials this relation becomes exact, and we have

$$dQ = \frac{pAds}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Expressed in words this tells us that the bend under these conditions is subject to a distributed load applied in the direction of the radius of curvature and proportional at any point directly to the product pA and inversely to the radius of curvature ρ. In the case of a circular bend ρ will be uniform, the radius of the bend. If non-circular, the intensity of the load will vary inversely as the values of ρ. Furthermore, it appears that for any small element of length along the mid-curvature line M, the resultant load is measured by the product pA multiplied by the length of the element, and divided by the radius of curvature ρ.

Thus in the case of a bend 24 inches mean radius in a pipe 10 inches internal diameter, and under a pressure of 100(pi2) the unbalanced pressure load per inch of length on the mean radius will be

$$\delta Q = \frac{100 \times 78{\cdot}54 \times 1}{24} = 327 \text{ (p)}.$$

A pipe bend under these conditions is therefore to be treated simply as a curved beam subject to a distributed load determined in accordance with (3) or (4), and with end reactions determined as previously developed for the bend as a whole. The case becomes reduced, therefore, to the mechanics of the beam and need not be considered in further detail here. It may be pointed out, however, that in the case of a bend of uniform radius attached rigidly to the tangent pieces at the ends of the bend, the unit load, measured as above, will result in longitudinal tension only, measured by the total load pA. That is, while the condition of the bend is exactly that due to a distributed load applied along the radius and measured

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as in (3) or (4), nevertheless the result of this is, when the ends are anchored, to produce simply a tensile stress in the bend. It is in fact readily seen that the case is entirely similar to that of a chamber of circular cross section under fluid pressure, and in which, as is well known, the shell is subject to tension only. In the case of the bend the load pA/ρ takes the place of the pressure in a cylindrical shell, and with the result of a pure tensional stress measured by the load pA divided by the cross section of the metal.

If the ends of the bend are not rigidly attached to the rest of the line, or in any event if they are not constrained by forces having longitudinal components and which can therefore balance the end pulls pA, the case becomes entirely changed and cross-breaking stresses may develop. Further reference will be made to this case in Secs. 49, 50.

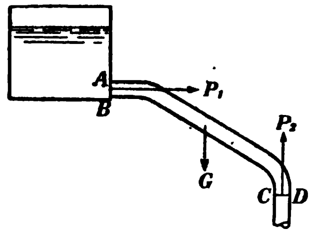

FIG. 90.—RESULTANT FORCE ON PIPE LINE ELEMENTS—STATIC CONDITIONS.

Case (*ad*). **Static Conditions with Influence Due to Differences of Level Significant.**—Let AC (Fig. 90) denote a part of a pipe line in which we cannot assume the pressure as uniform throughout. Assume ideal planes AB and CD forming a complete enclosure of the part under consideration. Now from hydrostatics we know as follows:

1. In a completely closed chamber under hydrostatic pressure there is no resultant force in a horizontal direction.
2. The only resultant force is in a vertical direction, and it is measured by the weight of the liquid.
3. The centre of application of such a resultant gravity force is at the centre of gravity of the liquid or at the centre of volume of the enclosure.

It follows that if the chamber of enclosure is not completely closed, the total unbalanced system of forces will consist of two parts:

1. The gravity component vertical in direction, equal to the weight and passing through the centre of volume of the chamber.
2. Pressure components represented by the pressure over the various openings, reversed in direction.

Thus in Fig. 90 the resultant will consist of the vertical gravity component G acting downward, the pressure component P_1 over the area AB acting to the right and the pressure component P_2 over the area CD acting upward. The unit pressures over AB and CD will in this case be different, and P_1 and P_2 must be computed accordingly. We are therefore led to the following general proposition, which indeed will include both cases (*ac*) and (*ad*).

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Given any part of a system of pipes, connections, fittings, etc., under hydrostatic pressure. Then for such part of the system we may represent the unbalanced force system as follows:

(1) Draw a vertical line downward through the centre of volume under consideration, and of length proportional to the weight of the water or fluid contained.

(2) Assume imaginary planes closing all openings, thus giving, constructively, a completely closed volume.

(3) Through the centre of pressure of each such area draw a line at right angles to the plane of the opening directed from without inward, and of length proportional to the total pressure on such area. In ordinary cases the centre of

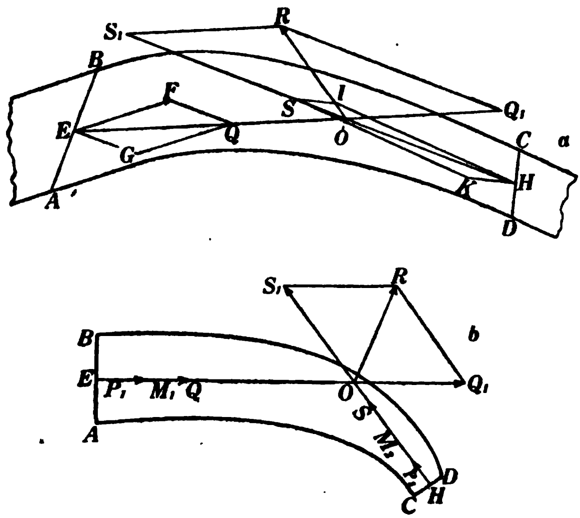

FIG. 91.—RESULTANT FORCE ON PIPE LINE ELEMENTS—GENERAL CASE WITH FLOW.

pressure may, without sensible error, be taken at the centre of gravity of the area.

The system of unbalanced forces will then be represented by the forces denoted by the lines (1), (3) above, which may be combined into a single resultant or a single resultant and a couple or treated as may be desired by the methods of elementary mechanics.

Case (*bc*). **Steady Flow Conditions with Influence Due to Differences of Level Insignificant.**—In order to determine the reaction between moving water in a pipe and the containing pipe, we may conveniently resort to a fundamental principle of hydraulics which may be stated as follows (see Appendix III).

In any hydraulic system containing water in motion, and where the dimensions are such that we may neglect the weight of the

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water as such, the force reaction of the water on the system will be given by the vector sum of the following systems of forces :

(1) The total pressures over the ideal sections bounding the system or element, reckoned from without inward and combined as vectors.

(2) The sum of the momenta per second at inflow and outflow, the former taken direct and the latter reversed and all combined as vectors.

In the case of water at rest system (2) disappears and we have left simply system (1) which reduces the case to one of static equilibrium as discussed under case (*ac*), and by the aid of precisely the same principle.

This proposition holds for cases including friction, so that if we know the conditions of flow, the force reaction between the water and any part or the whole of the system may readily be determined.

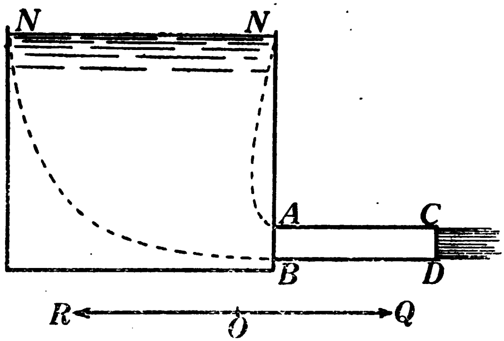

FIG. 92.—RESULTANT FORCE ON PIPE LINE ELEMENTS—FLOW FROM OPEN RESERVOIR.

As an illustration of the proposition in this general form let $ABDC$ (Fig. 91*a*) denote an element in a pipe line system through which water is flowing in the direction from AB to CD. Let p, v, A and M with subscripts 1 and 2 denote respectively the pressure, velocity, area and momentum at the sections AB and CD.

From the centre of the section AB we then draw lines EF and EG respectively in the line of flow and perpendicular to the plane of the section. Lay off on these lines distances EF and EG, representing respectively $M_1=wA_1v_1^2/g$ and $P_1=p_1A_1$, and find the resultant EQ. A similar construction at CD gives the resultant HS. The two resultants intersect at O. Then lay off $OQ_1=EQ$ and $OS_1=HS$ and find the final resultant OR as representing in magnitude and direction the total reaction of the water on the element in question.

Usually the bounding sections AB and CD may be taken perpendicular to the lines of flow, simplifying the construction as in Fig. 91*b*.

Certain special applications will be of further interest.

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Thus in Fig. 92 let the pipe AC be discharging through the area $CD=A$ with velocity v. In this case in order to complete the stream line system we must imagine stream lines continued from A and B up to the surface of the water as indicated by dotted lines. Then assuming discharge at CD into the atmosphere and measuring pressure above the atmosphere, we shall have $p=O$ over both faces NN and CD.

Again the momentum at NN will be sensibly zero while that at CD will be $M=wAv^2/g$. The resultant of systems (1) and (2) will therefore be a force wAv^2/g directed from right to left, or in the direction CA, and measuring the reaction of the water on the system.

These relations are indicated by the vector diagram where

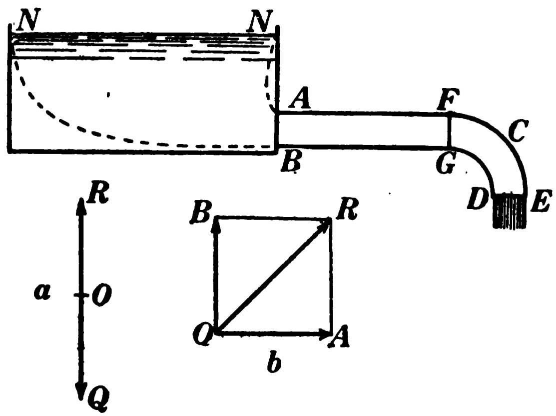

FIG. 93.—RESULTANT FORCE ON PIPE LINE ELEMENTS—FLOW FROM OPEN RESERVOIR.

OR=reaction of water on system and OQ=opposite and equal reaction of system on water.

Furthermore, if secondary losses are neglected we shall have $v^2/g=2H$ and $F=2wAH$=twice the static pressure over the area of discharge.

This is an expression of the well-known relation that neglecting losses due to friction and turbulence, the dynamic pressure due to a jet or stream is twice that of the static head necessary to produce the velocity of discharge.

If we have an elbow in the discharge line, as at C (Fig. 93) then using the same method as before we find the total force reaction measured by $OR=F=wAv^2/g$ and as indicated in the vector diagram (a).

For all cases where the system considered extends from a reservoir where the water is at rest, to one or more points of free discharge, it is clear that system (1) of the forces enumerated above will disappear and that the force reaction on such system will be measured

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by force system (2), the resultant of the forces representing the reversed momentum per second at the point or points of discharge. Thus let Fig. 94 represent a plan view of a system including reservoir and three discharge outlets. Draw lines from outlets 1 and 2 in the line of discharge and extending back to a point of intersection O. Then lay off forces OA and OB to represent the reversed momentum per second represented by the discharge from these orifices. Find the resultant OP and draw the line through the discharge orifice 3 back to an intersection O_1. Then from O_1 lay off $O_1P_1=OP$ and $O_1C=$ reversed momentum per second for orifice 3. Then the resultant O_1R will represent the final resultant force reaction on the system. These methods may be applied to any combination of elements involving a reservoir and one or more discharge orifices.

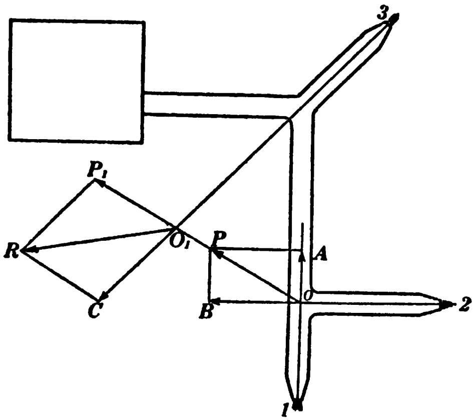

FIG. 94.—RESULTANT FORCE ON PIPE LINE ELEMENTS—CASE WITH MULTI-DISCHARGE.

Let us consider next the elbow C (Fig. 93). If we assume free discharge into the atmosphere the pressure over the section FG will be sensibly zero. Hence system (1) of the forces will disappear and we shall have the vector diagram at b, where OA=vector value of M at FG, OB=reversed vector value at DE, and OR=force reaction on elbow. Note again that OR will here be twice the value of the displacing force found for the same elbow under static conditions.

Next let the elbow C be fitted with a nozzle DE (Fig. 95). This will reduce the velocity of flow through the pipe and elbow and decrease the friction loss with corresponding increase in pressure.

The total force reaction will be found in the same manner as for Fig. 93, but its value will not be the same. The relation between the force reaction and the size of the orifice will be considered at a later point.

Taking first the elbow C between FG and IJ we shall have over

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the two areas FG and IJ a pressure $P=pA$, sensibly the same in numerical value and related as at OA, OB in the vector diagram at a. We also have M at inflow direct represented by AC and M at outflow reversed represented by BD. These combine together and give a final resultant OR for the force reaction on the elbow.

To find the value of $OC=P+M$ we have, in the general case, including friction, the relation

$$\frac{p}{w}+\frac{v^2}{2g}+\frac{Lv^2}{C^2r}=H.$$

Then $$P=pA=wA\left(H-\frac{v^2}{2g}-\frac{Lv^2}{C^2r}\right)$$

But $$M=wA\frac{v^2}{g}.$$

Hence $$OC=P+M=wA\left[H+v^2\left(\frac{1}{2g}-\frac{L}{C^2r}\right)\right]\ldots\ldots\ldots(5)$$

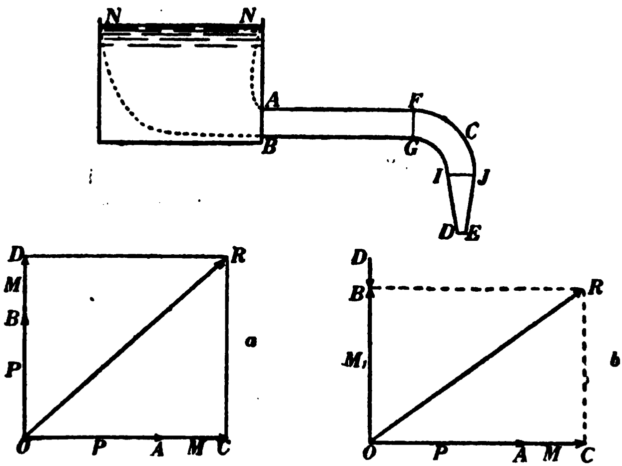

Fig. 95.—Resultant Force on Pipe Line Elements—Discharge from Open Reservoir through Elbow and Nozzle.

This equation shows that the value of $P+M$ will increase or decrease or remain independent of v according as the parenthesis $(1/2g-L/C^2r)$ is $+$, $-$, or 0.

Again, it should be noted that the total resultant OR may be viewed as the resultant of the two components $(P+M)$, one horizontal and the other vertical, acting jointly on the elbow C.

Next if we include the elbow and nozzle as one element, we shall have the same horizontal components P and M denoted by OA and AC (Fig. 95b).

For the pressure at DE we shall have zero and for the reversed

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momentum a value M_1 represented at OB, greater than M by reason of the smaller area and higher velocity. We shall then have the final resultant OR representing the total reaction on the combination of elbow and nozzle.

Again, if we lay up OD (Fig. 95*b*)$=OD$ of a we shall have the difference DB as the measure of the reaction between the nozzle and the elbow.

To determine the measure of these forces we proceed as follows:

Let a=area of nozzle.

$a/A=m$.

f=efficiency of nozzle.

u=velocity through nozzle.

v=velocity along pipe.

We have $M_1=wau^2/g=wAmu^2/g$.

We may find u as in Sec. 17 and thus express M_1 in terms of the conditions of flow. This will give

$$M_1=\frac{wAH\,(2mf)}{1+\dfrac{2gfLm^2}{C^2r}}\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

We may next express $(P+M)$ as in (5) but substituting for v its value mu, determined as in Sec. 17. This will give

$$P+M=wAH\left[1+\frac{m^2f\left(1-\dfrac{2gL}{C^2r}\right)}{1+\dfrac{2gfLm^2}{C^2r}}\right]\ldots\ldots\ldots\ldots(7)$$

Denoting the denominator in (6) by B and reducing we may express these values as follows:

$$M_1=\frac{wAH(2mf)}{B}$$

$$P+M=\frac{waH(1+m^2f)}{B}$$

$$\text{Hence}\quad (P+M)-M_1=\frac{wAH(1+m^2f-2mf)}{B}\ldots\ldots\ldots\ldots(8)$$

The values of m and f are always less than 1, and it is readily shown in such case that the parenthesis in (8) is always positive. Hence $P+M$ is always greater than M_1 and the difference gives the reaction of the nozzle, which is always downward as we should expect.

An interesting question arises as to the condition which will make M_1 a maximum in any given case. It will be clear that with a very large or m nearly 1, the velocity through a will be limited by the friction loss in the line. With a small value of a the velocity along the line will be less, the friction loss less and the discharge velocity greater. It may thus result that the maximum value of M_1, the

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reaction on a line fitted with a discharge nozzle, will not be found at the point of full opening.

Taking M_1 as in (6) and considering m a variable we readily find by the usual method for maximum and minimum the value

$$m=\sqrt{\frac{C^2r}{2gfL}} \quad \text{.....................(9)}$$

This gives the value of a in terms of A for the maximum value of M_1. It is also readily shown when this condition is realized that

$$u=\sqrt{gfH} \quad \text{.....................(10)}$$

$$v=C\sqrt{\frac{Hr}{2L}} \quad \text{.....................(11)}$$

$$\text{and} \quad \frac{Lv^2}{C^2r}=\text{friction head}=\frac{H}{2} \quad \text{...............(12)}$$

It is clear that, as defined, m is always less than 1. Hence in order to make such a maximum value of the reaction possible, we must have, from equation (9) $L>C^2r/2gf$. Otherwise as m is increased from 0 we should simply have a continuously increasing value of the reaction up to the point of $m=1$ or full opening.

Again, since the impulse of a stream on a fixed bucket, as in the case of an impulse water-wheel when the latter is at rest, is directly proportional to the reaction on the nozzle and pipe line, it follows that the above values determine the conditions for maximum starting torque on an impulse water-wheel. It is of interest to compare these relations with those of Sec. 20 giving the conditions for maximum power.

Case (bd). **Steady Flow Conditions with Influence Due to Differences of Level Significant.**—This is the same as the last case, but with the addition of the effect due to gravity on the water over considerable variations in level.

Reference is made to this case in Appendix II. We proceed as in the case (*bc*) but add a fourth system of forces representing the weight due to gravity acting vertically downward with the other systems as in the preceding case.

48. Load Due to Weight of Pipe or Element, and also of Contained Water

The weight of the pipe or element and also that of the contained water must, of course, always be supported. As to whether they will join in forming a stress-producing load in such degree as to require recognition will depend on the circumstances and geometrical arrangement of the pipe system and its supports. Generally speaking, the effect of gravity in producing pressure within the pipe line or pipe line element will be accounted for by the principles and methods already developed. When, however, the horizontal

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dimensions of the element or system under consideration are of any considerable extent, the weight of the element as a structure and the weight of the contained water must be included with other forces due to unbalanced pressure in order to find the final load. These loads due to weight are readily found and combined with the load due to unbalanced pressures by the principles of elementary mechanics.

49. Stresses in Expansion Joints

The principles previously developed find an important application in the case of expansion joints. Thus if the joint is made up, as in Fig. 96, with the internal diameter for the flow of the water equal

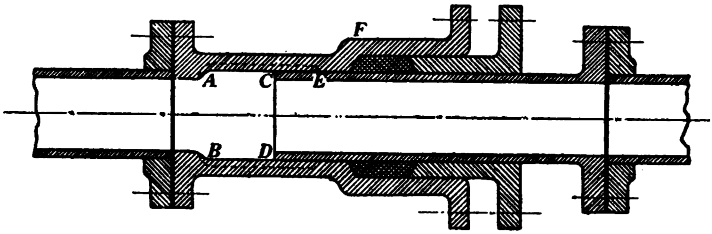

Fig. 96.—Expansion Joint.

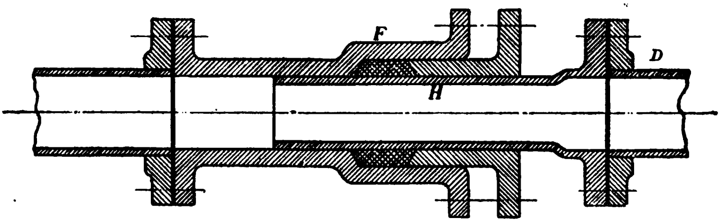

Fig. 97.—Expansion Joint.

throughout to that of the pipe except for the part between AB and CD, then it is clear that there will be a force tending to separate the two members of the joint and measured by the unit pressure p multiplied into the area of the annular ring of metal at CD. In the case of large pipes this might rise to a force of very considerable magnitude. Thus if diameter=48 inches, thickness of metal= 1 inch and p=100(pi2), we find the total load as above=15,000 pounds.

If the metal of part F is relieved between A and E as indicated by the dotted lines, such relief will make no difference in the resultant load tending to separate the parts of the joint. It is readily seen that the pressures on the two additional shoulders thus formed will be equal and opposite and will balance each other on the piece F.

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On the other hand, if the outside diameter of the sliding member H is made equal to the internal diameter of the pipe, as in Fig. 97, it is clear that there will be no axial force on the part F due to the liquid pressures at the joint, while the axial pressures on the part H will just balance and there will therefore be no resultant force tending to separate the two parts of the joint.

If, again, the internal diameter of the pipe D is made greater than the outside diameter of the slide H, the liquid pressures will give a resultant tending to push H into F, producing a tension which must be carried by the pipe lying to the right.

These cases all develop as simple applications of the principles already discussed, and no matter what may be the form or design of an expansion joint, the use of these principles will serve to give the resultant of the liquid pressures at the joint.

50. Combinations of Bends or Elbows with Expansion Joints

Cases of some interest may develop as the result of the combination of expansion joints and bends or elbows. In considering such cases it must be borne in mind that the expansion joint virtually cuts the pipe at the joint and that no longitudinal stress or support or constraint can be transmitted from one member of the joint to the other—at least so long as no additional guard bolts or other members are fitted. Actually, expansion joints are very commonly supplied with bolts connecting the two members and allowing a certain limited degree of freedom, but preventing the complete separation of the two parts. In this manner, and between fixed limits, changes due to temperature may be accommodated while the two members of the joint cannot become entirely disconnected.

Holding in mind the inability of the expansion joint by itself to transmit longitudinal force, together with the hydraulic principles discussed in the preceding sections, the various cases will admit of simple treatment.

Thus in the case of a bend, as at AB (Fig. 98*a*), fitted with expansion joints at A and B, the resultant force on the bend will be readily found by a simple application of the principles of Sec. 47, case (*ac*) or (*bc*), as may be required. This force will then be represented by some resultant KL and which must be carried by reactions at the ends A and B. These end forces or reactions cannot be longitudinal. They will instead develop as side pressures between the two parts of the expansion joint. Taking these reactions at right angles to the pipe at these points, we have then the bend acting as a beam under a hydraulic unbalanced pressure load distributed over the curved portion, and supported by two forces acting at right angles to the pipe at the expansion joints. The total amount of the unbalanced pressure load is readily found as developed in Sec. 47, and this will

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serve, as in the usual manner with beam problems, to determine the end reactions at A and B. The distribution of the load is determined as in Sec. 47 and is represented by a force acting along the radius equivalent to a pressure pA/r per unit length of arc, where p is the pressure in the pipe, A the cross-sectional area and r the radius of the bend.

In this manner the distribution of the load and the end reactions become known. The entire problem becomes, therefore, reduced to the mechanics of the beam, and need not be here treated in further detail.

It may be stated, however, without present proof that the maximum bending moment at the middle point of a circular bend

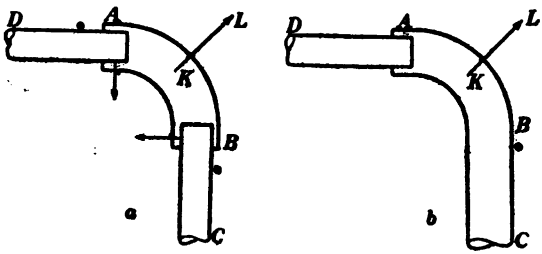

FIG. 98.—EXPANSION JOINTS COMBINED WITH ELBOW.

as developed by the application of the mechanics of the beam to this problem is shown to be

$$M=prA\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

where r=mean radius of bend and θ=half angle of bend.

For a 90° bend or elbow, as in Fig. 98*a*, this becomes:

$$M=\cdot 4142prA.$$

Thus with a 90° bend of 24 inches mean radius in a 10-inch pipe under a pressure of 100(pi2) and with the ends carried in slip joints, we shall have for the maximum bending moment at the middle of the bend the value:

$$M=\cdot 4142\times 100\times 24\times 78\cdot 54=78{,}000.$$

With metal about $\frac{1}{2}$-inch thick this would result in a stress of about 1800(pi2) in the outer fibre, a stress, therefore, by no means serious in itself.

In order that an elbow, as in Fig. 98*a*, may have support as assumed, the pipe must be tied or supported near the slip joints; otherwise the parts of the joint would separate completely.

If an elbow is fitted with a single slip joint, as in Fig. 98*b*, the condition as regards bending moment on the elbow is indeterminate. This results from the fact that the end reaction at A is indeterminate, depending on the stretch and flexibility of the pipe at B. The total

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force on the elbow will, however, be represented by a resultant KL, found as in Sec. 47, and the component of this parallel to DA will give a force which will tend to separate the parts of the joint and which must be carried by some tie or support near B or else by guard bolts at A preventing movement beyond a certain limit.

51. Case of a Long Pipe with Open Ends Carried in Slip Joints

An interesting case is presented by a slightly bent pipe, such as AB (Fig. 99), with the ends carried in slip joints and not maintained rigidly in line. We may thus assume the possibility of the pipe's

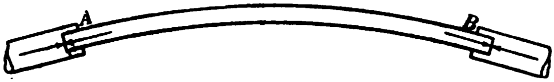

Fig. 99.—Resultant Force on Long Elastic Pipe.

assuming the form of an elastic curve as indicated in the figure. The balancing forces in this case will be represented by two forces P, P acting over the end sections and directed from within outward. Hence the resultant force on the pipe itself will be represented by two equal forces PP acting over the end sections and directed opposite, or from without inward. This places the pipe exactly in the condition of a curved column carrying on the end a load P.

We may also consider the pipe as in the condition of a beam

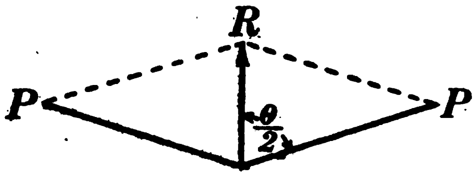

Fig. 100.—Resultant Force on Long Elastic Pipe.

loaded transversely with a distributed load, the resultant of which will be the same as for the two forces PP applied at the ends as above noted.

The law of distribution of this load has been determined in Sec. 47 and is given in equation (4). We have, therefore, for the transverse load Q per unit length along the pipe or beam the value

$$Q=\frac{P}{\rho}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

or, as noted, the transverse load varies along the line of the bend or curve, inversely as the radius of curvature of the bend.

Turning again to the pipe as a whole, the resultant of the two

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forces P acting on the ends and along the line of the axis at the ends will be given by a construction, as in Fig. 100, and measured by

$$R=2P\sin\tfrac{1}{2}\theta.$$

This resultant will tend to bend the pipe still further, and the actual result will depend on the relation between the magnitude of the resultant R, the law of the distribution of the actual forces along the length of the pipe, and the resistance of the latter to bending. The pipe may therefore be considered as in the condition of a column subjected to an end thrust P and liable to yield by buckling; or otherwise, to a beam subjected to a transverse load varying inversely as the radius of curvature of the pipe.

In order to develop the question of the relation of the strength of the pipe to the magnitude of this transverse load, we may employ the well-known relations in the theory of elastic beams between load, shear, bending moment, slope and deflection.

These are embodied in the following equations:

Let Q=load per unit length.
S=shear.
M=bending moment.
m=tangent of slope from straight line.
y=deflection from straight line.
I=moment of inertia of section.
E=coefficient of elasticity.
x=distance along pipe.
L=length of pipe.
p=unit pressure in pipe.
D=diameter of pipe.
t=thickness of wall.

Then
$$\left.\begin{aligned} S&=\int Q\,dx\\ M&=\int S\,dx\\ m&=\frac{1}{EI}\int M\,dx\\ y&=\int m\,dx \end{aligned}\right\}\quad\text{.........................}\quad(14)$$

Or conversely,
$$\left.\begin{aligned} &\frac{dy}{dx}=m\\ &\frac{d^2y}{dx^2}=\frac{dm}{dx}=\frac{M}{EI}\\ &\frac{d^3y}{dx^3}=\frac{d^2m}{dx^2}=\frac{1}{EI}\frac{dM}{dx}=\frac{S}{EI}\\ &\frac{d^4y}{dx^4}=\frac{d^3m}{dx^3}=\frac{1}{EI}\frac{d^2m}{dx^2}=\frac{1}{EI}\frac{ds}{dx}=\frac{Q}{EI} \end{aligned}\right\}\quad\text{...........}\quad(15)$$

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Now when the radius of curvature ρ of a beam is large and the curvature small, as we may properly here assume, we have, by a well-known property of plane curves,

$$\rho = \frac{d^2y}{dx^2}$$

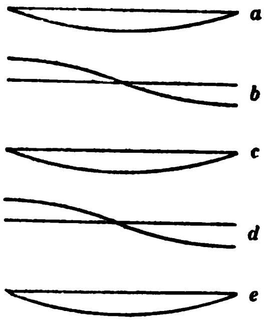

Fig. 101.
Resultant Force on Long Elastic Pipe.

For a curve such as Fig. 101*a*, where y is taken positive, downward, d^2y/dx^2 and hence ρ will be negative. The negative sign here is to be considered simply as having a directive significance. Hence in expressing Q in terms of P and ρ from (13) we must give ρ a negative sign in order that Q and P may both be considered positive. Having this algebraic relation in mind, we have, therefore,

$$Q = -\frac{P}{\rho} = -P\frac{d^2y}{dx^2} \quad \ldots\ldots\ldots\ldots (16)$$

Also we have in general (see (15)) :

$$Q = EI\frac{d^4y}{dx^4} \quad \ldots\ldots\ldots\ldots (17)$$

It will be noted that of these two equations (16) is restricted to the special case which we are here considering, while (17) is general. Combining we have

$$\frac{d^4y}{dx^4} = -\frac{P}{EI}\frac{d^2y}{dx^2}$$

Whence integrating twice, we have

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}y \quad \ldots\ldots\ldots\ldots (18)$$

The solution of this equation is the well-known sine curve, which we may take in the form

$$y = a \sin\frac{\pi x}{L} \quad \ldots\ldots\ldots\ldots (19)$$

Then differentiating twice, we have

$$\frac{d^2y}{dx^2} = -\frac{a\pi^2}{L^2}\sin\frac{\pi x}{L}$$

$$\text{or} \quad \frac{d^2y}{dx^2} = -\frac{\pi^2}{L^2}y \quad \ldots\ldots\ldots\ldots (20)$$

Comparing (18) and (20), we have

$$\frac{P}{EI} = \frac{\pi^2}{L^2}$$

$$\text{or} \quad P = pA = \frac{EI\pi^2}{L^2}$$

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Let D_1 and D_2 denote the external and internal diameters of the pipe and t the thickness of metal. Then we shall have

$$p=\frac{4\pi EI}{D_1^2L^2}\text{ .... }(21)$$

$$I=\frac{\pi(D_1^4-D_2^4)}{64}.$$

Where t is very small compared with the diameter we may take a mean value between D_1 and D_2 and reduce the value of I to $I=\pi D^3t/8$. Substituting in (21) and reducing we find the approximate value:

$$p=\frac{4{\cdot}935EDt}{L^2}\text{ .... }(22)$$

Turning now to (19) as the equation to the form assumed by the pipe, and taking successive derivatives, we have from (15) the following:

$$y=a\sin\frac{\pi x}{L}$$

$$m=a\frac{\pi}{L}\cos\frac{\pi x}{L}$$

$$M=-EIa\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}$$

$$S=-EIa\frac{\pi^3}{L^3}\cos\frac{\pi x}{L}$$

$$Q=EIa\frac{\pi^4}{L^4}\sin\frac{\pi x}{L}$$

These are illustrated in Fig. 101 a, b, c, d, e, and show that the successive values of load, shear, bending moment, tangent of slope and deflection are given by alternate sine and cosine curves with coefficients as indicated.

If now we assume the relation of (21) to obtain we may put for EI its value in terms of P and thus find for M, S, and z the following:

$$M=-Pa\sin\frac{\pi x}{L}=-Py$$

$$S=-Pa\frac{\pi}{L}\cos\frac{\pi x}{L}$$

$$Q=Pa\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}=P\frac{\pi^2}{L^2}y.$$

It will be seen that the mid-length value of y is a. It is also seen that p in (22) is independent of a. That is, the value of P or p in order that the assumed conditions may subsist is independent of the mid-length deflection a.

This means that no matter what the deflection, so long as it is not enough to involve any marked curvature of the pipe, that is, so

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long as we may consider the value of ρ to be given by d^2y/dx^2, so long will the pipe remain in neutral equilibrium in the form of a sine curve under the constant value of P or p as given above.

That is if the pipe is given any mid-length deflection a, so long as a is relatively small, the pipe may be expected to assume a sine curve with a for the maximum deflection and to remain in equilibrium under the forces operating. If then a is increased or decreased, so long as it still remains relatively small, the pipe will remain wherever it is placed and in equilibrium under the forces operating.

This value of p may therefore be taken as a critical value, defining the condition for neutral equilibrium under the general conditions assumed. If p is less than the critical value, then the pipe, if slightly displaced from a straight line, will return by the operation of its elastic forces. If p is greater than the critical value, then the deflection will increase beyond limit and flexure will result, at least unless other conditions step in to prevent. This limit value of the pressure is relatively high. Thus let $D=3$ inches, $L=120$ inches, $t=\cdot 1$ inch. Then from (22) we find $p=2879$(pi2).

The indications of the formulæ of the present section, and in particular of (21), giving the value of the critical load, have been verified experimentally by Fidler* with both copper and drawn steel pipes respectively 1·17 inches and 1 inch outside diameter by 1·00 inch and ·90 inch inside diameter, and 10 feet long.

52. Influence of Anchors, Piers, Ties, Abutments, etc., on the Development of Stress in Pipe Lines

In the preceding sections we have examined the various sources of load which may enter into the production of stress in pipe lines and pipe line elements. It is clear, of course, that these loads must all be carried in one way or another, but it does not follow that they will all enter fully into the production of stress on the pipe line itself. This will depend very largely upon the manner in which the pipe line is supported or constrained, and hence upon the extent to which such loads may be carried in whole or in part by the piers or anchors or other means of support or constraint.

Loads arising from balanced internal pressure must, in general, be carried by the pipe, or pipe line element subjected to such pressure. Loads arising from unbalanced internal pressure are very commonly carried, in some part at least, by various external means of support or constraint. If such unbalanced force is localized at a known point, such as the reaction from a stream issuing from an opening, the support or constraint can be applied in

* "Calculations in Hydraulic Engineering." Longmans, Green and Co., London and New York.

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the line of the resultant of such force, thus relieving the pipe line itself of any cross-breaking load. If the forces producing load of this character are distributed over a considerable extent of the line, as for example the weight of a horizontal line of pipe with its contained water, then support will naturally be supplied at appropriate intervals, thus placing the line in the condition of a continuous girder with stresses developed accordingly.

It is clear that parts of the system separated by points of complete constraint will represent independent systems so far as these various forces and the resulting stresses are concerned. Hence in determining the stress due to loads resulting from unbalanced forces, the following general program may be followed:

1. Note the separation of the system as a whole into parts by points of complete constraint.

2. Taking any one part thus set off, determine in magnitude, direction and line of application, the various unbalanced forces due to internal pressure.

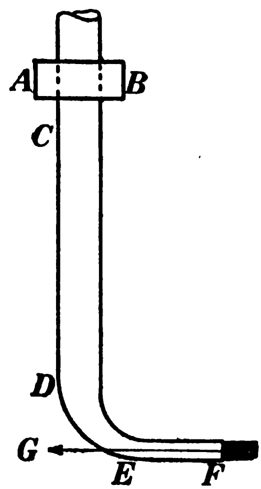

Fig. 102.—Reaction on Support.

3. Determine also the forces due to gravity on the pipe and its contents, in case such are of importance for the problem in hand.

4. In case there is but one point of constraint capable of resisting the forces in any one plane, then the problem with reference to such forces is entirely definite. The loads are known and they must all be carried at the one point of constraint. The problem from this point on is therefore one of the mechanics of materials and need not here be further considered.

5. In case there are two or more points of constraint which might share in resisting the forces in any one plane, then the problem is entirely indefinite as to the part of the load carried by the pipe and that carried by the points of constraint, and hence indefinite as to the stress developed at any point in the pipe due to the loads. In order to reach any definite result some assumption must be made regarding the manner in which the constraint is shared among these various points. With such an assumption made, the problem becomes one of simple mechanics as before.

A few simple illustrations will serve to show the application of these general principles.

In Fig. 102 suppose CD vertical and EF horizontal with AB as a point of support and constraint. Then the weight will be carried at AB as a direct load. Again the reaction due to the escaping jet at F will be represented by a force acting along the line FEG. This

will place the pipe in the condition of a cantilever beam with load at the end as determined by the magnitude of the reaction due to the jet. This reduces the problem to one of simple mechanics.

Again, in Fig. 102 suppose that *CDEF* is horizontal. Then there will be a vertical load perpendicular to the plane of the paper due to gravity acting on the pipe and its contents and a horizontal load acting along the line *FG*. This will give a resultant load in an oblique direction with the pipe acting as a cantilever beam.

Again, if we assume *CD* horizontal but the nozzle turned vertically down, the reaction of the latter will oppose gravity and we shall have the net resultant in a vertical line with the pipe acting still as

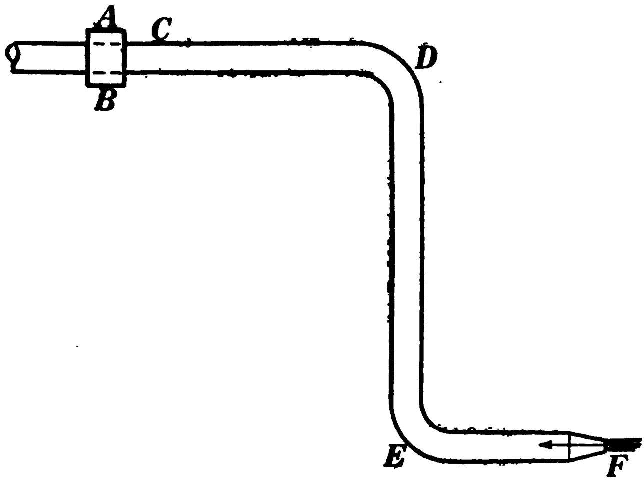

FIG. 103.—REACTION ON SUPPORT.

a cantilever. If in the last case the nozzle is turned up instead of down, the two loads will be additive instead of subtractive in their relation, with the pipe acting as a cantilever.

In these various cases, of course, the line of action of the resultant of the gravity forces will pass through the centre of gravity of the pipe and its contents, and that of the reactive forces along the line of flow at the nozzle backward from the outlet.

Precisely the same methods apply in the case indicated in Fig. 103.

In Fig. 104 the same arrangement is indicated as in Fig. 102 with the addition of an abutment of some sort at *G*. This will place the pipe in the condition of a beam fixed at *C*, supported at *G* and loaded at *D*. The force at *C* is readily determined by taking moments about *G* while the bearing reaction at *G* will be the sum of the forces at *D* and *C*. Precisely the same relations will obtain in case of a tie at *G* instead of a strut or abutment.

In case the point *G* is located at *D*, the point *C* will be relieved of load except as some stretch of the tie or yield of the abutment may

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permit a cross-breaking load on the pipe. The condition, therefore, is indefinite except as some assumption is made regarding the extent of stretch or yield, or regarding the degree in which the load is thus divided between the external constraint and the pipe.

In a case such as that of Fig. 105, with two points of support, one on either side of the elbow, the condition is definite if the supporting forces are in the nature of flexible ties. In such case the

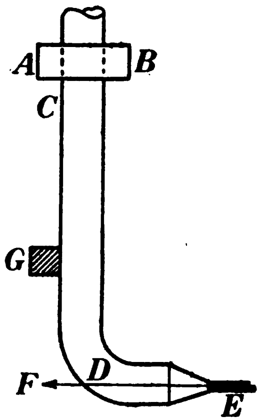

FIG. 104.
REACTION ON SUPPORT.

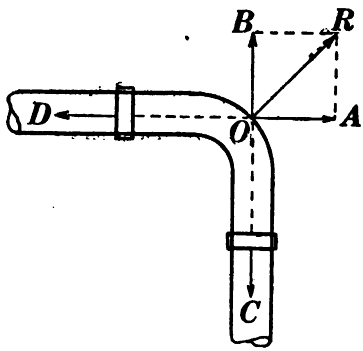

FIG. 105.
REACTION OF ELBOW ON FLEXIBLE TIES.

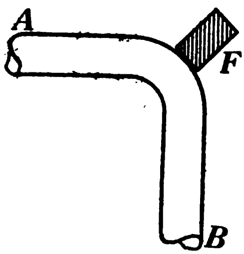

FIG. 106.
REACTION OF ELBOW ON DIRECT SUPPORT.

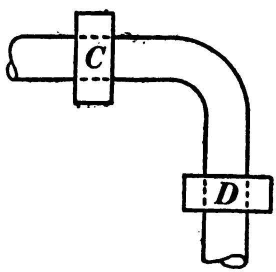

FIG. 107.
REACTION OF ELBOW ON CONSTRAINING PIERS.

force D alone can oppose the load OA and the force C alone can oppose OB. This condition will therefore definitely determine known total tension loads at C and D.

If, however, the elbow is supported against an abutment F (Fig. 106), the case becomes indefinite. If F can be assumed to carry the entire load, there will be no load transmitted to the pipe to be carried at any other point. Otherwise, as F may yield in some

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degree, load will be thrown on the pipe which must be carried at some other point.

Again, if the supports at C and D (Fig. 107), are of the nature of anchor blocks giving complete constraint, then the problem becomes indefinite. Either point is capable of carrying the load due to the elbow, and hence the actual amount carried at each point will be indefinite except as some special assumption is made.

Such cases of partial support or constraint are often furnished in the case of buried pipes by the weight of earth cover or by the resistance to lateral or longitudinal movement furnished by earth filling.

In all such cases of uncertain distribution of support or constraint, judgment alone can be relied on to determine some reasonable basis of division with a safe margin to allow for the measure of uncertainty involved.

In the case of cast-iron pipe with bell and spigot joint, especial attention must be given to the proper support or constraint of elbows in order to prevent the danger of separation at the joint. This is of particular importance in high-pressure fire lines at hydrant settings and elsewhere where the forces developed might seriously endanger the integrity of the line. To safeguard such points suitable strap and rod ties are commonly fitted, preventing movement of such a character as to render possible the opening up of the joint.

53. Stresses in Connections and Fittings

In the preceding paragraphs we have considered the subject of pipe line elements, connections and fittings with special reference to the unbalanced forces which may develop, and with emphasis on the loads which such forces may throw on the pipe line itself. We have now to consider briefly the stresses which may develop in the connections and fittings themselves.

From the hydraulic standpoint there is no sharp line of demarcation between the pipe and a fitting or connection, such as an elbow, angle, Y branch, Tee, valve or nozzle. They are all parts of a continuous water conduit and hence subject to the same fundamental hydraulic laws and principles. We therefore proceed to the determination of the stress on any section of such an element by considering the part lying between such section and the nearest point of constraint, and determining the load on such part by the principles discussed in the preceding paragraph. If the part thus cut off by the section under consideration is without constraint, the problem is simplified by the elimination of any considerations of the share of the load which the constraint may carry. From the load thus determined, the stress in the section is then to be determined by the usual methods of the mechanics of materials.

In many cases the parts may be considered as without constraint

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in one or both directions at right angles to the sections under consideration. In such cases the tension over any such section due to the direct loading resulting from the internal pressure will result as follows:

Let A=projected area of surface subjected to pressure on the side of the section not subject to constraint.

a=area of section (metal).

p=unit pressure in chamber.

Other notation as above.

Then from the principles already adduced we have

$$pA=Ta.$$

$$\text{or} \quad T=\frac{pA}{a} \qquad (23)$$

Thus if a constant value of T is to be maintained we must maintain a constant relation between A and a for the various sections which we may cut across the chamber. This will usually be neither practicable nor desirable on account of the existence of other stresses, as we shall see below. In any event it will result that the maximum value of T will be found where there is the maximum value of A/a; or otherwise, on the section where the area of metal is least in proportion to the projected area subject to load.

In any such case, therefore, it is only necessary to seek out by trial the section where the area of metal is least in proportion to the projected area subject to load and to find T as above. This will give the maximum direct stress in tension due to internal pressure.

In all such chambers of irregular form, such as Y branches, valve bodies and elbows, there will be certain sections which are noncircular in form. In Y branches in particular, certain parts approach a flat or only slightly curved form. In all such cases the stress along any filament lying between two such parallel sections will not be wholly tension. A bending stress with its accompanying shear will be set up, and the unit stresses from these must be combined with the stress in tension due to direct loading. Usually the forms of such chambers are not such as to permit of direct investigation and indirect and approximate methods must be used. We have two reference forms to which the actual forms may usually be approximated. These are the ellipse as a form of section and the flat plate.

When parallel sections for some little distance are nearly uniform in size and form and approximate to an ellipse, we may apply the formulæ for the strength of a pipe of elliptical section.*

These are as follows:

Let a and b denote the two semi-axes of the ellipse (see Fig. 108).

* "Résal Traité de Mécanique," Vol. V, p. 134.

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Assume a dimension of unity or 1 inch in a direction perpendicular to the plane of the section.

Let c = excentricity $=\sqrt{1-(b/a)^2}$.

C = a constant depending on the ratio of b to a and given approximately by

$$C = \cdot 333 + \cdot 167 \frac{b}{a}$$

M_A = bending moment at end of long axis.

M_B = bending moment at end of short axis.

p = unit internal pressure.

Then we have

$$M_A = \frac{pa^2c^2}{2} C \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(24)}$$

$$M_B = \frac{pa^2c^2}{2}(1-C) \quad \ldots\ldots\ldots\ldots\ldots \text{(25)}$$

In the above equations the inch and the pound are the units.

The bending moment at A will develop tension on the inside and compression on the outside, while that at B will develop tension on the outside and compression on the inside. At A, therefore,

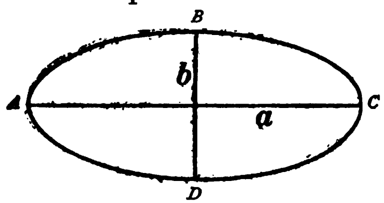

Fig. 108.—Stress on Elliptical Section.

the tension due to the direct load (p over the diameter AC) must be added to the tension on the inner fibres due to M_A, in order to give the total maximum tension at A, which will be on the inner fibres. Similarly at B the tension due to the direct load (p over the diameter BD) must be added to the tension on the outer fibres due to M_B, in order to give the total maximum tension at B, which will be on the outer fibres.

By the use of these equations an approximate value may be developed for the maximum stress in any such elliptical section.

Where the form approaches that of a flat plate over a certain area, the necessary support is usually developed by the use of a system of intersecting ribs, thus forming a series of cells, usually triangular or four-sided, the flat base of which is intended in each case to be self-supporting as a flat plate. In this mode of design, therefore, the ribs are intended to act as girders in carrying the load, while the elementary flat or nearly flat areas thus formed are expected to be self-supporting between the ribs, and to contribute to the support of the load as a whole.

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Two questions thus arise:

1. The thickness of the flat plate between the ribs.
2. The dimensions and spacing of the ribs.

In many cases, however, the thickness is fixed by other considerations or by relation to other parts, and question (1) only remains.

The design of these features is necessarily by empirical formulæ, as follows:

Thickness of Flat Plates supported by Ribs.

Let A=area of cell or element supported by ribs at boundary, (i2). It is here assumed that no two dimensions of A differ widely.

p=pressure (pi2).

t=thickness (i).

C=constant, about 100 for cast iron and 160 for cast steel.

Then

$$t=\frac{\sqrt{pA}}{C} \qquad (26)$$

Supporting Ribs.—Regarding the design of the supporting ribs, two questions arise:

1. The spacing of the ribs.
2. The dimensions of the ribs.

Only the most general indications can be given regarding these matters, and reference should be made, if possible, to successful designs of similar character and operating under generally similar conditions.

The spacing of the ribs is usually made from 8 to 16 or 18 inches, varying with the size. The thickness should be not far from the thickness of the body or shell and the height may vary from 4 to 6 or 8 times the thickness at the highest part, often tapering or fading off to nothing at the ends, according to the features of the design. As a general guide, the results to be applied with judgment, use may be made of the formula:

$$\frac{pAL}{24}=\frac{kI}{y_o} \qquad (27)$$

Where p=pressure (pi2).

A=area of a strip considered as supported by one rib through from one end to the other. Normally A will equal the product of the distance between the ribs by the extreme length (i2).

L=length of rib from end to end (i).

k=stress developed in outer fibre (pi2).

I=moment of inertia of section of rib (i4).

y_o=distance from neutral axis to outer fibre (i).

In taking values of k, I and y_o, the thickness of the metal constituting the body or shell may be added to the height of the rib proper. Thus for illustration, suppose an area 15 inches wide and

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60 inches long considered as supported by a rib 2 inches thick and 10 inches total height at centre. What will be the safe pressure, allowing a working stress of 16,000 pounds in the steel? We find $pAL/24=2250p$ and $kI/y_o=533333$. Whence $p=237$ (pi2).

As noted above, however, the results of no formula alone should be accepted without judgment and comparison with similar cases if possible.

54. STRESSES OF JOINT FASTENINGS, FLANGES, BOLTS, ETC.

Stresses in joint fastenings may be either tension or shear. Wherever a longitudinal pull develops in a pipe line, the joint fastenings must, of course, carry such pull as a load in tension or shear, depending on their disposition relative to the joint—shear in a riveted joint and tension in a flange joint with bolt fastenings. Similarly where a bending stress develops, the fastenings will be thrown into either shear or tension. In all such cases the principles of the present chapter will serve to determine the load at the joint, at least so far as it is determinate, and the problem is thereby reduced to one of the mechanics of materials and may be treated accordingly.

55. STRESS DUE TO BENDING MOMENT IN SPANS

A long pipe line supported at frequent points, insofar as its relation to deformation through gravity forces is concerned, operates as a continuous girder.

The mechanics of a continuous girder or beam shows that the maximum bending moment occurs at the pier (considered as a point of support), and is

$$M=\frac{WL}{12} \qquad (28)$$

where W=weight between supports and L=length. We have, then, from the familiar beam formula:

$$\frac{kI}{y_o}=\frac{WL}{12} \qquad (29)$$

Where k=stress in outer fibre of beam.
I=moment of inertia of section.
y_o=distance from neutral axis to outer fibre.

For a cylindrical shell about a diameter, we have

$$I=\frac{\pi D^3 t}{8}$$

$$y_o=\frac{D}{2}$$

Also $W=\dfrac{w\pi D^2 L}{4}+\sigma\pi DtL$ (pounds).

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Where all dimensions are in inches, w=weight of one cubic inch of water and σ=weight of one cubic inch of steel.

Hence we find

$$\frac{k\pi D^2 t}{4}=\frac{w\pi D^2 L^2}{48}+\frac{\sigma\pi D t L^2}{12}$$

$$\text{or}\quad kt=\frac{wL^2}{12}+\frac{\sigma t L^2}{3D}$$

$$\text{and}\quad k=\left(\frac{w}{12t}+\frac{\sigma}{3D}\right)L^2$$

But $w=\cdot0361$ and $\sigma=\cdot2835$.

Substituting these values we have

$$k=\left(\frac{\cdot00301}{t}+\frac{\cdot0945}{D}\right)L^2 \text{ (pi2)}\ldots\ldots(30)$$

In the preceding formulæ the values of I and of w are expressed on the assumption that the thickness of the pipe is small compared with the diameter. This assumption is usually permissible in pipe line problems. It may be noted, however, that the value of D used should correspond to the mid-thickness of the shell. If higher accuracy is desired or if t is not small compared with D then we must use the following:

$$I=\frac{\pi(D_1^4-D_2^4)}{64}$$

$$w=\frac{w\pi(D_2^2 L)}{4}+\frac{\sigma\pi(D_1^2-D_2^2)L}{4}$$

where D_1 and D_2 denote the outside and inside diameters respectively.

56. Combined Stresses

In various cases combinations of stresses may exist, such, for example, as tension or compression combined with shear, tension or compression combined with bending, tension or compression combined with torsion, bending combined with torsion.

In order to find the maximum intensity of stress in such cases recourse must be had to the principles and methods of mechanics as developed in textbooks on that subject. The principles of the present section will serve to develop the conditions of the pipe or pipe line element as to the magnitude and location of the loads. Beyond this point the problem becomes one of mechanics.

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CHAPTER V

MATERIALS, CONSTRUCTION, DESIGN

THE main purpose of the present work is the discussion of the various hydraulic problems which may arise in connection with the transport of liquids through pipe lines. No attempt is made therefore to present, with any degree of fullness, discussion of constructive features. In fact, a presentation of the constructive features of pipe lines and their mountings and attachments, including valves, elbows, Y's, expansion joints, etc., and treated with reasonable fullness from the standpoint of description and general discussion, would require a volume in itself.

The purpose of the present chapter is therefore to present briefly, and from the standpoint of description and general discussion, mention of the more important structural elements of pipe line construction, and with some reference to the more important problems which may arise in connection with them, but without attempt to approach a comprehensive treatment of this phase of the subject.

57. MATERIALS

The materials commonly employed for the construction of pipe lines are steel, cast iron, wood stave and reinforced concrete.

Steel is employed in two forms, plate or sheet steel and cast steel.

Plate or sheet steel is employed for two classes of pipe as follows:

(*a*) Commercial pipe as commonly employed for piping steam, air, gas, water, etc., and in sizes from $\frac{1}{8}$ inch to 12 or 15 inches inside diameter and up to 30 inches outside diameter.

(*b*) Pipe made up of steel plates with longitudinal joint either riveted or welded, with diameter from 16 or 18 inches to 7 or 8 feet, and with thickness of plate to suit the special requirements of the case.

Cast steel is employed for bends, elbows, Y branches, flanges, saddles and other like connections or accessory mountings, and also occasionally for short connecting lengths of pipe.

Cast iron is employed in the form of lengths (usually 12 feet), with the well-known bell and spigot form of joint, and in commercial sizes and thicknesses of metal up to 84 inches diameter, and in special sizes and thicknesses of metal to suit the requirements of

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the case. Cast iron is also employed for bends, elbows, Y branches, flanges, saddles, and other like connections or accessory mountings, but where the pressure requirements are relatively light. Where the pressure requirements are moderate to high and in the best grade of work for all pressures except the very lowest, such items should be made of cast steel.

Wood stave pipe with steel rod or wire band reinforce is employed for moderate pressures, and where the special characteristics of such pipe may meet the requirements of the situation.

Reinforced concrete has been only sparingly employed for pipe line construction. With sufficient steel reinforce it may readily be made adequate in strength for moderate pressure requirements. Due, however, to the fact that the tensile strength of concrete is low and that full assurance cannot be had of the simultaneous development of stress in both concrete and steel each in proportion to its safe strength, it is necessary in practice to supply steel circumferential reinforce sufficient to carry the full internal load by itself and without assuming aid from the concrete.

In the case of moderate to high pressures, therefore, where the pressure is the determining element with regard to thickness of wall, there would be no advantage in using concrete except for its value as a protective coating for the steel. In the case of light pressures, however, where the amount of steel required for strength is far less than that required for stiffness, resistance to external collapse, and for durability under corrosion, the combination of adequate reinforce steel for strength under internal pressure with concrete for stiffness and for protection, may present advantages sufficient to justify its use.

Broadly speaking, however, steel or cast-iron pipe with like fittings and connections furnish the typical or representative practice in pipe line construction. Furthermore, while the general principles of pipe line flow are independent of size, many of the special problems considered in the present work imply the larger sizes of pipe line, such as would be typical of power plant practice or of the transport of large volumes of liquid over long distances.

We may now consider in some further detail the principal types and forms of pipe and pipe line construction.

58. Commercial Pipe

This is placed on the market in three grades as regards strength, known respectively as *standard*, *extra strong* and *double extra strong*. Commercial sizes of standard pipe as shown on recent manufacturers' lists show nominal sizes ranging from $\frac{1}{8}$ inch to 15 inches rated by inside diameter and sizes ranging from 14 to 30 inches rated by outside diameter. The latter is commonly designated as O.D. pipe. For the pipe rated on inside diameter the actual diameter

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differs somewhat from the rated diameter, usually in excess, especially in the smaller sizes. The actual outer diameter of O.D. pipe agrees with the nominal rating.

Standard engineering handbooks may be consulted for details regarding the characteristics and dimensions of such pipe.

In the case of extra strong and double extra strong, the added thickness is placed on the inside, thus reducing the inside diameter, but leaving the outside diameter the same, and thus suited to the same fittings and screw connections as standard pipe. In so far as standard fittings and connections may be considered adequate, they may therefore be used with the extra and double extra strength pipe. If otherwise, special fittings and connections are required, the tapping size will in any event be the same as for standard sizes of pipe.

Engineering handbooks or dealers' lists may be consulted for details regarding the characteristics and dimensions of such pipe.

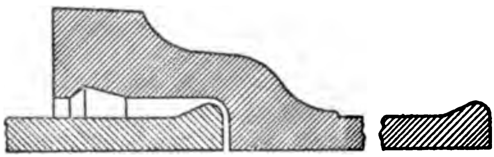

FIG. 109.—BELL AND SPIGOT JOINT.

O.D. pipe is made as noted above in diameters listed by the manufacturers, from 14 to 30 inches, and in varying thickness according to service required, from $\frac{1}{4}$ to $1\frac{1}{8}$ inches.

The usual connections and fittings for commercial pipe, such as bends, turns, elbows, tees, Y branches, valves, etc., are too well known to require detailed consideration in the present work. Full information regarding these matters is furthermore available in the various manufacturers' or dealers' lists.

59. CAST-IRON PIPE. COMMERCIAL SIZES AND STANDARDS

Commerical cast-iron pipe is made in a wide variety of diameters and thicknesses of metal, in accordance with varying requirements and varying standards of design and manufacture. The nominal size is measured on the inside, and the varying thickness of metal affects therefore only the outside diameter.

The usual manner of connecting successive lengths of such pipe is by means of the bell and spigot joint formed with space for calked metallic lead as packing material, as shown in Fig. 109.

Engineering handbooks or dealers' lists may be consulted for details regarding the characteristics and dimensions of such pipe.

In addition to the standard bell and spigot form of joint, various

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forms of special joint are occasionally employed. Thus in Fig. 110 are shown two forms of flexible joint, lead packed. The form shown at *a* is more commonly employed, while that shown at *b* is more expensive and is intended for large pipe under relatively high pressure. So-called "Universal" pipe, as shown in Fig. 111,

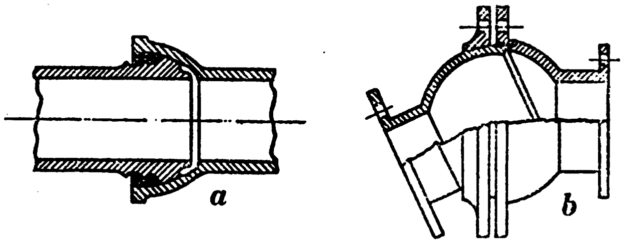

FIG. 110.—SPECIAL FORMS BALL AND SOCKET JOINTS.

is fitted with an inside and outside taper or cone joint with machined surfaces, giving an iron on iron contact. The slope of the tapers are slightly different, thus allowing some degree of flexibility while still remaining tight. The two parts of the joint are drawn together by bolts carried in lugs as shown.

A great variety of formulæ have been proposed and employed, giving the relation between the diameter, thickness and pressure

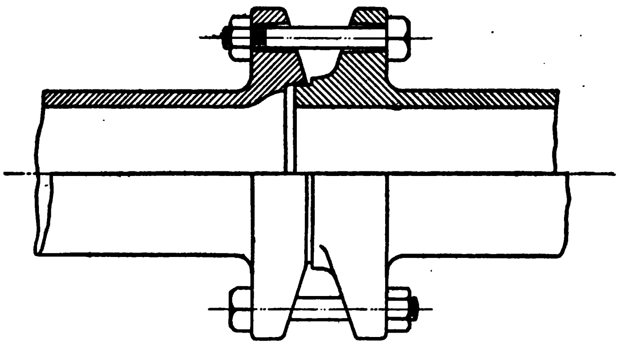

FIG. 111.—UNIVERSAL PIPE JOINT.

or head for safe operating conditions. Among these Fanning's, which has had, perhaps, as wide acceptance as any, may be put in the following form :

$$t=\frac{(p+97{\cdot}2)d}{7220}+\frac{1}{8}$$

where t=thickness (i).
p=pressure (pi2).
d=diameter (i).

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This formula, which gives results agreeing well with the figures of standard practice, implies a wide variation in the actual working stress for varying sizes and pressures. From the form of the expression it is clear that the thickness t provides for the following combination :

1. A working pressure p with a working stress of 3610 (pi2).
2. An excess pressure of 97·2 (pi2) (or in round number 100 (pi2)) as the result of shock or other unusual conditions, and with the same working stress.
3. An excess thickness of $\frac{1}{8}$ inch to allow for corrosion and wear, accidents of manufacture, etc.

In addition to the standard thickness of cast-iron pipe and fittings suited to meet ordinary requirements, specially heavy grades are listed by large manufacturers and intended to meet the requirements of specially high pressures, the thickness and other dimensions being graded to the working pressure to be carried, usually in 100-pound steps up to 400 or 500 pounds per square inch. Manufacturers' lists may be consulted for the details of such extra heavy pipe.

60. Sheet Steel Pipe

Sheet steel pipe is made up in lengths according to the available dimensions of steel plate, and joined length to length either by circumferential riveted joints or by means of flanges or other special form of joint. Longitudinal joints are either riveted or welded. As a general rule each length or section of pipe is made up of a single plate wrapped around the circumference, and hence with but one longitudinal joint per section.

Taking first the form with riveted joints we shall pass in rapid review the chief constructive features.

The available materials are sheet steel plates in widths up to 8 or 10 feet, in lengths up to about 20 feet, and in thicknesses increasing by sixteenths up to $1\frac{1}{2}$ inches or more if desired.

It is shown in mechanics that in the case of a cylinder under internal pressure the stress along a longitudinal ideal section is twice that along a circumferential section. It follows that the chief effort, in the matter of fastenings, must be directed toward the development of the highest possible efficiency in the longitudinal riveted joint. The only exception to this general rule is found in the case of pipes subjected to light pressure where strength is not the ruling factor in determining the thickness. An illustration will make clear the considerations involved. Assume a diameter of 24 inches and a pressure of 30 (pi2) with a safe actual working stress of 15,000 (pi2) and a longitudinal joint efficiency of 80 per

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cent. From mechanics we have for the thickness of a pipe under internal pressure, the formula:

$$t=\frac{pd}{2Te}$$

where p=pressure (pi2).
d=diameter (i).
t=thickness (i).
T=safe working stress in joint.
e=joint efficiency.

Substituting in this formula we find $t=\cdot03$.

It appears, then, that so far as strength alone is concerned a thickness of ·03 or, say, $\frac{1}{32}$ inch would be sufficient under these conditions.

Such a pipe, however, would lack stiffness and resistance to external local forces or to collapse under external pressure in case the pressure within should ever fall below the atmosphere. Furthermore, there is no provision against corrosion or wear, except as contained in the factor of safety. Thus with an ultimate strength of steel plate at 60,000 (pi2) the factor of safety when new is 4. Suppose now a thickness of no more than ·01 inch to disappear under the influence of corrosive action. This means the loss of one-third the available metal and the reduction of the factor of safety to 2·67. With $\frac{1}{64}$ inch thickness removed by corrosion, only one-half the original metal remains and the factor of safety has fallen to 2. The serious results of such a condition are plainly apparent, and in order to provide for a reasonable life under corrosive action and also to give stiffness and strength under external load, it is necessary in all such cases to add arbitrarily to the thickness. For the case mentioned, $\frac{1}{8}$ or $\frac{3}{16}$ inch would be considered the minimum thickness permissible. For all such cases it appears, therefore, that strength under internal pressure is not the ruling factor in the determination of thickness, and hence that the realization of the highest possible efficiency in the longitudinal joint is of less importance than in cases where, under high pressures, strength under such pressure becomes the ruling factor.

Longitudinal Joints.—For the longitudinal joints of steel plate pipe line sections, a great variety of design may be employed, according to the efficiency to be considered significant in the special case.

Thus when stiffness and allowance for corrosion determine a thickness considerably in excess of the requirements for strength against internal pressure, a single riveted lap joint may be employed. A double riveted lap joint will give a somewhat higher efficiency and a joint more easily made and kept watertight. Next in order, and where it may be desirable to maintain a more truly circular form of section, comes the single or external butt strap joint with either single or double riveting, as indicated in Fig. 112. The efficiency of

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the single riveted lap joint will range about ·55 or ·56 and of the double riveted lap joint about ·70. The efficiency of the single and double riveted single butt strap joints will range about the same as for the corresponding lap joints, the only advantage being in a more truly circular form of pipe section in the latter case.

Where high efficiency is desired, as in all cases where thickness for strength against internal pressure is the determining feature, the lap or single butt strap joints should not be employed. For such cases double butt straps with the three or even four rows of riveting on each side are employed, raising the efficiency of the joint to from ·80 to ·90 or more according to the particular design of joint employed.

A detailed discussion of the theory and manifold forms of riveted joints is outside the scope of the present work. In Figs. 113, 114, however, are shown diagrams of approved forms of such joints as may be applied to the longitudinal seams of steel pressure pipes.

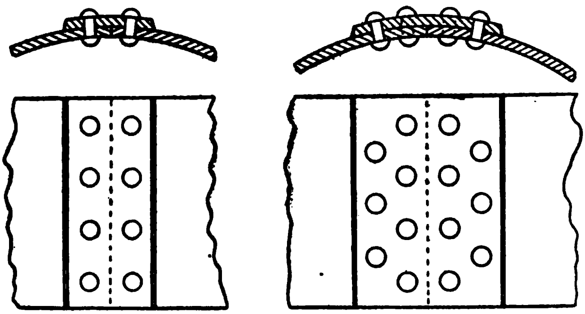

FIG. 112.—RIVETED JOINTS.

In connection with these diagrams a few indications may be given regarding the examination of any proposed form of joint for strength and efficiency.

It should first be noted that such procedure is in considerable degree arbitrary in character, since we do not know the influence of the friction between the plates and of various other factors which may affect the relation between the load and the manner in which it is carried by the component elements of the joint. With the usual conventions, however, the joint may be examined as follows.

Obviously the resistance to rupture in all possible ways should be examined, and the strength for each such possible method compared with the strength of the plate as a whole. Actually only three possible modes of rupture are commonly considered.

1. Rupture by tearing between the outer rivets of the rivet pattern.
2. Rupture by shearing the rivet sections.
3. Rupture by crushing due to the bearing load between the rivet and the metal of the plate.

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The unit of the joint commonly taken is the element of the rivet pattern covering a distance equal to one space in the outer row of rivets, as AB, Figs. 113, 114.

Let p=pitch of rivets in the outer row.
t=thickness of plate.
d=diameter of rivet.

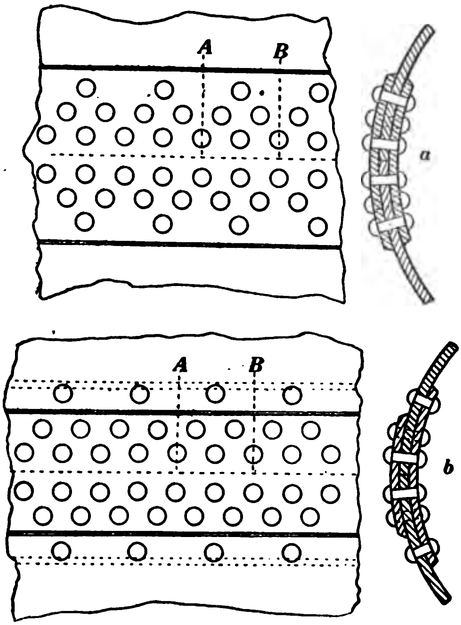

FIG. 113.—RIVETED JOINTS FOR LONGITUDINAL SEAMS.

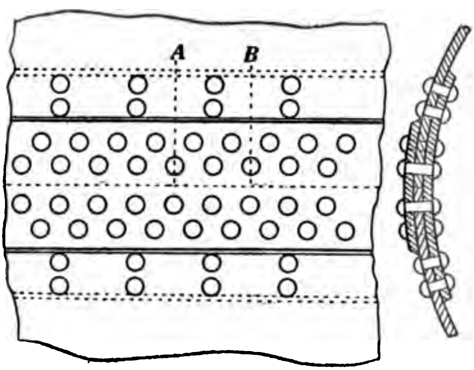

FIG. 114.—RIVETED JOINT FOR LONGITUDINAL SEAMS.

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m=number of rivets in single shear in one element of the rivet pattern.

n=number of rivets in double shear in one element of the rivet pattern.

T=tensile strength of plate.

S_1=shearing strengths of one section of rivet in single shear.

S_2=shearing strength of one section of rivet in double shear.

C_1=crushing strength for rivets in single shear.

C_2=crushing strength for rivets in double shear.

Then for the total strength of plate of width p we have

$$(1)\quad ptT.$$

For the strength against tearing along the line between the two rivets in the outer row we have

$$(2)\quad (p-d)tT.$$

For the strength against rupture by shear we have

$$(3)\quad (mS_1+2nS_2).$$

For the strength against failure by crushing we have

$$(4)\quad dt(mC_1+nC_2).$$

The various efficiencies will be given by dividing the various expressions (2), (3), (4) by (1).

Obviously the lowest value must be considered as ruling for the joint in question.

The highest economy of joint is obtained when the proportions are such as to give equal values to all three efficiencies. This, however, is not always practicable, though in most cases this condition may be closely realized.

For steel plates and rivets the following values may be employed for the various strength factors above noted.

$$T=60{,}000$$
$$S_1=44{,}000$$
$$S_2=45{,}000$$
$$C_1=90{,}000$$
$$C_2=110{,}000$$

The distance of the centre line of the row of rivets nearest the edge of the plate should be from 1·5 to 2 times the rivet diameter.

The rivet diameter d should be from 1·2 t to 1·4 t.

In multiple staggered riveted joints the minimum distance between rows of rivets should be ·6 to ·8 the minimum pitch.

Welded Longitudinal Joints.—In place of riveted joints, welded longitudinal joints have, during the past decade, come into extended and approved use. Such joints are made lap-welded in gas-heated furnaces and with approved technique in the process show efficiencies of 90 per cent and better. Such form of joint is especially suited to the heavier thicknesses found necessary for the lower ends of high pressure penstocks and similar designs. Where the thickness would exceed 1 inch, the equivalent strength may be made up by a main shell with welded bands shrunken on, three or four inches wide and with an equal distance in the clear between bands. In

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this manner, pipe for the heaviest service may be made up in welded form.

Welded pipe, due to the higher joint efficiency as compared with riveted pipe, has the advantage of thinner plates and less weight for the same diameter and head or greater diameter for the same thickness and head. Due to the absence of rivet heads it has also the advantage of better hydraulic conditions of flow, as referred to in Sec. 61.

Circumferential Joints.—As already noted in the case of a cylindrical shell subject to internal pressure, the stress along a longitudinal line or section is twice that along a circumferential line or section. Or otherwise, a cylindrical shell under internal pressure has twice the strength against rupture along a circumferential line that it has against rupture along a longitudinal line. It follows that with equal efficiency in both longitudinal and circumferential joints the factor of safety will be twice as great against rupture along a circumferential line as compared with rupture along a longitudinal line. It results that, so far as strength alone is concerned, there is no occasion for using the especially high efficiency riveted joints which are required for the longitudinal seams. The efficiency of a properly proportioned single riveted joint is usually found about 55 per cent. Such a joint in a circumferential seam would therefore represent a factor of safety equal to that for a longitudinal joint with efficiency of 110 per cent were such efficiency possible. Or otherwise, with the very best efficiency of longitudinal joint possible, even supposing it to reach 100 per cent, and with a single riveted circumferential joint of 55 per cent efficiency, there will still remain an excess factor of safety with regard to rupture along the circumferential joint, and if tested to destruction, rupture will occur along the longitudinal joint.

It should be noted further that the development of full stress along a circumferential line in a cylinder under internal pressure presupposes a cylinder with closed ends and without external constraint. The condition of a pipe with open ends and through which is flowing a stream of water is far from fulfilling these specifications. In fact it is readily seen that in a typical pipe line, stress along a circumferential line will only be developed as a result of some combination of the following conditions and of which (4) must be a constituent element.

(1) Bends, turns or elbows.

(2) Changes in size.

(3) Closed valves.

(4) Such freedom from constraint at or near the conditions (1), (2) or (3) as to permit end movement relative to some other part of the pipe definitely anchored in place, thus developing a lengthwise pull on the pipe as a whole and a resultant stress along a circumferential line.

With actual pipe lines these conditions are likely to obtain in

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varying degrees, but it is unlikely that they will be such as to permit the development of the full lengthwise pull due to the internal pressure over a closed end of the pipe, and hence of the development of the full stress along a circumferential line (see Chap. IV, Sec. 52).

All of these considerations show, so far as we are concerned with strength against internal pressure, the greater relative importance of high efficiency in the longitudinal as compared with the circumferential joints, and the sufficiency of a properly proportioned single riveted lap joint.

The circumferential joint must, however, provide for other requirements quite independent of strength under internal pressure. These are:

1. Strength under bending stress to which the pipe may be subjected and general stiffness, coherence and continuity of strength.

2. Watertight closure of the joint. While the single riveted joint can be made watertight under normal working conditions, it is much easier to make a double riveted joint tight, and the latter is much less liable to develop leaks under irregular shifting stresses due to changes in the temperature conditions. The double riveted joint is also markedly superior to the single riveted in stiffness and in giving strength to the pipe under bending stress. For these reasons the double riveted form of joint is to be recommended for the circumferential seams in all high-grade work.

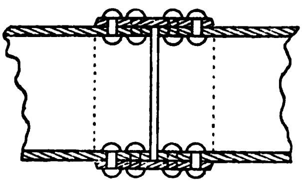

Fig. 115.—Riveted Joint for Circumferential Seams.

In connecting the successive sections of the line choice will lie between two modes of construction:

1. In and out sections, or alternate sections differing in radius by an amount equal to the thickness of the plate, and thus adapted to form the lap circumferential joint by the slipping of the smaller section into the larger.

2. Sections all of the same radius or diameter, butting together at the end and connected by an outside butt-strap, thus making a double lap joint (see Fig. 115).

The use of in and out sections has the advantage of a slight saving in cost and of being easier to calk from the inside. It has, however, the disadvantage of a periodic change in pipe diameter at every section, alternately large and small, so that there will be at each joint a definite loss of head (see Sec. 9) due to this cause. The use of uniform sections with outside butt-strap avoids this loss and thus furnishes much better hydraulic conditions than with alternating diameters.

To realize this advantage, however, the space left for calking, if

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any, between the ends of the sections must be filled in, otherwise there will be presumably little to choose between the two forms of pipe. We shall again refer to this subject at a later point.

By reason of the possibility of better hydraulic conditions, the use of sections of uniform diameter and with outside butt-straps is to be recommended in high-grade practice.

61. Hydraulic Conditions in Riveted Pipe Lines

The features which may enter into the production of eddy losses in riveted pipe lines are the following:

1. Rivet heads in both longitudinal and circumferential joints.
2. Longitudinal butt-strap ends.
3. Abrupt changes in size at the ends of sections, in case in and out sections are used.
4. Abrupt enlargement in size between the ends of section of uniform diameter, in case the ends are not butted closely together, or in case the space left between the ends is not otherwise filled in.

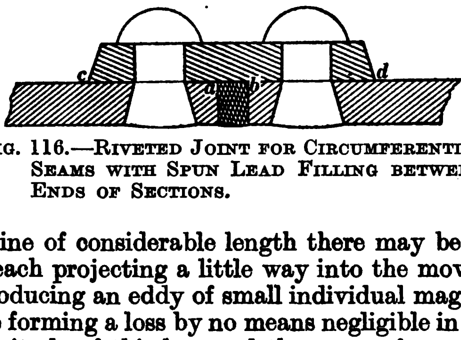

Fig. 116.—Riveted Joint for Circumferential Seams with Spun Lead Filling between Ends of Sections.

In a pipe line of considerable length there may be thousands of rivet heads, each projecting a little way into the moving stream of water and producing an eddy of small individual magnitude, but in the aggregate forming a loss by no means negligible in amount. The possible magnitude of this loss and the means for reducing it to a minimum have not attracted the attention which they deserve. Instead of the full projecting head, as is too commonly employed, the use of a countersunk nearly flush head or point, as shown in Fig. 116, would be advantageous. This gives the effect of smooth ship plate riveting and provides improved hydraulic conditions as compared with the projecting head. Pipe rivets are commonly driven, heads inside and points outside. This is primarily as a matter of convenience. With such mode of riveting, the rivet for the results of Fig. 116 must be formed with a special head. If, on the other hand, the rivets are driven head outside and point inside, the usual form of rivet will serve and it only remains to countersink the hole at the proper angle, head up, as in ship work, and chip off the excess. While the saving by such form of riveting can scarcely be estimated with any approach to accuracy, it seems well assured that it is by no means negligible in amount.

Longitudinal butt-strap ends have already been referred to. In

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common practice these are often shifted through a certain angle at each joint, usually 90°. This gives a series of ends, two for each section of pipe, all unshielded and all operating to produce loss through eddies and turbulence. There seems to be no necessary reason for shifting the straps. It cannot add essentially to the strength of the line. If then the straps are lined up along a single element of the cylindrical pipe, they produce simply a change in the form of cross section and doubtless introduce a slight loss along their longitudinal edges, but the loss due to their ends may be practically eliminated.

The influence of the abrupt change in size at the ends of sections where the in and out system is used, has been referred to previously. While not directly measurable and only to be inferred by comparison, the loss due to such ends is undoubtedly far from negligible. It may be obviated by the use of uniform sections with outside butt-straps, assuming proper care of the space between the ends as noted below.

The influence due to the sudden enlargement between the ends of pipes fitted with joints, as in Fig. 115, has been already referred to. If this space is left open, a loss will result and the possible advantage of the uniform diameter of sections will be largely, if not wholly, lost. To avoid this, the space should either be filled with some material such as spun lead, or the joints may be trimmed and butted close with electric welding in lieu of calking, as referred to below. The advantage, in respect of the hydraulic conditions of flow, offered by welded longitudinal joints in place of riveted joints, has already been noted in Sec. 60.

62. Calking of Riveted Seams

The calking of a riveted seam is much the more effective when applied on the side under pressure. It is obvious that it is easier to stop a small leak in a seam on the entering side rather than on the issuing side. Hence in pipe work inside calking is much more effective than outside calking. On the other hand, with the pipe under pressure, local leaks or imperfect sections of the joint can only be closed from the outside. For these reasons it may be recommended to carefully and thoroughly calk all seams on the inside before applying the pressure. Then for the closure of such small residual leaks as may show, or for the closure of small leaks which may develop with the pipe in service external calking may be resorted to.

In this connection reference may be made to the circumferential joint formed by an outside butt-strap, as shown in Fig. 116. If the section ends are butted close together it will not be practicable to calk on the inside at the angles a and b. On the other hand, unless the plates are trimmed with great nicety, they will not butt together closely all the way around, and a crack or opening will be left

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between the plates of varying width. To permit of calking at *a* and *b* the plate ends may be separated a distance somewhat less than the thickness, as shown in the figure. A special tool may then be employed for calking at *a* and *b*, thus insuring, as nearly as may be, a watertight closure of the joint.

This, however, leaves an opening between the plates sufficient to form an eddy of sensible magnitude as a result of the sudden enlargement in size of conduit. This again may be closed by calking spun lead into the opening. This is metallic lead in the form of fine threads, and forms an admirable material for filling such spaces. If the edges of the plates are slightly undercut, it will aid in holding this in place. In any event, the undercut developed by the calking at *a* and *b* will operate to form an anchor at the bottom for such lead filling. In this manner all requirements may be met; the joint may be calked on the inside and a smooth continuous surface provided for good flow conditions for the water.

63. Electric Welding at the Joints in Lieu of Calking

Modern developments in the art of electric welding with metal electrodes furnish an admirable substitute for the time-honoured practice of impact calking. By this means the entire line of contact between the two plates forming the joint may be closed and the plates locally united by welded metal filling. The use of local electric welding instead of calking may be strongly urged in all high-grade practice. It unquestionably gives the nearest approach to absolute assurance of a watertight joint. There is presumably less choice than with calking as to application on the inside or outside. Application on either side, as may be convenient, will be effective, or if extra assurance is desired, both inside and outside seams may be treated. A further advantage of electric welding in lieu of calking is realized with circumferential joints, as in Fig. 116. Here the seams at *a* and *b* are readily treated, or otherwise the electric calking may be restricted to the outside seams at *c*, *d*.

Welding by the oxy-acetylene process may also be applied to the same end, but it is somewhat less convenient in application for this particular purpose, and it will also be usually found somewhat more expensive in use.

64. Pipe Joints and Connections

The connection of the successive lengths of riveted pipe by means of circumferential riveted joints has already been referred to in Sec. 60. When the pipe is to withstand high pressure, as at the lower end of a high head line, or at the pump end of a high-pressure pumping line, some form of flange joint is often employed. In the

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case of welded pipe, a number of special forms of joint have been developed in addition to the regular type of flange connection.

Fig. 117*a* shows a standard form of flange joint in which the flanges are riveted to the sections of pipe.

In Fig. 117*b* the flanges are likewise riveted to the pipe, but the form of packing is peculiar and of special value for very high pressures. As shown in the figure, a groove of section narrowing

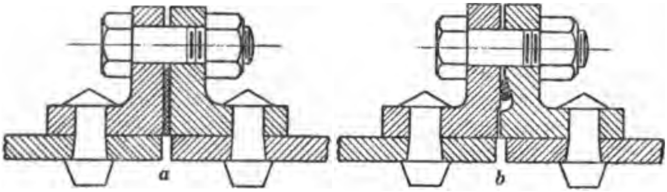

FIG. 117.—FORMS OF FLANGE JOINTS.

toward the outside is formed in one of the flanges. Within this groove is fitted a ring of soft, solid rubber packing. The water has free access to the base of this groove and the pressure acts on the rubber ring tending to drive it more and more closely into the narrow end of the groove, thus effectually packing the joint even against the very highest pressures employed. This form of packing joint is also employed with entire success against pressures of

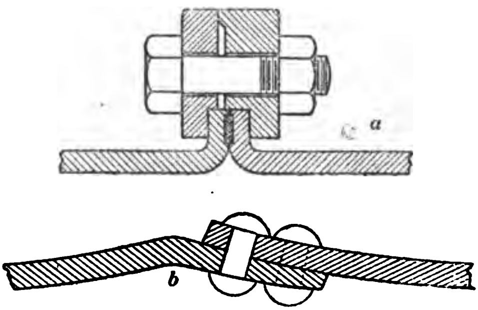

FIG. 118.—FORMS OF FLANGE JOINTS.

several thousand pounds per square inch—far above any pressures liable to be met with in pipe line practice.

In Fig. 118*a* the ends of the sections of pipe are flanged out and are pinched together between specially formed ring flanges as shown. When the nuts are properly set up on the through bolts holding the two parts of the flange together, the projecting rib on the outer edge insures a definite compression on the flanges and on the packing material between them. The individual rings in the case of such a construction must be made in two parts in order to get them into place back of the flange on the pipe.

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Fig. 118*b* shows a favourite form of joint for thin and medium thickness of large welded pipe.

Fig. 119*a* shows a form of connection which gives in effect an expansion joint at each joint of the line. The construction will be clear from the diagram.

Fig. 119*b* shows an excellent though somewhat expensive form of

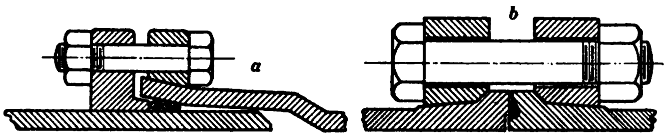

FIG. 119.—FORMS OF FLANGE JOINTS.

joint for high-pressure work. The ends of the pipe sections are thickened up by special treatment in fabrication, and are turned up to form as shown. Within the thickened end is formed a tapering groove for soft rubber ring packing, similar to that in Fig. 117*b*. The action of the flange rings on the sloping surfaces formed on the pipe is evident from the diagram.

Fig. 120 shows a form of joint suited to very heavy pipe. Here

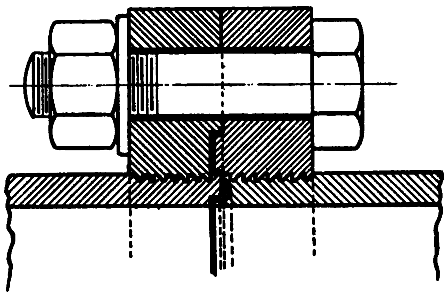

FIG. 120.—FORM OF FLANGE JOINT.

the flanges are threaded on to the pipe, and a groove or recess for soft rubber ring packing is formed in the metal of the pipe itself without thickening.

65. WOOD STAVE PIPE

See Fig. 121. This pipe is made in two forms, machine banded in lengths, and continuous. Machine-banded pipe may range from 2 or 3 inches up to 24 inches in diameter and in lengths up to 20 or 24 feet. It is banded with heavy wire wound on by machine under appropriate tension. For connecting together successive lengths, different types of coupling may be employed.

In one type of such coupling a coupling band is used, the inside

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diameter of which is only slightly less than the outside diameter of the pipe. The ends of the pipe are then slightly reduced in diameter so that a tight joint may be made with the coupling band. The latter is then forced for half its length on to the end of one of the lengths of pipe and the next length is forced into the other side of the coupling, thus completing the joint. A coupling band of this character is made up the same as a short length of pipe with wire or steel-rod banding to give it the necessary strength. In fact such coupling bands are usually banded up to perhaps double the strength of the pipe, in order that they may stand the stress which develops from forcing them on to the end of the pipe.

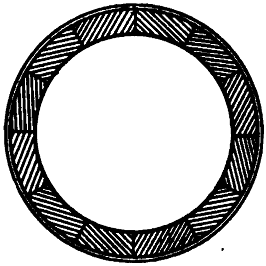

FIG. 121.—SECTION OF WOOD STAVE PIPE.

A second form of coupling shows a similar band or sleeve, but with inside diameter somewhat less than the outside diameter of pipe. The latter at the ends is then reduced in diameter to two-thirds or one-half thickness of the wood for the length of bearing, and the joint is assembled as before.

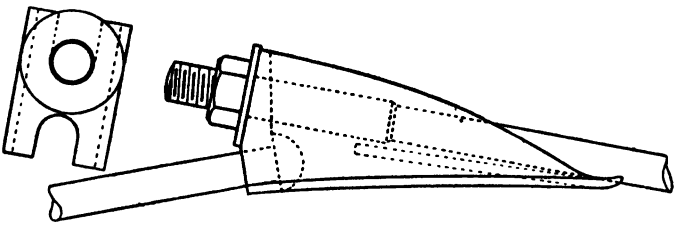

FIG. 122.—SADDLE FOR ROUND ROD TIES—WOOD STAVE PIPE.

In a third form, used only for small pipes and light pressures, one end of a pipe length is reduced in diameter to about half the thickness of the wood and a corresponding counter bore is made in the companion end of the adjacent length. These are then forced together and the joint is complete.

Instead of wood couplings, cast-iron couplings of suitable form are sometimes employed. For elbows, Ys and Tees, cast-iron fittings are commonly employed.

Continuous wood-stave pipe is made by assemblage in place, breaking joints with the successive staves so that the entire line becomes a continuous structure without joint or break in continuity. The banding of such pipe consists of separate ring bands, usually of round steel, threaded at one or both ends, fitted with a suitable saddle or end fitting and set up with nut, as shown in Fig. 122. Such pipe is made in diameters ranging from 12 inches to 100 inches and more.

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The following thicknesses of staves for machine-banded pipe have been generally adopted by manufacturers of this style of pipe.

Diameter.	*Thickness.*
3 inch.	1 inch.
4, 5, 6 ,,	$1\frac{1}{16}$,,
8, 10 ,,	$1\frac{1}{8}$,,
12, 14 ,,	$1\frac{3}{16}$,,
16, 18, 20 ,,	$1\frac{1}{4}$,,
22, 24 ,,	$1\frac{5}{16}$,,

For continuous stave pipe the thickness of the stave increases somewhat with the size and with the pressure. The variation with the pressure has two purposes:

1. To secure increased general rigidity and solidity of construction with advancing pressures.
2. To allow for the necessary bearing pressure between the sides of the staves.

The following table gives the general range of thicknesses employed:

Diameter,	*Thickness.*
Under 25 inches.	$1\frac{3}{8}$ inches.
24–26 ,,	$1\frac{7}{16}$,,
28–34 ,,	$1\frac{1}{2}$,,
36–44 ,,	$1\frac{9}{16}$,,
46–54 ,,	$1\frac{5}{8}$,,
56–72 ,,	2 inches to $3\frac{1}{2}$ inches
74 upward	$2\frac{5}{8}$,, $3\frac{5}{8}$,,

In pipe of this construction the band, when the pipe is under load, must carry the total load due to the internal pressure plus the reaction due to the compression between adjacent staves. Some degree of such compression is necessary in order to insure a water-tight joint with the pipe under load. The bands must therefore be put on with such initial tension as will insure the necessary degree of residual compression when under full load pressure. When the pipe is not under load the tension in the band will be that due to the original tension with which it is set up. When pressure is brought on the pipe the tendency will be to force the staves outward, relieving the compression but increasing the tension in the bands so that under actual working conditions the total band tension must equal, as above noted, the sum of the load due to water pressure plus the reaction due to the compression.

Mr. A. L. Adams,* in recognition of this general principle, has developed a formula connecting the pressure, size and spacing of bands, thickness of stave, etc. He assumes the extra load due to edge compression in the staves to be measured by 1·5 times the water pressure over the contact area of the staves plus an allowance

* "Trans. Am. Soc. C.E., 1899," p. 27.

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of 100 (pi2) for swelling of the wood when wet. On this assumption the formula develops as follows:

$$f=\frac{S}{(R+1\cdot5t)p+100t}$$

Where f=band spacing (i).
S=tensile load carried by band (p).
R=internal radius of pipe (i).
t=thickness of stave (i).
p=water pressure (pi2).

Mr. D. C. Henny* considers that the allowance of 100 (pi2) in the Adams formula for the swelling of wood when wet is unnecessary, and prefers the formula

$$f=\frac{S}{(R+1\cdot5t)p}$$

The bands, when in the separate or ring form, as for continuous pipe, are upset at the thread ends so as to give full section of metal at the root of the thread.

The working stress in the band may be taken from 12000 to 15000 (pi2) according to character of service.

In addition to the tensile load carried by the band itself due consideration must be given to the question of the bearing or crushing load on the wood under the band. For redwood (*Sequoia sempevirens*) a bearing value of 700 (pi2) may be employed. For Douglas fir (*Pseudotsuga Mucronata*) bearing values of 800 to 1000 (pi2) are employed. In determining this value it is assumed that one-half the diameter of the round rod or wire bears against the wood. On this assumption and denoting the bearing pressure by B and diameter of band by d, we have for the relation between B and S the equation

$$B(R+t)\frac{d}{2}=S.$$

In selecting the banding there are two variables, the spacing of the bands and the diameter of the rod. If there were no limitations to either of these, values could always be selected which would give any desired combination of values of tensile and bearing stress. Both dimensions must, however, be limited. The spacing cannot be greater than 8 or 9 inches and only then for very low pressures. Again, bands smaller than $\frac{1}{2}$ to $\frac{5}{8}$ inch corrode too rapidly, while those greater than $\frac{3}{4}$ inch are too stiff and difficult to readily handle. Within these limits the desired dimension will not always secure both bearing value and tensile stress as desired and hence it may become necessary to design for both and take the final dimensions according to which of the two controls.

For the smaller sizes of pipe, for example 12-inch diameter and less, the bearing value is likely to be the controlling feature and

* *Ibid.*, p. 68.

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must therefore be carefully considered. For the larger sizes of pipe the stress due to the pressure is usually found to rule and the bearing pressure may therefore be neglected in the computation. Where any doubt may exist, however, the determination should be made in both ways and the safer value taken.

Table XXV shows the size and spacing of bands recommended

TABLE XXV

Diameter of Pipe (inside)	BANDS								
	Diameter	Spacing in Inches for Heads in Feet							
		25 feet	50 feet	75 feet	100 feet	125 feet	150 feet	175 feet	200 feet
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
10	3/8	9 5/16	6 5/8	5 1/8	4 1/16	3 7/16	2 15/16	2 5/8	2 5/16
12	3/8	8 3/8	5 15/16	4 9/16	3 11/16	3 1/8	2 11/16	2 3/8	2 1/8
14	3/8	7 11/16	5 3/8	4 1/16	3 5/16	2 3/4	2 3/8	2 1/8	1 7/8
16	3/8	7 3/16	4 15/16	3 3/4	3	2 1/2	2 3/16	1 7/8	1 11/16
18	3/8	6 3/8	4 3/8	3 5/16	2 11/16	2 1/4	1 15/16	1 11/16	1 1/2
20	3/8	6 1/8	4 1/8	3 1/8	2 1/2	2 1/16	1 13/16	1 9/16	1 3/8
22	3/8	5 7/8	3 15/16	2 15/16	2 3/8	1 15/16	1 11/16	1 7/16	1 5/16
24	1/2	9 3/16	6	4 1/2	3 9/16	2 15/16	2 1/2	2 1/4	1 [illegible]
27	1/2	8 11/16	5 5/8	4 1/8	3 1/4	2 3/4	2 5/16	2	1 13/16
30	1/2	7 15/16	5 1/8	3 3/4	3	2 7/16	2 1/8	1 13/16	1 5/8
33	1/2	7 1/2	4 13/16	3 1/2	2 3/4	2 1/4	1 15/16	1 11/16	1 1/2
36	1/2	7 1/16	4 3/8	3 1/4	2 9/16	2 1/8	1 13/16	1 9/16	1 3/8
39	1/2	6 11/16	4 1/4	3 11/16	2 7/16	2 [illegible]	1 11/16	1 1/2	1 5/16
42	1/2	6 3/8	4	2 [illegible]	2 1/4	1 7/8	1 9/16	1 3/8	1 1/4
48	1/2	5 3/4	3 9/16	2 9/16	2 [illegible]	1 11/16	1 7/16	1 1/4	1 [illegible]/16
54	5/8	9 3/8	5 11/16	4 1/16	3 3/16	2 5/8	2 3/16	1 15/16	1 11/16
60	5/8	7 3/8	4 5/8	3 3/8	2 11/16	2 1/4	1 7/8	1 5/8	1 7/16
66	5/8	6 7/8	4 5/16	3 1/8	2 7/16	2	1 3/4	1 1/2	1 5/16
72	5/8	6 9/16	4 1/16	2 15/16	2 5/16	1 7/8	1 5/8	1 3/8	1 1/4
84	5/8	6	3 5/8	2 5/8	2	1 5/8	1 7/16	1 1/4	1 1/16
	3/4	8 5/8	5 1/4	3 3/4	2 7/8	2 3/8	2	1 3/4	1 9/16
96	5/8	4 7/8	3 1/8	2 3/16	1 3/4	1 7/16	1 1/4	1 1/16	
	3/4	7	4 3/8	3 1/8	2 1/2	2	1 3/4	1 1/2	1 5/16
108	3/4	6 1/2	4	2 7/8	2 1/4	1 7/8	1 9/16	1 3/8	1 3/16
120	3/4	6 1/16	3 11/16	2 5/8	2 1/16	1 11/16	1 7/16	1 1/4	1 1/16

Band Spacing for Wood Stave Pipe.

for service under various pressures as indicated in terms of water head.*

It thus appears that in such a combination of wood staves and steel bands the latter provide the necessary strength under internal pressure while the staves provide for a watertight inclosure, insure the necessary stiffness and general coherence as a structure, at the same time protecting the bands against corrosion by the water.

* Furnished by Mr. J. D. Galloway.

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As already noted in Sec. 59, it is not practicable to reduce the thickness of steel pipe below some minimum value which according to size may range from $\frac{1}{8}$ to perhaps $\frac{5}{16}$ or $\frac{3}{8}$ inch. This minimum thickness is necessary regardless of stress due to internal pressure and in order to secure stiffness, resistance against collapse under external pressure and by way of excess material to provide adequate life under corrosion extending over a period of years.

It thus results that under appropriate conditions, a combination of wood staves and steel bands may provide advantageous construction for light or medium pressures.

By way of example assume a pipe 24 inches diameter under a pressure of 50 feet of water. Then from Table XXV we find per foot of length two $\frac{1}{2}$-inch bands giving a net section of ·40 (i2) per foot of pipe.

On the other hand, for an all-steel pipe under these conditions $\frac{1}{8}$ to $\frac{3}{16}$ inch would be considered a minimum thickness, at least having in view a reasonable life. This would mean a cross section of 1·5 to 2·25 (i2) of steel per foot of length for the all-steel pipe.

It therefore appears in this case that strength alone can be secured by the use of one-fifth the amount of steel which an all-steel pipe would require. It then remains to be determined whether the combination per foot of ·40 (i2) of steel section with the necessary wood staves will furnish a more economical or more satisfactory pipe than the use of 1·5 to 2·25 (i2) of steel section per foot as required by all-steel pipe. This will naturally depend on special or local conditions and on the special advantages which the wood pipe may be able to offer.

Among others the following special features may be noted.

Wood stave pipe is not subject to corrosion or tuberculation, as in the case of iron or steel. It furthermore resists corrosion or damage under the attack of acidulated water or many chemicals which cannot be safely handled in iron or steel pipe.

Regarding frictional resistance, the normal coefficients are somewhat better than for steel pipe. As noted in Sec. 7, when new, a value of $n = \cdot 011$ may be employed, or values of C varying from 120 to 135 according to size. Due also to the absence of corrosion the coefficients remain higher than for iron or steel pipe of the same age. The ultimate state of the interior surfaces of wood stave pipe, in common with many forms of water conduit tends toward the development of a sort of gelatinous slime covering with a value of n perhaps ·012 or ·013 and with values of C increased accordingly.

Due to the elasticity of the wood and of the banding it is not liable to rupture when the contained water is frozen. The necessary expansion is taken up by the wood and by the banding.

In the matter of durability, wood stave pipe of California redwood (*Sequoia sempervirens*) has often given results comparing favourably with wrought iron or steel. It should, however, be kept

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filled with water. Alternate wetting and drying out of wood stave pipe will bring about rapid deterioration. Wood stave pipe, as a rule, decays from the outside, and it is therefore desirable to have such pipe exposed where it can be examined and kept painted with a suitable protective covering. The usual coating is hot tar with an outer coating of sawdust. If this coating can be kept intact and the pipe kept filled with water the life should range to twenty years and upward.

The steel wire or rod banding must also be protected from corrosion by suitable protective coating, or otherwise the security of the pipe against rupture will be endangered.

66. Reinforced Concrete Pipe

Reinforced concrete pipe has already been briefly mentioned. The field of usefulness is restricted to light pressures, as for wood stave pipe, and for the same reason. It is, in fact, clearly seen that the concrete in the one case exercises the same function that the wood stave does in the other; it supplies stiffness and protection against corrosion.

In addition to circumferential reinforce, a certain amount of longitudinal reinforce must also be provided in order to give longitudinal coherence and strength, together with the needed strength under possible bending stresses.

One serious limitation of concrete pipe is in the difficulty of making it watertight, especially under any but the lighter pressures. The consideration of ways and means for rendering concrete watertight, or as nearly so as possible, is aside from the main purpose of the present work and the point can only be noted here in passing. The fact, however, must not be forgotten, and the difficulty of an entirely satisfactory realization of watertight concrete and of the avoidance of small cracks and consequent leakage must be considered as constituting a serious limitation to the practicable use of such materials in pipe line construction.

67. Design

The given quantities in the usual problem of design are the following:

1. The rate of flow or quantity of water to be handled.
2. The head under which the line is to operate, and the general profile and topography of the line.

The principal quantities to be determined are the following:

1. The diameter or the general distribution of diameter along the length of the line.
2. The determination of thickness and its distribution along the length of the line.

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3. Details of joints and connections.
4. Design of mountings and fixtures.
5. Design of piers and anchors.

Diameter of Line—Economic Size.—The determination of the diameter of a pipe line is an economic problem. Whether the line is to be used for the delivery of power to a water wheel or for the carrying of water or other liquid under a pressure head, the following fundamental elements are involved:

(1) The lost head resulting from the flow through the pipe and the annual value of the head thus used up.
(2) The size and weight of the pipe, the cost of the same installed in place and the annual charges on such cost.

It is obvious that as the pipe is larger, item (1) will decrease and item (2) increase; and inversely as the pipe is smaller.

Further, it is evident that each of these items must be viewed in the light of an expense or cost and that the sum of the two represents the total annual cost chargeable against the pipe line itself.

The economic problem therefore reduces itself to the finding of such a size of pipe as shall make the sum of these two items a minimum. In Appendix IV will be found a brief mathematical discussion of the general problem of the economic determination of a variable element in an engineering design.

In the case of a pipe line under specially restricted and simple conditions it is possible to express algebraically the various quantities involved, and thus to derive an algebraic formula for the economical diameter in terms of the ruling conditions of the problem. It will be instructive to examine briefly this special case, in particular for the light which it sheds on the general character of the relation between the economic diameter and the controlling elements of the problem. The special conditions of the assumed case are as follows:

1. The diameter is uniform.
2. The gradient is uniform.
3. The thickness bears a constant relation to the product of the pressure by the diameter.
4. The weight per lineal foot bears a constant relation to the product of the thickness by the diameter.

In the typical pipe line problem none of these conditions is fulfilled. The diameter commonly decreases in steps from upper to lower end, and for reasons to which further reference will be made at a later point. The gradient is rarely uniform, but more commonly follows, in some measure, the accidents of the topography. The thickness, instead of showing a uniform relation to the product of pressure and diameter (that is to the stress due to internal pressure), varies necessarily by steps according to the thickness of plates commercially available, and at the upper end, as we have previously

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seen, there is likely to be a thickness in excess of that required for strength alone.

In spite of these differences between the actual and the assumed case, the indications of the latter are highly instructive, and may serve further to give a very close first approximation in many actual cases.

Let X=annual charges on investment in pipe.
Y=annual value of the head used up in friction.
a=cost of pipe per pound installed in place (dollars).
b=rate of interest for fixed charges on pipe (decimal).
c=value of one horse-power year (dollars) estimated at lower end of pipe line.
w=density of water (pf3).
σ_1=density of steel (pf3), with allowance for laps, butt straps, rivet heads, etc.
D=diameter of pipe (f).
e=efficiency of longitudinal joint.
T=safe working stress in longitudinal joint (pi2).
C=coefficient in Chézy formula.
V=rate of flow (f3s).
L=length of line (f).
H and H_o as in Fig. 123.

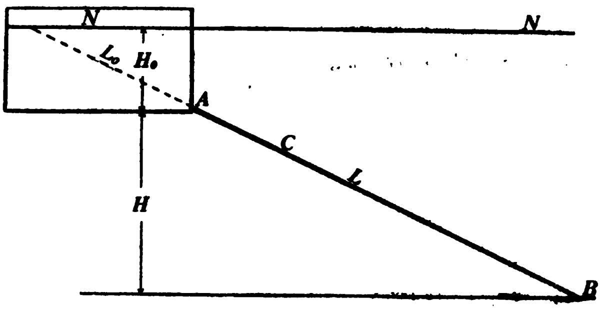

FIG. 123.—DESIGN OF THICKNESS FOR SHELL OF PIPE LINE.

Case 1.—For the above-indicated simple case we then readily find as follows :

$$\text{Thickness at lower end}=\frac{w(H+H_o)D}{2eT\times 144}\text{(f)}$$

$$\text{Thickness at upper end}=\frac{wH_oD}{2eT\times 144}\text{(f)}$$

$$\text{Mean thickness}=\frac{w(H+2H_o)D}{4eT\times 144}\text{(f)}$$

$$\text{Circumference}=\pi D$$

$$\text{Hence}\quad X=\frac{ab\pi w\sigma_1(H+2H_o)LD^2}{576eT}\quad\ldots\ldots\ldots\ldots(5)$$

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$$\text{Lost Head} = \frac{64V^2L}{\pi^2C^2D^5} \text{ (see Sec. 6)}$$

$$\text{Hence} \quad Y = \frac{64cwV^3L}{550\pi^2C^2D^5} \quad \dots\dots (6)$$

$$\text{Put} \quad P = \frac{\pi w\sigma_1(H+2H_o)L}{576eT}$$

$$S = \frac{64wV^3L}{550\pi^2C^2}$$

We then have

$$X = abPD^2 \quad \dots\dots (7)$$

$$Y = \frac{cS}{D^5} \quad \dots\dots (8)$$

$$\text{Then} \quad \frac{dX}{dD} = 2abPD$$

$$\frac{dY}{dD} = -\frac{5cS}{D^6}$$

To find the conditions for a minimum of the sum of X and Y we put the sum of the derivatives equal to zero and solve for D. This gives:

$$D = \left(\frac{5cS}{2abP}\right)^{\frac{1}{7}} \quad \dots\dots (9)$$

Restoring the values of S and P, and collecting all numerical values into one term we have

$$D = 1{\cdot}2726\left(\frac{ecTV^3}{abC^2\sigma_1(H+2H_o)}\right)^{\frac{1}{7}} \quad \dots\dots (10)$$

The value of σ_1 is intended to allow for the excess of weight of the actual pipe over that of a shell with geometrical volume πDtL. The value to be used will therefore exceed the weight of a cubic foot of steel in the same ratio as the weight of an actual foot of pipe exceeds that of a volume πDt.

Tables of weights based on good design show that for lap-riveted pipe the value of σ_1 will be about 600, while for butt-strap pipe it will rise to values about 700, relatively larger in each case for thin plates and smaller for thick plates. More accurate values for any given case may be derived from the weights in Tables XXVI and XXVII.

In this same connection, Merriman* gives a formula for the weight of lap-riveted pipe as follows:

$$w = 12{\cdot}5Dt + 10.$$

Where w=weight in pounds per lineal foot and D and t are taken in inches.

The indications of this formula agree fairly with the weights in Table XXVI.

* "Civil Engineer's Hand Book."

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The form of (10) shows that D varies directly with eTc and V, and inversely with a, b, C and $(H+2H_0)$. The physical interpretation of these relations will prove interesting to the reader. We also note that the variation with all of the factors except V is very slow. With V it is a little less than as the square root. Thus a change of, say, 20 per cent in any of the factors except V will affect D by a little more than 2½ per cent. The same percentage change in V will affect D by about 8 per cent. The variation of D with a change in the various factors in (10) except V is then very sluggish or, in other words, any given value of D will hold approximately over a very considerable range of variation in these various factors. To change in V, the value of D is more quickly responsive as noted. It may also be noted that equal percentage changes in certain of the factors will leave the economic diameter unchanged. Thus equal changes in a and c will not affect the value of D.

It may furthermore be readily shown by substitution from (9) in (7) and (8) that when the economic value of D is employed we shall have $X=2{\cdot}5Y$. That is, the total fixed charges on the pipe will be 2·5 times the annual value of the total lost head.

Equation (10) thus serves to establish the economic value of a constant D on the assumption of a value of t varying and at all points proportional to a head varying from H_0 to (H_0+H). An ideal line thus determined, would therefore extend over this range of head, with continuous change in t from one end to the other, while remaining fixed in diameter as determined in (10).

It will be noted that the value of D thus found is independent of L, this term occurring on both sides of the final equation for D and thus cancelling out. Otherwise we note that both X and Y necessarily vary, each directly with L and hence the relation which makes their sum a maximum or minimum is independent of L. This means, in effect, that the value of D thus determined is independent of the line gradient.

The line must, however, extend from head H_0 to head (H_0+H), and hence the minimum length must be H, implying a vertical line With any other gradient the length will be greater, but no matter what the gradient may be (assumed constant) the value of D will give the economic diameter for a pipe of uniform size extending between these limits of head and with t continuously varying with the total head at the given point.

In the case of a varying gradient, the total line may usually be divided into a series of parts, each with a sensibly uniform gradient. Then by the use of the same equation (10) the economic diameter for each section or part may be found, using for H_0 the total head at the upper end of the section and for H the difference of head covered by the section. This will, in general, give as many different diameters as there are sections, continuously greater as we go from the lower to the upper end of the line.

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Case 2.—**Tapering Pipe Line with t continuously proportional to $D(H_o+H)$.** The assumption of a uniform diameter of line from one end to the other necessarily implies a limitation in the search for the most economic of all lines. The resulting D will indeed be the economic value on this assumption, but it does not follow that the removal of this restriction may not result in an economic result of still higher value.

To develop the possibilities of a varying diameter let us seek to determine the economic value for an element of the pipe extending from total head (H_o+H) to $(H_o+H+\Delta H)$, where ΔH is a small or (at the limit) differential element of head.

In equation (10) the part played by H_o here becomes (H_o+H), while H becomes ΔH. At the limit then $(H+2H_o)$ becomes $2(H_o+H)$, and we have

$$D=1{\cdot}2726\left(\frac{ecTV^3}{2ab\sigma_1 C^2(H_o+H)}\right)^{\frac{1}{7}} \ldots\ldots\ldots\ldots (11)$$

In this equation, H is, of course, to be taken as a variable with values ranging from O to the value as in Fig. 123 at the bottom of the line; or otherwise (H_o+H) is to be taken as the total head at the given point, ranging from one end of the line to the other.

It is readily seen that this equation will give, from one end of the line to the other and for corresponding values of the total head (H_o+H), a series of values of D, increasing from bottom to top. The value at the bottom of the line where (H_o+H) is the total head, as in Fig. 123, will be less than the value given by equation (10); while the value at the top, where (H_o+H) becomes H_o, will be greater than the value by equation (10).

By suitable investigation it may be shown that, for a section of uniform gradient, the ratio of the weights in the two cases is given as follows:

$$\frac{W_2}{W_1}=\cdot 9567\frac{(H_o+H)^{12/7}-H_o^{12/7}}{H(H+2H_o)^{5/7}} \ldots\ldots\ldots\ldots (12)$$

Where W_2=weight of tapered pipe
W_1=weight of uniform pipe.

By a suitable investigation, it may also be shown that the ratio of the values of the friction head in the two cases, h_2 and h_1 is the same as for the weights.

It may also be readily seen that with these values, W_2 and h_2 will each be less, respectively, than W_1 and h_1, their values becoming equal when $H=0$, that is, when the line becomes horizontal and in operation under the head H_o.

Equation (11) gives therefore, in the general case, an economic value of D corresponding to each value of the total head (H_o+H) from top to bottom of line. With a profile of the line laid down, the distribution of D along the length is readily determined, and thus all characteristics of the line become known.

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Case 3.—**Pipe Line with Stepwise Variation in Thickness.**—Actual pipe lines cannot be made with thickness of shell varying continuously with the head or with the product of head and diameter. They must rather be made with thickness varying in steps in accordance with the commercial materials available. Steel plate is usually obtainable in steps of one-sixteenth inch, and assuming such or similar steps it becomes of interest to examine the problem of the economic diameter of line with such a stepwise distribution of thickness.

For any one element or section of the line let

t=thickness of shell .... (feet)

ΔL=length of section with constant thickness t .... (feet)

We have then, using the same general notation as before:

$X = ab\pi Dt\sigma_1 \Delta L.$

While Y has the same value as before with the substitution of ΔL for L.

Treating these in the same manner as before for the minimum value of $X+Y$ we find

$$D=\left(\frac{32cwV^3}{55\pi^3C^2ab\sigma_1 t}\right)^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

or with $w=62{\cdot}4$

$$D=1{\cdot}027\left(\frac{cV^3}{abC^2\sigma_1 t}\right)^{\frac{1}{6}} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

It will be again noted that the value of D is independent of the element of length ΔL and for the same reason as in equation (10). It results that the value of D thus found is the economic value for this constant value of t, and from the lowest point where the head and thickness sustain the proper relation for strength, upward as far as this value of t is carried.

Suppose then that t_1, t_2, t_3, etc., denote a series of thicknesses—differing, for example, by $\frac{1}{16}$ inch or $\frac{1}{192}$ foot. Let D_1, D_2, D_3, etc., denote the corresponding values of D found by (14). Then the lowest point at which any given combination of D and t will fulfil the conditions for strength will be given by the relation:

$$wD(H+H_o)=288etT \quad \ldots\ldots\ldots\ldots\ldots (15)$$

There will thus result a series of values of H which we may denote by H_1, H_2, H_3, etc. Then t_1 being the thinnest plate, it appears that the successive sections of pipe will extend over ranges of head as follows:

D_1, t_1 from $H=H_1$ to $H=0$
D_2, t_2 from $H=H_2$ to $H=H_1$
D_3, t_3 from $H=H_3$ to $H=H_2$

If for any reason t_2 or t_3 were taken as the minimum thickness, then similarly the combination D_2, t_2 or D_3, t_3 would extend from $H=H_2$ or H_3 to $H=0$. Otherwise it appears that beginning at the point where $H=H_3$, for example, the combination D_3, t_3 will extend

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upward as far as the thickness t_3 is carried. Naturally this will be only until the next combination D_2, t_2 can be made available. In this manner the range of head is determined for each combination of D and t.

With the profile of the line laid down, the lengths of section corresponding to these various ranges of head H_1, (H_2-H_1), etc., are readily determined, and thus the line becomes known in all its characteristics.

Influence of Variable Flow on Economic Size.—In the various cases of economic design thus far considered it is noted that the lost power is proportional to V^3, the cube of the rate of volume flow. Hence in the case of variable flow the mean loss will be proportional to the mean cube of the variable rate of flow, or approximately to the mean cube of the variable power developed. If, therefore, we have given a curve showing the typical or accepted variation of flow through any unit period of time, as per day or week or month or year, we may cube the various ordinates of such a curve and find in any convenient manner the mean cube. This taken for V^3 in the preceding formulæ will then give the conditions for economic design.

General Comment on Economic Formulæ.—With reference to the various formulæ developed above for the economic diameter of a pipe line, there will result, in most cases, certain departures from any mathematical ideal as contemplated by a formula. These departures arise chiefly in connection with the relation of the weight of the pipe to the defining dimensions D and t, also in the varying price per pound which may exist as between lap-riveted and butt-strap riveted pipe, also in the effects arising from irregular profile and also in the influence on pipe line cost, of the specials required to connect together sections of different diameters.

For these various reasons, any indication regarding economic diameter as given by formula should be checked with reference to the influence which such factors may have on the final values of the two terms X and Y, and on their rate of change for a small change in the diameter of the line.

Reference should also be made to further reasons which may exist for decreasing the diameter of pipe toward the lower end, and quite independent of the problem of economic design. These are found in the increased difficulty of rolling heavy plate to circular form with increase in the thickness; and also in the increasing difficulty of riveting up and handling extra heavy plates in the field. It is true that the difficulty in rolling heavy plate increases with the decrease in diameter as well as with the increase in thickness; nevertheless, the thickness factor is on the whole the more important, so that while the decrease in diameter will partly offset the gain due to decreased thickness, there will result, at least within limits, some balance in facility of fabrication for the reduced size of pipe. Again, while there is no absolute limit to the thickness of

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plates which can be worked and assembled in the field, yet it is a well-known fact that the practical difficulties rapidly increase from 1 inch or $1\frac{1}{4}$ inches upward. Broadly speaking, plates $1\frac{1}{4}$ to $1\frac{3}{8}$ inches in thickness are close to the limit of effective and satisfactory work in the field. If therefore the circumferential joints are to be riveted rather than flange joint connected, it is desirable to hold the thickness not to sensibly exceed $1\frac{1}{4}$ inches. But the thickness with any given pressure is directly proportional to the diameter. Hence by a suitable reduction in diameter at the lower end the thickness may be brought within an acceptable limit.

Thus by way of illustration. Given a total static head of 1400 f. Then $p = \cdot 433 \times 1400 = 606$. Suppose that the use of equation (14) in this case indicates for the lower end of the line a diameter of 60 inches with a thickness of $1\frac{3}{8}$ inches, and suppose that this thickness is considered undesirable and that it should be reduced to $1\frac{3}{16}$ inches. This will correspond to a diameter of 52 inches. These values will not be economic, but may be considered necessary, due to the reasons noted above. The question will then arise as to what extent this departure from economic conditions may be compensated for in the remainder or in the upper parts of the line.

A first approximation to such compensation may be arrived at as follows:

The economic lower diameter is 60 inches with $1\frac{3}{8}$ inches thickness. Suppose the minimum thickness employed at upper end to be $\frac{5}{16}$ inch. Then the range of thickness is in the ratio of 5 to 22. Equation (14) shows that the economic diameters vary as the sixth root of the thickness. Hence the diameter range will be $(4 \cdot 4)^{\frac{1}{6}} = 1 \cdot 28$ and if the lower economic diameter is 60 inches the upper would be 76·8. If then instead of an economic line with diameter tapering from 60 inches to 76·8, we substitute a line with taper from 52 to say 84 inches, thus giving approximately the same mean diameter, there will be an approach toward compensation and such a distribution will give at least a first approximation toward the most economic line under the limitations imposed—a distribution of diameters which may be checked by one or two trial layouts on either side of that proposed.

Determination of Thickness.—The thickness of the metal forming the shell of the pipe must be adequate to meet the following requirements:

(1) The maximum static or steady condition pressure to which the pipe is likely to be subjected in operation.

(2) Any excess pressure due to shock or water hammer to which the pipe may be exposed.

(3) Collapsing pressure due to a reduction of the pressure on the inside to less than the atmospheric pressure.

(4) Stiffness and rigidity and strength to meet such bending stresses as may be set up due to the action of the pipe as a beam.

(5) Any reasonable special stresses to which the pipe may be

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subjected as a result of expansion and contraction under temperature changes.

(6) Special stress due to distortion of the cross section of pipe from true circular form, or failure to realize this form as in lap-riveted pipe.

(7) The usual variations to be expected in the strength of the plates resulting from the accidents of manufacture.

(8) Corrosion and wear, in order that the pipe may have a lifetime of reasonable length and without a too rapid decline of the factor of safety.

In order that requirements (3), (4), (8) may properly be met, a lower limit is usually placed on the thickness, regardless of the strength called for by the other requirements. This lower limit, as elsewhere noted, is usually not far from one-quarter inch. At some point in the line as C, Fig. 123, the thickness required to insure the necessary strength against internal pressure will be $\frac{1}{4}$ inch. Then from this point to the upper end of the line the thickness is held uniform at this value, while from this point down it is increased in accordance with the strength requirements of the case. It must not be assumed, however, that a thickness of $\frac{1}{4}$ inch will insure against collapse under external excess pressure. This will depend on the diameter. This thickness will, however, with the usual sizes, give a fair collapsing strength under a slight excess of external pressure and the air relief valve, as noted in Sec. 74, must be depended on to prevent any greater external excess than can be safely carried by this thickness.

Requirements (1) and (2) relate directly to stress resulting from internal pressure. In order to meet these requirements with due allowance for the others, as in all engineering work, it is customary to base the design on the load which can be determined with some degree of certainty and to include all other unknown loads and all margin of uncertainty in the so-called factor of safety. In the present case we may design primarily with reference to requirement (1) and depend on the factor of safety to provide sufficient margin for all of the other requirements and for the general outstanding margin of assurance desired; or, otherwise, we may add to the pressure representing requirement (1) a certain amount, often taken from 50 to 100 (pi2) and representing requirement (2). The factor of safety need then include only the remaining items with the outstanding margin of assurance.

With modern approved appliances for controlling excess pressure due to shock (see Sec. 75), the amount to be anticipated should not be too great to permit of merging with the other uncertain quantities under a fairly liberal factor of safety.

The preceding remark regarding requirement (3) should not be here forgotten. The factor of safety is not expected to necessarily insure against collapse. It may or may not, depending on the thickness used. In a pipe under high head it will undoubtedly be

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thick enough at the lower end to withstand full collapsing pressure, but may not be toward the upper end where the thickness approaches the lower limit assigned for the line.

As a still further or somewhat different factor of safety, a constant thickness is sometimes added, as in Fanning's formula for cast-iron pipe (see Sec. 58), in order especially to provide for numbers (7), (8).

Assuming that all requirements other than (1) are to be covered by the factor of safety, it becomes of importance to inquire what values may be given to this factor. Experience indicates that values from (4) to (5) will insure a safe and satisfactory design. Furthermore, since the longitudinal joint is the weakest part of a riveted or welded pipe under internal pressure, the use of such factors of safety implies actual working stresses in the longitudinal joint of from 12,000 to 15,000 or 16,000 (pi2), assuming normal soft steel plates with an ultimate tensile strength of about 60,000 (pi2). With a joint efficiency of e this will imply an actual working stress in the plate itself, along a longitudinal line, reduced in the ratio$=e/1$.

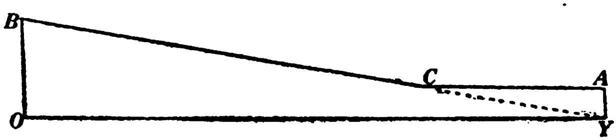

FIG. 124.—LAYOUT OF THICKNESS ON PRESSURE HEAD.

Passing now to actual methods of design we note first the formula from mechanics :

$$pD=2teT \qquad (16)$$

where p=pressure (pi2)
D=diameter (i)
t=thickness (i)
e=efficiency of longitudinal joint
T=safe working stress in metal of joint (pi2).

Transforming into terms of thickness, we have

$$t=\frac{pD}{2eT} \qquad (17)$$

But $p=wH$, where H=head at the given point and $w=\cdot 433=$ factor for transforming head of water in feet into pressure in pi2 Substituting we have

$$t=\frac{wHD}{2eT} \qquad (18)$$

It thus appears with fixed values of e and T, that the thickness will vary directly as the product HD ; or with constant diameter, it will vary directly as the head H.

In selecting the actual thickness to be used, convenient use may be made of the diagram of Fig. 124. First assuming the diameter uniform, lay off the line OY to represent the head $(H+H_0)$ (Fig. 123).

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OB is then laid off to represent the value of t at the lower end, assuming an actual joint stress 12,000 to 16,000 (pi2) as may be decided upon. The line BY, using OY as a base, will then give the corresponding value of t at any elevation. At C this line runs into the minimum thickness line AC, which is laid off at the desired distance from OY. The actual values for any given elevation will then lie along ACB. The indications of such a diagram may usually be very closely realized by commercial sizes varying by 16ths of an inch. Such a diagram set up on the sheet with a profile of the line will indicate at a glance the lengths and locations corresponding to the various thicknesses to be used.

In case the diameter varies, increasing from bottom to top, the variation will not be continuous but stepwise. The diagram corresponding to Fig. 124 for such a case will therefore show a series of steps as in Fig. 125, where three different values of D are indicated.

To solve equation (18) for quick approximate values, a form of

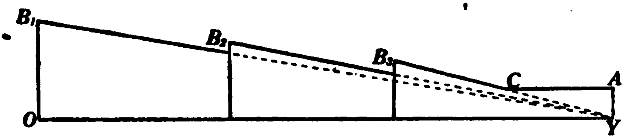

FIG. 125.—LAYOUT OF THICKNESS ON PRESSURE HEAD.

straight-line diagram may be conveniently employed as illustrated in Fig. 126. In this diagram AB and CD are parallel lines with OO a perpendicular between them. The values of t in steps of $\frac{1}{8}$ inch, for example, are laid off on OB with any convenient unit. The scale for e is laid off similarly on OD, and for D on OC. Then OA becomes the axis of H and the unit of the scale will be given by the relation :

$$\text{Unit of } H = \frac{(\text{Unit of } t) \times (\text{Unit of } e) \times \cdot 433}{(\text{unit of } D) \times 2T}$$

Thus if 1 inch of thickness be represented by 3·2 inches on the axis OB, 100% efficiency by 5 inches on OD, 100 inches diameter by 5 inches on OC, and putting $T=16{,}000$, then unit of $t=3{\cdot}2$, unit of $e=5$, unit of $D=\cdot 05$ and unit of $H=\cdot 00433$ or 100 feet$=\cdot 433$ inch, and the scale may be laid down accordingly.

The diagram then fulfils the condition that any two lines drawn, as shown, and meeting on the axis OO will fulfil the conditions of (18) and thus either one of H, t, e or D may be readily found once the other three are known.

In this general connection Tables XXVI and XXVII will be of value, the former giving the weight of lap-riveted pipes and the latter of double butt-strap triple-riveted pipes, each for various diameters and thicknesses of plate as noted.

These weights are based respectively on 6-foot courses and 8-foot courses, each with the usual allowances for overweight of

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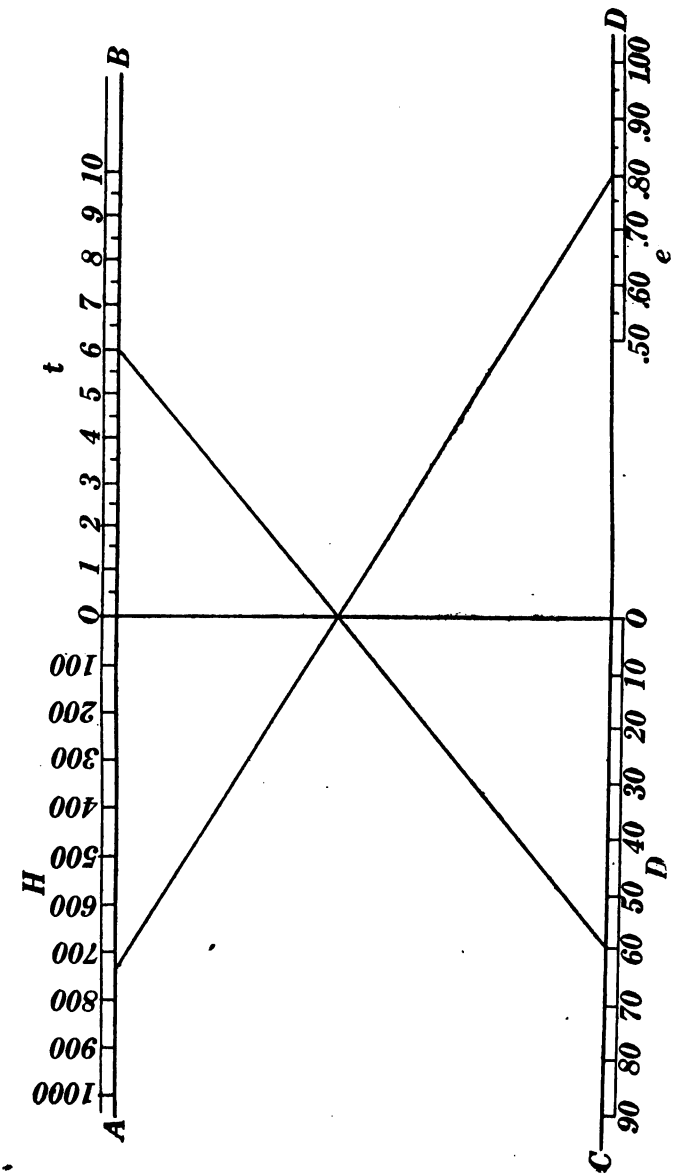

FIG. 126.—DIAGRAM FOR DETERMINATION OF THICKNESS OF PIPE LINE SHELL.

Head H, unit 1 foot. Diameter D, unit 1 inch. Thickness t, unit $\frac{1}{8}$ inch.

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rolled plates, and include the calculated weights of the joints and rivets and a coating of asphalt.*

TABLE XXVI

Weights, in Pounds per Foot, of Lap-Riveted Pipes
Thickness of Plates (inch).

Diameter (inside) inches	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16
18	33·9	49·5	64·7	78·6	93·6	109·9	126·7	147·0
20	37·4	54·6	71·0	86·5	102·7	120·5	138·6	160·7
22	41·1	59·7	77·6	94·3	112·1	131·1	150·9	174·4
24	44·5	64·4	83·8	102·0	121·1	141·7	162·8	188·0
27	49·9	72·2	93·7	113·9	135·1	157·8	181·0	208·4
30	55·3	79·8	103·3	125·3	148·7	173·5	199·1	228·8
33	60·8	87·6	113·1	137·2	162·7	189·7	217·4	249·4
36	66·0	95·1	122·3	148·7	176·5	205·4	235·4	269·9
39	71·3	102·5	131·6	160·3	190·4	221·5	253·6	290·2
42	76·6	110·2	141·8	172·2	204·0	237·4	271·5	310·7
48	87·3	125·3	161·0	195·5	231·8	260·3	307·9	351·9
54	98·0	140·6	180·2	218·9	259·2	301·1	344·1	392·8
60	108·8	155·8	199·5	242·5	286·9	333·0	380·4	433·9
66	119·4	170·8	218·7	265·6	314·5	364·9	416·6	474·6
72	130·0	186·1	237·8	288·8	342·0	396·7	452·8	515·6

TABLE XXVII

Weight, in Pounds per Foot, of Butt-Strap Riveted Pipes.
Thickness of Plates (inch)

Diameter (inside) inches	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
24	205·9	230·1	255·7	279·6	307·1	350·3	384·8	426·5	450·3
27	225·8	253·2	280·0	306·7	336·8	384·2	419·1	464·7	490·3
30	245·6	275·3	304·7	333·1	365·5	416·6	455·0	502·4	531·0
33	265·7	297·5	329·3	359·7	394·2	450·0	489·7	540·3	571·8
36	286·2	320·7	353·8	386·4	423·5	482·9	525·3	579·4	612·5
39	305·5	342·3	378·8	413·8	453·4	515·3	571·7	618·5	652·8
42	326·0	365·0	403·6	441·0	482·1	547·6	595·0	655·8	693·0
48	367·5	411·3	454·2	495·7	543·7	616·7	667·9	734·3	777·5
54	407·4	455·7	503·6	549·4	602·6	681·8	738·7	811·0	857·9
60	446·8	500·9	553·8	605·0	661·0	748·4	809·0	886·6	940·3
66	487·6	545·3	604·0	657·6	720·6	814·2	879·9	964·5	1020·7
72	527·9	589·4	652·6	711·8	780·0	881·0	951·4	1039·5	1102·1

* See paper by Mr. J. D. Galloway, "Trans. Am. Soc. E.C., 1915," Vol. LXXIX.

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Thickness of Plates (inch)

	1-1/16	1-1/8	1-3/16	1-1/4	1-5/16	1-3/8	1-7/16	1-1/2
27	524·4	566·0						
30	567·0	611·9	653·1	686·4				
33	609·9	657·8	702·5	736·6	776·1	809·2		
36	654·1	704·9	751·3	788·2	831·8	864·9	908·8	949·9
39	690·3	750·9	799·2	838·1	884·1	920·4	964·8	1009·9
42	739·3	796·3	846·5	889·4	936·1	976·4	1025·2	1071·3
48	826·6	891·2	946·9	994·5	1046·9	1091·6	1144·7	1195·9
54	912·0	983·7	1044·7	1096·8	1153·5	1204·4	1260·9	1318·5
60	1004·9	1076·1	1142·2	1199·6	1261·2	1318·2	1380·9	1443·7
66	1086·5	1169·1	1239·3	1301·7	1369·5	1427·1	1494·8	1561·7
72	1171·6	1260·6	1336·1	1401·7	1473·5	1539·9	1612·9	1682·4

68. EXPANSION AND CONTRACTION IN PIPE LINES DUE TO CHANGES OF TEMPERATURE

The well-known formulæ of physics give us the following:

$$L_2 = L_1(1 \pm at)$$

Where L_2 and L_1 are the two lengths, t is the *change* in temperature and a is the coefficient of linear expansion.

Taking t in Fahrenheit degrees we have for a as follows:

Cast Iron	·0000056
Wt. Iron and Steel . . .	·0000064
Brass	·0000100
Copper	·0000089

It is often more convenient to take the expansion for $L_1 = 100$ feet $= 1200$ inches and $t = 100$°F. This gives numbers 120,000 times the above, as follows:

Cast iron	·66 (i)
Wt. Iron and Steel . . .	·768 (i)
Brass	1·20 (i)
Copper	1·07 (i)

As a closely approximate value we may take for steel pipe an expansion of ¾ inch per 100 feet per 100°F.

In the case of an empty pipe, temperature changes of 100° are by no means impossible, especially in countries where the skies are clear and the sun bright. The total expansion of a line of pipe of considerable length is therefore a quantity requiring definite and careful consideration. Thus a line 1000 feet long will show a total expansion for 100° rise in temperature of approximately 7·5 inches; for a rise of 50° only, an expansion of 3·75 inches, and otherwise in proportion.

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69. Stresses in Pipe Lines due to Expansion and Contraction

Suppose a pipe line of length L rigidly held between abutments so that it cannot expand. Let the temperature rise t degrees, what stress will be developed ? We may treat this problem by assuming one of the abutments removed and the pipe free to expand, and then ask how much stress will be developed by compressing it at the higher temperature back to the original length.

Denote the stress by T. Then by definition of the coefficient of elasticity E, we have

$$\frac{\Delta L}{L} \text{ or } \frac{L_2-L_1}{L_1}=\frac{T}{E}$$

$$\text{But} \quad \frac{L_2-L_1}{L_1}=\frac{L_1 at}{L_1}=at$$

$$\text{Hence} \quad T=Eat.$$

Inserting values for E and a for the four materials as above we have in each case an equation of the form,

$$T=Bt$$

where B has values as follows :

Cast Iron . .	56 to 112	(higher values for lower values of t).
Wt. Iron and Steel .	192	
Brass . . .	100	
Copper . . .	140	

It thus appears, for example, that rise of temperature of 100°F. in a steel pipe if held between abutments in such manner as to prevent expansion will result in a compressive stress of about 19,000 (pi2). Similarly in the case of a fall of temperature in a pipe held between fixed anchors and prevented thereby from contracting. Precisely the same conditions will develop as above with the pipe under tension instead of compression, and the coefficients relating tension to change in temperature will be the same as above.

In the case of riveted pipe with sections connected with circumferential riveted joints, the sectional area of the rivets in shear will normally be less than that of the pipe itself in tension or compression. Let m denote the ratio of the area of section of shell to the section of rivets. Then the stress which must be carried by the rivets (except as partly taken by the friction of the plates) will be m times as great as the T above. Since the value of m may vary from 1·3 to 1·5 it is clear that under extreme conditions of temperature change stresses may develop which will result either in the rupture of the circumferential joints or in straining them beyond the elastic limit. Similar conditions may develop with flange joints under tension.

The relief of pipe line from stress when subject to temperature change is realized through some form of slip or expansion joint (see Secs. 49, 77).

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70. Erection of Steel Pipe Lines

The chief points involved in the erection of a steel pipe line are the following:

(1) The supply of suitable provision for carrying the down-hill thrust of the line and for safeguarding the permanent connections at the lower end (power units, pump discharge, etc.) from undue stress or disturbance due to such thrust, before the line has finally settled into its permanent condition.

(2) Safeguarding the line at all points against undue stress due to changes of temperature.

(3) The proper support and constraint of the pipe at all points during erection and before all parts are completely assembled.

(4) Hydrostatic test in sections of not too great length so that any weakness or faulty construction may be promptly discovered and remedied.

Certain of these requirements are of special significance only in the case of pipes laid on steep slopes, as with power plant penstocks. Others are more general in character. In what immediately follows we shall have especially in view the case of steep-slope construction.

The above various requirements in the case of steep-slope construction are usually best met by erection from the lower end upward. Requirement (1) should be met by the provision of an especially strong and reliable anchor block capable of safely carrying all the down-hill thrust that can in any way be anticipated. In addition, many good engineers prefer to leave out, between the lower end of the line and the permanent connections, a short flange-joint filler piece which is not put in until the line is installed and has had time to settle into permanent condition. This filler piece may then be cut to the proper length and inserted in place. In any case, however, the anchor block should be installed between the line and the permanent connections, as a means of absorbing all down-hill thrust and of shielding them from the stress which such thrust might produce.

Requirement (2) will be met without additional features in case the design contemplates an adequate supply of expansion joints. In such cases, however, it must be boine in mind that the pipe fixed at any given anchor block will push up the hill and draw back down the hill as the temperature rises and falls. Care should therefore be taken, and especially when the pipe is empty and in climates subject to wide fluctuations of temperature, to avoid any temporary constraint which might hamper or prevent such temperature movements.

In case the pipe, when exposed to the sun, can be filled with water, the temperature movements are greatly reduced, usually to a negligible amount. It is therefore always desirable to keep the pipe from the lower end upward filled with water as the erection

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proceeds, both with reference to the reduction of temperature movements and for purposes of strength and leakage tests as referred to below.

Again, in cases where the pipe is to be buried or where for other reason expansion joints are omitted, the same care must be exercised during installation and while the pipe is uncovered, to keep the line filled so far as possible and to avoid hampering or preventing such movements as accidental changes in temperature may produce.

Again, if for any reason a line without expansion joints is placed under complete constraint at two distant points and while still uncovered, special care must be taken to safeguard the pipe against undue temperature stress. Filling with water will usually meet every requirement. If this is not practicable, temporary shelter from the sun may be necessary. These various special precautions regarding temperature movements are, of course, the less necessary as the line between points of constraint is curved or provided with angles or bends.

Requirement (3) calls only for the exercise of normal engineering judgment and care. The tendency toward movement under gravity or any other forces which may be involved, must be carefully evaluated, and if the features of the permanent design are in any degree inadequate during the process of erection they must be supplemented with suitable means according to the requirements of the case.

Requirement (4) calls for periodic hydrostatic tests. To this end there are required the following:

1. Means for closing the section of line to be tested.
2. Means for applying the test pressure desired.

The lower end of the line may usually be closed without difficulty. If it is connected through to permanent units, such as a waterwheel or pump, there will be an intervening valve, the closure of which will meet all requirements at this point. If it is not connected through to permanent units, the lower open end will usually be provided with a flange. In such case a test blank flange should be provided, by means of which the lower end may be temporarily closed for test purposes and also for holding water in the line as installed, for reasons as noted above.

In case there is an open plain end without flange, closure may be effected by means of a temporary bulkhead (see Sec. 78).

71. Piers and Anchors

Pipe lines require definite points of support in order that the weight when filled with water may be carried without danger of serious deformation through sagging. Likewise at critical points, especially where changes in direction are made, and at intermediate

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points on long straight grades (especially when steep), definite points of complete constraint or anchorage are required.

Forms and Construction of Piers.—In modern engineering practice piers are commonly made of concrete, a 1 : 2 : 4 or 1 : 2 : 5 mixture of cement, sand and crushed rock representing standard proportions. Where the pier is intended to serve simply as a support for the weight of the pipe it is usually made of sufficient width to embrace about one-third or sometimes nearly one-half of the circumference.

Where several lines are carried along parallel and closely adjacent, the piers will naturally combine into a series of continuous concrete walls underlying the pipes at suitable intervals and formed with properly rounded depressions to receive and constrain the pipe.

Due to expansion and contraction resulting from changes in temperature, the pipes must slide back and forth on the supporting surface of the pier. This slipping movement will be greatly facilitated with relief of stress, both in the pipe and in the pier, by the

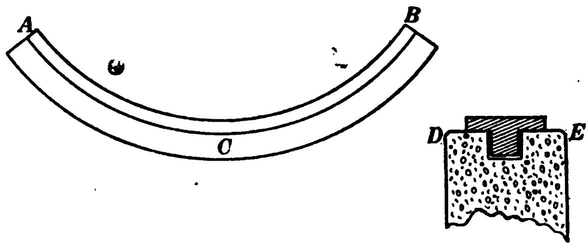

Fig. 127.—Saddles for Support of Pipe Line.

use of some form of metal bearing surface. A convenient and effective form of bearer for this purpose is shown in Fig. 127. AB is a cast-iron ring formed to the curvature of the pipe and with a rib C on the convex or lower side. This rib is received in a groove formed in the concrete pier, and the ring AB is thus kept in place during the travel of the pipe back and forth. The friction of steel pipe on cast iron is much less than on concrete, and the pipe will therefore slip with comparatively small resistance on the surface of support. Furthermore, the slipping of a pipe back and forth directly on the concrete surface will tend to break off and crumble the pier at the edges DE, and in case an iron ring support is not used, the edges of the piers at D and E should be well bevelled away in order to save them from such action of the pipe.

In the setting of piers in pipe-line construction, the point of primary importance is that of sub-foundation. The pier is intended to furnish a secure and dependable point of support for the pipe line. This cannot be realized unless the pier itself has also a secure and dependable foundation, either on bed rock or on hard pan or other stratum not liable to yield or shift or to suffer damage from

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local wash. In the best practice the piers should be anchored into the sub-foundation (preferably bed rock) by vertical steel rods. Old railroad rails, or material otherwise possibly scrap, may often be utilized to excellent advantage in this manner. This anchoring of the piers into the sub-foundation is of special importance on relatively steep grades. Instead of anchoring the pier into the bed rock by steel rods, as above suggested, it will sometimes be preferable to excavate into the rock and thus anchor the pier by pouring the concrete into the cavity, formed preferably with rough and overhanging sides. The entire question of the sub-foundation for piers and of the extent to which they should be tied or anchored into such foundation is one for the exercise of good engineering judgment, having in view the purpose of the pier and the local conditions applying to the case.

Anchors.—In addition to definite points of support, pipe lines require, at critical points such as angles and Y branches and at certain intervals on long straight grades, definite points of attachment or constraint.

These are intended to furnish fixed points in the line for the anchorage of expansion joints and relative to which expansion and contraction can take place. Likewise they are intended to supply, especially on long uniform gradients, a suitable number of points of complete constraint relative to movement of the pipe in all directions.

In approved practice anchor blocks are made of concrete of the same mixture as for piers, and preferably with sufficient steel reinforce to insure strength and coherence under any stresses which can be anticipated. The question of sub-foundation is, of course, of special importance, and wherever practicable the anchor block should go down to bed rock or to an entirely dependable formation. Regarding anchorage to or into the sub-foundation, the same remarks apply as in the case of piers. Here, however, such anchorage should be considered as absolutely essential. In no other way can safety be assured against longitudinal movement under the stresses to which the line is subject.

Special means must also be provided for securing the pipe to the block in order that the latter may fulfil its purpose. Such modes of attachment are usually of two kinds.

(*a*) A circumferential strap passing over the upper part of the pipe and secured by bolts or equivalent fastenings anchored into the block.

(*b*) A projecting rib or flange, formed by an angle or T bar riveted around the pipe and bedded in the block.

Very commonly the concrete of the anchor block is carried up and over so as to entirely surround the pipe. A strap as in (*a*) acts primarily to prevent lateral displacement. When properly designed and set up, however, it may also be depended on to give security against longitudinal movement. A ring or flange as in (*b*) acts

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primarily to prevent longitudinal movement. If, however, the concrete block with suitable steel reinforce is carried over the pipe, the latter will operate as a band or tie, thus providing security against movement in both directions.

In the design of anchor blocks, as with piers, much will depend on the local conditions and special circumstances of the case. The engineer must simply hold in mind the ultimate purpose of the structure, and then adopt such means as the conditions may indicate.

72. Relative Advantages of Buried or Unburied Pipe

In many cases, such, for example, as municipal water supply, convenience will require that pipe lines be buried. In other cases, as, for example, a water-power penstock line on a rock hill side, the expense of burying may be practically prohibitive. In such cases the matter may be determined by necessity and independent of questions of relative advantage or disadvantage otherwise. In many cases, however, the conditions will be such as to admit of either mode of treatment as may be judged most advantageous. The various points of advantage and disadvantage may be summarized as follows:

Regarding all matters related to inspection, repairs, upkeep generally and length of serviceable life, the advantages will lie with the unburied pipe. Such lines may be readily inspected as to leaks, condition of protective covering and condition generally. Repairs such as the calking up of leaky seams, are readily carried out. Repainting and general maintenance are also carried out under convenient conditions. Naturally all of these conditions, if taken advantage of, will make for the maximum serviceable life of the line. On the other hand, the buried line cannot be inspected or repaired or repainted on the outside, and these conditions will naturally reduce the serviceable life of the line.

With regard to stresses developed under changing temperatures and the necessity of making provision for them by slip joints or otherwise, the advantage lies with the buried pipe. As noted in Sec. 70 with the line continuously full of water, such stresses are commonly reduced to a negligible amount. But pipe lines must be unwatered from time to time for internal inspection or repair and under these conditions, if exposed to the sun, changes in length may develop with severe stress, unless such changes are adequately accommodated by slip joints or suitably located bends. With buried pipe the line is protected from the direct action of the sun and the variation in temperature of the pipe, even if not carrying water, will be very much reduced. Under these conditions the stresses developed under changes of air temperature are usually

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negligible, and the design will not therefore require the provision of special expansion joints, as in the case of unburied pipe.

With regard to cost of laying there is no certain advantage with either side. The cost of ditching and backfilling will be saved in the case of the unburied pipe. Even here, however, a certain amount of ditching will be required in order to eliminate small irregularities in the ground. On the other hand, the cost of pier supports and anchors will normally be less for buried than for unburied pipe. Piers and anchors cannot in all cases be dispensed with in the case of buried lines. Careful judgment will be required in accordance with the circumstances of the case. If the trench is carried through a hard permanent formation, the requirement of support, as such, will be adequately met. Anchors for steep side hill work will, however, be required at points to be selected with judgment. If the trench is carried through soft uncertain formations, then both piers and anchors will be needed, distributed and spaced according to judgment.

73. Protective Pipe Coatings

The problem of the protective covering of steel or iron pipes is fundamentally the same as for all iron and steel structures. Whether open or buried, corrosive agencies begin immediately to attack unprotected surfaces and operate unceasingly to reduce the metallic wall of the pipe to the condition of crumbling oxides and metal salts.

Such destructive agencies attack both the internal and external surfaces of the pipe, the former according to the possible chemical reactions between the metal of the pipe and the liquid carried and the latter according to the soil or earth formation in which the pipe is laid, alternations of moisture and dry-out, etc.

The conditions vary somewhat according to whether the pipe is of cast iron or plate steel and whether buried or above ground.

Cast-iron pipes are usually buried. There is no fundamental reason for this except that under the conditions specially suited to cast iron as regards pressure and type of service, convenience usually requires them to be placed underground. There is therefore no opportunity for periodic examination and repainting or retreatment. Whatever is done must be done when the line is installed.

Fortunately cast iron is relatively resistant to the attack of corrosive agencies and under normal conditions a life of twenty years or upwards may be anticipated.

Any comprehensive treatment of the problem of pipe-line corrosion and its prevention is beyond the purpose of the present work, and we can only note briefly the fundamental conditions to be observed in carrying out such measures.

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The best protective coatings fall into two general classes—chemically neutral carbon or graphite paints and coatings of a bitumastic or asphaltic character.

Where paints are applied the surface should be thoroughly cleaned of all scale, dirt, oil, or grease. A coat of red lead is then commonly applied as a base and the carbon or graphite paint over this. Especial care should be taken to secure the maximum of surface dryness as the paint is going on. Paint will not effectively adhere to a moist or wet surface. The careful cleaning of the surface is most important and the expenditure of a sum for cleaning, even approximating that of the paint itself, will be fully justified by the longer life of a coat of paint carefully laid on to a thoroughly cleaned and dry surface.

In the case of bitumastic or asphaltic coverings, the most favourable conditions are determined by a thoroughly cleaned surface, warm or hot liquid and a warm or hot dry surface. In the case of small pipe (cast-iron commercial pipe, etc.) the hot dip process is commonly employed, effectively realizing the temperature conditions indicated above. In the case of large pipe the covering must be applied by brush; but even here, hot liquid and a warm pipe surface are important aids in securing a closely adhering and effective covering. The best of such coatings when properly applied give a hard, enamel-like, closely adherent surface which will, for a long period of time, resist ordinary corrosive action.

All steel pipe lines, especially if buried, should be protected in the most effective possible way when installed. Especial care should be taken to see that the protective coating, whatever it may be, covers the pipe completely, that it is allowed to become dry and hard before backfilling and that it does not become abraded during the process of filling the trench.

Steel pipe, if uncovered, should be likewise carefully painted or treated. In particular, care should be exercised to see that the pipe where it lies in the saddles is well covered, as also the saddles themselves. The sliding back and forth due to temperature changes will inevitably abrade any coating which can be applied, but if the surfaces are liberally covered with protective material, the condition will be better than if left bare.

Regarding the dipping process, it may be noted that cast iron is better adapted to this method than is steel. The looser texture and rough surface of cast iron seems better adapted to furnish a bond with the dip than is the relatively smooth surface of steel plate. On this account many engineers of experience prefer, for steel plate pipe, a paint coating, properly laid on as noted previously, as furnishing on the whole the best protection against corrosive agencies.

Where pipe is laid in concrete and where it is desired that the concrete and iron shall bond together, as in an anchor block, the iron surface should be left clean and unpainted in order to permit bonding with the concrete.

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74. Air Relief Valves

In the operation of a pipe line, conditions may arise which, at certain points, may drop the hydraulic grade line below the level of the pipe; see Sec. 14. This means that the pressure within the pipe will be reduced below the atmosphere and danger of collapse of the pipe may result. To prevent the development of such a condition or to control the subpressure within safe limits, an air relief valve may be fitted. This is, in effect, a form of safety valve opening inward and admitting air inside the pipe, thus preventing the pressure from dropping below an assumed safe limit. In the examination of this problem there are two main questions.

1. What is the maximum drop in pressure which may be considered safe in the case of a pipe with given diameter and thickness?

2. What aggregate area of air valves will be required in order to prevent the drop in pressure exceeding this safe limit.

The first of these involves simply the question of the strength of the pipe under an excess external load; the second involves the hydraulic characteristics of the line and the problem of the flow of air through the valve opening.

Regarding the strength of large cylindrical pipe against collapse, there is great uncertainty so far as direct experimental evidence goes. Existing formulæ give the most widely divergent results. It is well established, however, that the double thickness at the joints—lap or butt-strap—gives an added element of strength, and that a uniform pipe without such local stiffening rings would collapse under a lower pressure than actual pipe with such local stiffening furnished by the joint doubling. Taking advantage of this fact, relatively thin pipe is sometimes stiffened further against collapse by a riveted circumferential angle iron, in the middle of each length, thus furnishing a definite stiffening ring at these points.

Among the various formulæ proposed, the following by Love may be taken as giving an indication of the collapsing pressure for long uniform tubes or pipe.

$$p=65{,}000{,}000(t/D)^3 \qquad (19)$$

p=pressure (pi2).
t=thickness (i).
D=diameter (i).

This formula does not take account of any support derived from the doubling at the joints, and hence actual pipe is likely to show strength greater than as indicated by the formula. The error, therefore, is likely to be on the side of safety.

If Love's formula is taken as primarily applicable when $L/D=6$ or more, and if the collapsing strength for lesser values of L/D is taken as varying inversely as the square root of the length (all other

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things the same), then an indication of the collapsing strength of pipe with angle-iron stiffening may be found as follows :

$$p_1 = 2{\cdot}5p\sqrt{\frac{D}{L}}$$

Where p is the strength as derived from (19), D is diameter and L is length between angle-iron supports, both in the same units of measure.

Thus for illustration for $t = {\cdot}25$ and $D = 50$ we shall find $p = 8$ (pi2).

If, however, we have angle-iron stiffening spaced at 100-inch intervals we shall have $\sqrt{D/L} = {\cdot}7$ and $p_1 = 1{\cdot}75p = 14{\cdot}0$ (pi2).

Strength against collapse is also affected adversely by any departure from a circular cross section. For this reason, other things equal, a pipe with lap-riveted longitudinal joint may collapse at lower pressure than if of true circular section.

There is much need of further experimental data on the collapsing

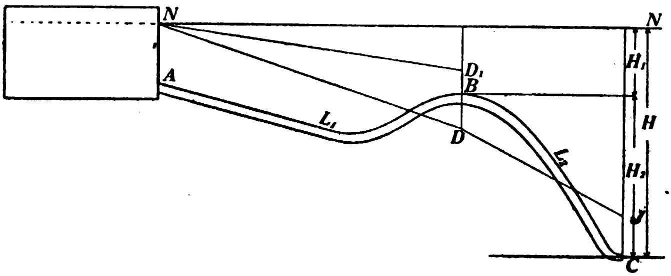

FIG. 128.—DESIGN OF AIR RELIEF VALVES.

strength of large riveted steel pipes under external load, and no present formula can be depended on to do more than give some indication of the probable load which such a pipe may safely bear.

The second part of the problem, involving the hydraulic characteristics of the line, together with the assumed emergency condition, includes a range of possibly variable quantities and conditions so wide as to make a general solution scarcely practicable. Typical cases will, however, serve to indicate the way in which an approach to a treatment of the problem may be made.

In Fig. 128 let ABC denote a pipe line considered in two sections, L_1, L_2, as indicated. For simplicity we may take the diameter uniform throughout the length from A to C. The necessary modification for variable diameter will be readily made by reference to Sec. 8.

Then from Chapter I (45) for steady flow conditions and with no air-valve opening we shall have, for the velocity in the line :

$$v = \sqrt{\frac{H}{\frac{1}{2gfm^2} + \frac{\Sigma L}{C^2 r}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

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For the resultant pressure head at B we shall have

$$\frac{p}{w}=H_1-\left(\frac{1}{2g}+\frac{L_1}{C^2r}\right)v^2 \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Now suppose that this value of v is one which gives a hydraulic grade line NDJ with pressure head at B, as in equation (21), measured by BD below the atmosphere.

Next suppose that it is desired to raise the pressure head at B up to D_1, giving for AB a hydraulic gradient ND_1 instead of ND. This new pressure will then tend to reduce the velocity in AB below, and to raise that in BC above the original velocity v.

Let q denote the new pressure in the pipe at B and v_1 and v_2 the new velocities in AB and BC.

We shall then have at B under the new conditions in L_1

$$\frac{q}{w}+\frac{v_1^2}{2g}=H_1-\frac{L_1v_1^2}{C^2r}$$

Whence $$v_1=\sqrt{\frac{H_1-q/w}{\frac{1}{2g}+\frac{L_1}{C^2r}}} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Likewise at C under the new conditions in BC we shall have, just outside the nozzle or opening,

$$\frac{u^2}{2gf}=\frac{v_2^2}{2gfm^2}=H_2-\frac{L_2v_2^2}{C^2r}+\frac{q}{w}+\frac{v_1^2}{2g}$$

Whence $$v_2=\sqrt{\frac{H_2+q/w+v_1^2/2g}{\frac{1}{2gfm^2}+\frac{L_2}{C^2r}}} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In this equation no account is taken of any loss of head resulting from the more or less abrupt change of velocity from v_1 to v_2.

With $q=p$ we should find $v_1=v_2$. With $q>p$ we shall have $v_2>v_1$ and the difference in velocity must be made up by the inflow of something at B. If the something were water, the problem would be simple. The amount of such inflow would be measured by (v_2-v_1) multiplied by A, the c.s. area of pipe, and it would only remain to provide suitable means for securing the inflow of water at this rate. In the actual case, however, air is the substance the inflow of which is depended upon to maintain the pressure conditions desired, and such inflow is to be realized as a result of the difference between the pressure of the atmosphere and that within the pipe at the point B. The air thus drawn into the pipe is susceptible to volume changes in accordance with the pressure changes between the atmosphere and within the pipe at B, and thence down the line to the point of exit at C. The presence of this variable element in the contents of the pipe greatly complicates the problem of pipe flow and renders analytical treatment exceedingly difficult, at least without some further experimental results bearing on the phenomena of such mixed flow.

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In the absence of any ready basis for precise treatment, guidance may be usually obtained by neglecting the change in the volume of the air along the pipe between B and C, by assuming the mixed flow to be given by the same formulæ and coefficients as for water alone, and in order to cover the divergence between such assumptions and a more exact hypothesis, by allowing a generous factor of safety.

If then a is the aggregate area through the air valves and z the inflow velocity, we shall have

$$za=(v_2-v_1)A$$

$$\text{or} \quad a=\frac{(v_2-v_1)A}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

$$\text{or} \quad \Sigma d^2=\frac{(v_2-v_1)D^2}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Where Σd^2 is the sum of the squares of the diameters of the valves and D is the diameter of the line.

The velocity z as a function of the difference in pressure between the atmosphere and within the pipe at B, is given by Table XXVIII in which, however, the coefficient of inflow or efficiency of the valve considered as an orifice, is taken at 0·60. We have thus at hand through equations (22), (23), Table XXVIII and (25) all requirements for a solution of the problem.

A numerical example will illustrate the procedure.

$$\text{Given} \quad H_1= 115 \text{ (f)}.$$
$$H_2= 285 \text{ (f)}.$$
$$L_1= 800 \text{ (f)}.$$
$$L_2=1000 \text{ (f)}.$$
$$D= 4 \text{ (f)}.$$

Thickness of plate at $B=\cdot 25$ (i),

Then from equation (19) it appears that such a pipe would be in danger of collapse under a subpressure of about 9 pounds per square inch.

Let it be proposed to maintain the subpressure at 4 pounds per square inch as a safe margin. Then from Table XXVIII, with an inflow coefficient of ·60 and at sea-level, the velocity of inflow under this head$=569$ (fs).

Suppose that a rupture of the pipe at C or other emergency condition gives a discharge opening equivalent to half the area of the pipe and that the discharge coefficient f for the opening may be taken at 0·80. Assume also the Chézy coefficient $C=110$.

Then in (20) we shall have

$$H=400$$
$$L_1+L_2=1800$$
$$f=\cdot 80$$
$$m=\cdot 50$$
$$C=110$$
$$r=1$$

Substituting and reducing we find

$$v=42{\cdot}02.$$

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TABLE XXVIII

Depression (pounds per square inch)

Altitude feet	2	4	6	8
0	392	569	718	856
1000	399	582	735	878
2000	407	594	752	900
3000	416	607	770	922
4000	424	620	788	945
5000	433	634	806	970
6000	442	648	825	995
7000	451	662	845	1023
8000	460	677	865	1050
9000	470	692	887	1079
10000	480	708	909	1107

Velocity of inflow of air through air valve (fs).
Coefficient of discharge f is assumed $=\cdot60$,

and from (21) for the resultant pressure head we find

$$\frac{p}{w}=-29\cdot2f=-12\cdot66 \text{ (pi2)}.$$

If then we put $q=-4$ (pi2) or $q/w=-9\cdot23$ (f) we shall find from (22)

$$v_1=39\cdot00$$

and from (23) $v_2=43\cdot21$.

Hence $v_2-v_1=4\cdot21$.

Then from Table XXVIII $z=569$.

Hence we have from (25) $\Sigma d^2=\dfrac{4\cdot21\times2304}{569}=17\cdot05$.

This would indicate a single valve about 4·5 inches diameter or two 3-inch valves.

If this assumed condition were the most serious to be anticipated, then the provision of air values as indicated should be adequate. On the other hand, it is always possible that the entire lower end of the pipe might rupture out in such a way as to give an outlet opening equivalent to the entire c.s. area of the pipe. In this case we have $m=1$, and assuming $f=1$ we should find in the above case $v=49\cdot34$ and with this velocity, for the pressure head at B,

$$\frac{p}{w}=-83\cdot8 \text{ (f)}.$$

If, again, the pressure is to be maintained at -4 (pi2) and the pressure head at $-9\cdot23$ (f) we shall have for v_1 the same condition and value as before, viz. $v_1=39\cdot00$, while from (23) we shall find $v_2=55\cdot23$.

Hence $v_2-v_1=16\cdot23$

and $\Sigma d^2=\dfrac{16\cdot23\times2304}{569}=65\cdot72$.

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This would indicate, say, two 6-inch or three 5-inch valves.

Suppose, again, we assume a condition as in Fig. 129 where AB is a long line, BCD a so-called "inverted siphon" and DE a relatively short section.

Suppose complete rupture at C as the most serious possible case. The condition of reduced pressure will develop quickly at B, but the velocity v_1 through L_1 will be slow in developing due to the long length and time lag. We may then take as an extreme case an assumed velocity in L_1 of v_0, the original steady motion value, and for v_2 in BC the value resulting from equation (23) assuming the pressure at B maintained at the desired limit value q below the

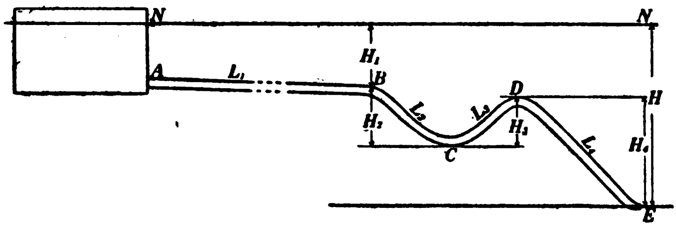

FIG. 129.—DESIGN OF AIR RELIEF VALVES.

atmosphere. We shall then have the basis for a determination of v_2-v_1 and for Σd^2, the aggregate square of the diameters for the air valves at B.

Thus for numerical values let us take

$$L_1=21600$$
$$L_2=800$$
$$L_3=600$$
$$L_4=1000$$

$$H_1=100$$
$$H_2=200$$
$$H_3=160$$
$$H_4=300$$
$$f \text{ at } E=\cdot 90$$
$$m \text{ at } E=\cdot 0771$$
$$q/w \text{ at } B=-9\cdot 23$$
$$C \text{ (Chézy coef.)}=110$$
$$D=2 \text{ (f)}.$$

Then the overall length=24000 (f) and the overall head=440 (f) and from (20) we shall find for the steady motion velocity before rupture, $v_0=8$ (fs).

Then, after rupture at C, we apply equation (23) to the conditions in L_2 putting $v_1=v_0=8$ and f and $m=1$. We thus find:

$$v_2=36\cdot 02.$$

$$\text{Then} \quad v_2-v_1=28\cdot 02$$

$$\text{and} \quad \Sigma d^2=\frac{28\cdot 02\times 576}{569}=28\cdot 36.$$

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This indicates at B a single valve about 5·5 in diameter or two 4-inch valves.

Again, at D we shall have water flowing in both directions—down DE to the discharge end and down DC to the point of upture. In such case we shall have in equation (25) the numerical sum of the two velocities instead of their difference. Suppose again the value of q at D to be maintained at—4 (pi2).

Then adapting equation (23) to the conditions in L_3 and the discharge at C, assuming f and $m=1$ we shall have in the formulæ, 160 for H_2, 600 for L_2, zero for v_1 and $-9{\cdot}23$ for q/w. Whence we find $v_3=36{\cdot}24$.

Again adapting the same equation (23) to the conditions in L_4 and the discharge at E we shall have in the formula, 300 for H_2, 1000 for L_2, zero for v_1, $-9{\cdot}23$ for q/w, ·9 for f and ·0771 for m. We then find $v_4=9{\cdot}73$.

The incoming air must supply the volume corresponding to both of these velocities.

$$\text{Hence we have} \quad v_3+v_4=45{\cdot}97$$

$$\text{and} \quad \Sigma d^2=\frac{45{\cdot}97\times 576}{569}=46{\cdot}54$$

This implies one 7-inch valve or two 5-inch valves.

75. Pressure Relief Valves and Breaking Plates

These items of pipe-line equipment are intended to operate as safeguards against the development of an undue excess pressure as a result of sudden valve movement under the various conditions discussed in Chapter III. Pressure-relief valves fall under two general classes according as they are intended to operate automatically, consequent upon a slight or limit rise in pressure, or as they are attached to some part of the control mechanism of a power unit thus deriving their movement from the movement of the latter. Valves of the latter type fall rather under the category of hydraulic power plant equipment and therefore lie beyond the scope of the present work.

Valves of the former type may be considered as an item of pipe-line equipment and as such merit brief notice.

While there are many types and structural forms of such valves, their operation depends on substantially the same basic principles. Fig. 130 shows in diagrammatic form the characteristics of one of the best of such automatic valves.

The valve itself is seated under a slight excess of pressure due to a difference in area between the valve and the balance piston. The main pipe line is connected through a small pipe to the space under the operating piston as shown. In this connection, and not shown in the drawing, is a valve under the control of a small pilot valve held normally closed under a balance of pressure between the main

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pipe line and the expansion tank. On the arrival of a pressure wave along the pipe line this balance is disturbed, the valve in the pipe connection is opened and the excess pressure builds up under the operating piston, resulting in the opening of the main valve and the relief of the pressure.

On the return toward normal pressure the pressure in the expansion tank will determine the return of the pilot valve, the closure of the connection through the small pipe and the resultant closure of the main valve due to the over balance of area. Any desired degree of retardation in the movement of the main valve is secured through the adjustable by-pass connecting the two ends of the operating cylinder.

All such valves depend for their operation on an initial rise in pressure, and the assumption of effective operation depends on the

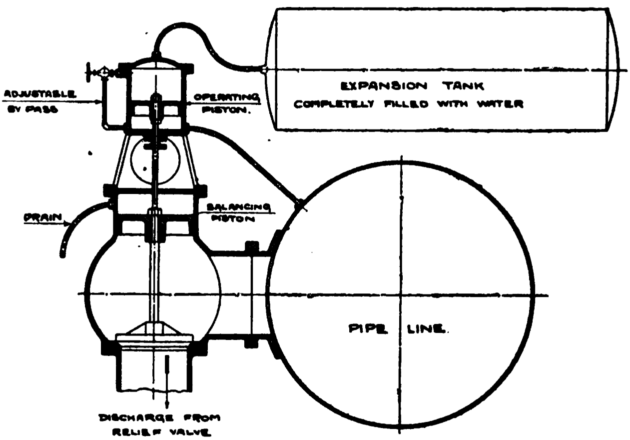

FIG. 130.—AUTOMATIC PRESSURE RELIEF VALVE. *

possibility of an adequate response to the initial pressure rise within the time permitted by the character of the pressure-time history. References to Chapter III will show the conditions under which the pressure rise may be expected to be abrupt in time or gradual. If the initial rise in pressure is sufficiently slow the operating parts of the valve may have abundant time in which to perform, each its function, and thus the effective operation of the whole may be secured. If, on the other hand, the case is one of very abrupt pressure rise, as in the case of the closure of a pipe line running with valve opening nearly or quite the full size of pipe and hence nearly on " gravity flow " (see Chapter III, Fig. 48), the rise of pressure just at the instant of valve closure is exceedingly rapid, constituting in effect a hammer blow, and no valve could be expected to operate in such time as to safeguard the line from such blow.

* Under Patents of the Pelton Water Wheel Co.

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Too much dependence should not therefore be placed on such automatic valves, and in particular their time characteristics of operation should be closely studied in connection with the probable time histories of the pressure shocks which are to be anticipated, and assurance should be obtained that within the probable period of pressure rise to the desired limit value there will be time for the effective operation of the valve in relief of pressures beyond such limit.

Breaking Plates.—As a further safeguard in connection with the rapid rise of pressure due to water ram, the use of the breaking plate should not be overlooked. This is a plate of appropriate area, bolted on at the lower or delivery end of the line, somewhat as a manhole cover-plate, but designed and intended to rupture under an excess pressure well below that which the pipe itself may be expected safely to bear. The area of the opening covered by such plate may be preferably some two or three times the nozzle or normal discharge area, thus insuring an immediate relief of pressure consequent on the rupture of the plate.

A first approximation to the design of such a plate may be made through equation (26) of Chapter IV, using for p the over pressure under which it is desired that the plate should break, and for the denominator, values some four times those given for safe operation. It is, however, always desirable to have such design checked up by actual test under the limit pressure desired, and in order to eliminate the uncertainties of all empirical formulæ and the unknown influence due to slight variation in the physical properties of the materials employed.

76. MANHOLES AND COVERS

In order to permit of access to the interior of large pipe lines for examination, painting, re-caulking, etc., manholes with cover plates are provided at occasional points in the line. The metal around such a hole is reinforced by a suitable doubling plate with inner steel casting for cover-plate joint. The cover is fitted up and secured in place making joint against the inner surface of the reinforce casting, entirely in accordance with the practice familiar in steam boilers and which need not be here described in detail.

77. EXPANSION JOINTS

The purpose of expansion joints has already been discussed in Secs. 69, 70. In the best practice a given section of the line which is to be treated as a unit in the matter of expansion and contraction is stabilized at its lower or delivery end by a suitable anchor block and provided at its upper end with an expansion joint. The pipe then creeps up and down the slope or back and forth along the line from the anchor block to the joint.

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The characteristic or essential elements of an expansion joint are indicated in Figs. 96, 97. The mode of operation and general character of construction aside from full structural details will be evident from the figures. As noted in Sec. 50, such joints are sometimes provided with guard bolts to prevent complete separation of the two parts of the joint in case of an extreme temperature drop.

In all cases the special hydraulic forces which develop as a result of the introduction of an expansion joint must be carefully examined and care taken to provide the necessary support or constraint to the two parts of the joint in order to prevent separation under the operation of such forces.

78. Test Flanges and Test Bulkheads

In connection with a program of test on large pipe lines it may become necessary to close temporarily an open end of the line. For this purpose a special so-called "test flange" is employed. This consists in effect of a cover plate, usually of cast steel, suitably ribbed or reinforced to safely bear the anticipated load, and provided with flange at the rim for connection to the corresponding flange on the open end of the pipe. If there is no such flange connection on the pipe it may become necessary to provide a corresponding "companion flange" and rivet it to the open end in order to realize the closure.

The general manner of dealing with the problem of structural design in the case of such a test flange has been outlined in Sec. 53.

It may also become necessary, in connection with the same program of test, to segregate out a special part or section of the line for individual test. To this end one or possibly two test bulkheads may be employed. A test bulkhead is some form of structure built up in such fashion as to permit of location within the pipe at the desired point and provided with means for deriving from the shell of the pipe the necessary support under the anticipated load. In the case of light pressures such a structure may be built up of timber and made tight by oakum calking or other like means. If the line is made up of in and out sections, support for the bulkhead may usually be derived from the end of the pipe section of smaller diameter. If of wood, the bulkhead at the rim would, in such case, need a metal plate reinforce in order to safely carry the load against the end of the inner pipe without crushing. If the pressures are heavier, some form of steel bulkhead will be preferable, fitted with an adjustable packing ring for making the joint against the shell and with hinged feet which may swing out and bear against the end of the smaller pipe or against a group of rivet heads, thus carrying the load. With the test concluded these feet may then be swung in, the joint unpacked and the bulkhead removed or shifted along to the next location as desired.

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In cases where the pipe is of uniform diameter on the inside, thus offering no convenient end for the support of the load, but where the pressures are not too extreme, the bulkhead may be supported against cross timbers secured by wedging friction against the sides of the shell. The principle involved in this mode of support is illustrated in Fig. 131. *AB* is a heavy timber carried at *A* on a plate of steel as a local reinforce to the shell of the pipe. At *B* it rests on a plank cut slightly wedging. Under these conditions pressure against *A* would promptly dislodge the timber, while, on the other hand, pressure at *B* would develop the wedging action giving an end thrust at *A* and corresponding reaction at *B*, with consequent friction grip on the shell of the pipe. The end *B* of such a timber is therefore a point capable of carrying a heavy thrust in the direction of the arrow. Four or six such timbers

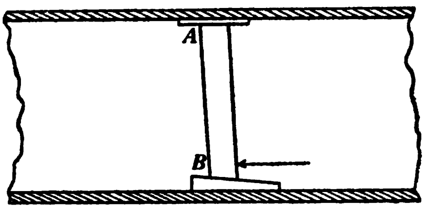

FIG. 131.—TESTING BULKHEAD.

spaced around the circle of the pipe will then give a series of points such as *B*, and from which, by suitable blocking or thrust timbers, the support may be carried back to the bulkhead itself.

79. PIPE-LINE VALVES

The general subject of pipe-line valves is too extended to permit of adequate discussion in the present work. It is indeed a subject to which an entire volume might well be devoted. No attempt will be made therefore to discuss the subject here, either from the descriptive or the design standpoints.

Space may be permitted, however, for a few words relating to the importance of valve design and to the main problems which present themselves in this connection.

The importance of a design which shall insure safe and reliable operation over a long period of time cannot be well exaggerated. There has, in many cases, been a tendency to design valves for large pipe-line work—valves up to 4 or 5 feet in diameter—simply as an overgrowth of ordinary small pipe valves and without adequate recognition of the severe conditions of service under which such large hydraulic valves are intended to operate.

A large valve of this character must meet adequately the following major requirements.

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1. The body or casing with its supporting ribs must be of sufficient thickness and character of design to insure, under the highest over-pressures to be anticipated, not only safety against rupture but also against any sensible deformation of any of the parts, such as might determine either leakage or binding and jamming of moving parts.

2. Strength of moving parts and of operating gear adequate to close the valve under full flow of water.

3. Bearing areas sufficient to insure operation of all sliding or rubbing surfaces over long periods of time and under the highest pressures anticipated, without danger of seizure or abrasion. The proper selection and use of metals for the two parts of a mutually sliding pair (as a valve gate and its seat) will aid in marked degree in the realization of this requirement.

Pipe-line valves are made in three general forms:

1. Gate or slide valves.
2. Butterfly valves.
3. Bulb or nozzle valves.

These forms will all be familiar to those in contact with hydraulic work, and with individual interpretation the above conditions may be applied to all of these forms or types of valve.

In particular it may be recommended to accept designs predicated upon the actual conditions to be met and carried out by those familiar with such work rather than so-called stock designs developed to meet, as a stock article, a certain average range of operative conditions, but frequently lacking in features which may have special importance for the particular problem in hand.

80. Pipe-Line Fittings, Ys, Bends, etc.

In accordance with the express purpose of the present work, as noted at the head of this chapter, no attempt will be made to discuss, in a descriptive manner, the various forms of pipe-line fittings. Brief reference has been made in Chapter IV to certain of the problems of structural design which may arise in connection with such forms. The material commonly employed in all large high-pressure work is cast steel, reinforced and ribbed as the characteristics of the case may require.

In the case of all such large fittings, as in the case of valves, and especially under high pressures, use may be recommended of special designs developed by those familiar with such work rather than of any form of so-called stock or standard design developed without special reference to the peculiar conditions of the case in hand.

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CHAPTER VI

OIL PIPE LINES

81. Physical Properties of Oil Affecting Pipe Line Flow

Due to the greatly increased value of the viscosity of oils, as compared with that of water, the problem of the flow of oil in pipe lines presents certain special features which will be briefly considered in the present chapter.

The general problem of the flow of viscous liquids in pipe-lines is included within that of the general problem of pipe line flow as discussed in Appendix I.

For many pumping problems it may be convenient to transform the expression for pressure gradient, as in Appendix I, (4), into pounds per square inch per mile of pipe line. As defined, the pressure gradient G is in pounds per square foot per foot of line. We have therefore to divide by 144 and multiply by 5280. We may also reduce D to inches by introducing the factor 12, thus giving finally

$$G_m = \frac{440 f\sigma}{D} \frac{v^2}{2g} \qquad (1)$$

Where G_m = gradient in (pi2) per mile of line.
f = coefficient as in Appendix I.
σ = density (pf3).
D = diameter (i).
v = velocity in (fs).

For any given case it becomes, therefore, necessary to determine for the conditions of operation, the values of μ and σ, the viscosity and density, and thence to proceed as indicated.

The density of oils is usually indicated by the so-called gravity on the Baumé scale, taken at some standard temperature, usually 60° F.

In order, therefore, to determine the value of the argument $Dv\sigma/\mu$, the following information is required:

(1) The diameter D.
(2) The velocity of flow v.
(3) The relation between gravity in Baumé degrees and density σ.
(4) The relation between density at standard temperature of 60° and density at any specified working temperature.

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(5) The value of the viscosity μ at the specified temperature, or if this is not known directly, such relations between viscosity, gravity and temperature as will make possible some estimate of the viscosity.

With this information at hand both μ and σ may be determined or estimated and thence the value of the abscissæ $Dv\sigma/\mu$. Thence with due regard for the roughness of the pipe, we may assume a value of f and thence from (1) determine either the friction head or the pressure gradient per mile.

Relation (3) is provided by the formula

$$\sigma=\frac{8736}{130+B} \qquad (2)$$

where B=gravity on Baumé scale
and σ=density in pounds per cubic foot.

Relation (4) is provided by the formula

$$y=y_1-\frac{121{\cdot}2-y_1}{2713}(t-60°) \qquad (3)$$

where y_1=density in pounds per cubic foot at 60° F.
and y=density at temperature t.

This is an empirical relation which has been found to agree closely with observations on American petroleum oils.

Relation (5) cannot be definitely developed simply because observation shows that the viscosity of oils is not determined by gravity and temperature alone. Viscosity apparently depends on the number and proportion of the various complex constituents of which the oil is composed, and on their physical states, as well as on the overall resultant gravity and temperature.

Broadly it is found that viscosity increases with increasing density or decreasing gravity Baumé, and with decreasing temperature. The relation of viscosity to gravity is, however, irregular in special cases and subject to occasional exception.

With regard to the variation with temperature it is usually found, starting with a high temperature, that the increase of viscosity is at first slow, the rate of increase rising rapidly as the temperature drops.

Professor W. R. Eckart of Stanford University has pointed out that the relation between viscosity and temperature t when plotted on double logarithmic scales shows a close approximation to a straight line, at least over the working range of temperatures for which the liquid may be said to retain its identity. At high temperatures, the more volatile constituents will begin to vaporize, and at very low temperatures certain constituents may begin to solidify and separate out, thus in either case changing the character of the liquid itself. Between these limits of temperature, however, Professor Eckart has shown by a large number of cases that within the limits of observational error the straight-line relation may, for all practical purposes, be assumed to hold.

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It thus results that if the value of μ for a given oil is known for two temperatures, the logarithmic plot may be drawn as a straight line between these points and extended over the working range of temperatures, thus giving a direct and practical form of relation between viscosity and temperature, and specifically, the value of μ for any specified temperature within the working range.

Likewise in the relation between density and viscosity at a fixed temperature, there is evidence of a slow rate of increase of viscosity with density at low values of the density, followed by increasing rate with increasing values of the density.

For numerical values we have the following:

A. C. McLaughlin* gives diagrams showing the relation between temperature F. and absolute viscosity for a series of American oils from which the following values may be drawn:

TABLE XXIX

Temperatures Fah.	*Viscosity*†
60°	·060 to ·300.
80°	·040 to ·250.
100°	·020 to ·160.
120°	·016 to ·080.
140°	·013 to ·045.

The densities of the oils to which these values refer range from 56·8 (pf3) to 60 (pf3). (Gravity Baumé 24° to 16° app.)

From tests made by Cooper on 60 samples of California petroleums R. P. McLaughlin‡ gives values for the relation between viscosity and gravity from which the following tabular values are derived:

TABLE XXX

Baumé Gravity	Lbs. per cub. ft. Density at 60° Fah.	Viscosity at 60° Fah.†	Viscosity at 185° Fah.†
36°	52·6	·0040	·00076
34°	53·3	·0050	·00090
32°	53·9	·0070	·00120
30°	54·6	·0103	·00160
28°	55·3	·0145	·00206
26°	56·0	·0206	·00260
24°	56·7	·0280	·00320
22°	57·5	·0370	·00380
20°	58·2	·0760	·00440
18°	59·0	·2640	·00650
16°	59·8	—	·01120
14°	60·7	—	·02450
12°	61·5	—	·14800

* "Journal Am. Soc. Mech. Eng., 1915," p. 263.
† The units involved in these values are the poundal, foot, second.
‡ "Journal, Am. Soc. Mech. Eng., 1915," p. 264.

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The very rapid increase of viscosity with density will be noted for 60° beginning about density=57 and for 185° beginning about density=60.

From a number of tests made on California oils Dyer* gives for oils of four different gravities values of the viscosity for varying temperatures from which the following tabular values are derived.

TABLE XXXI

Gravity Baumé	18°·2	18°·6	15°	12°·1
Density, lbs. per cub. ft. at 60°	58·95	59·59	60·26	61·48
Temp. Fah.	Viscosity† μ			
50	·7360	—	—	—
60	—	1·0530	—	—
75	·2450	·4970	1·5670	—
100	·1220	·2230	·3440	—
110	—	—	—	1·6610
125	·0670	·0990	·1500	·6140
150	·0365	·0490	·0750	·2110
175	·0242	·0245	·0310	·1020
200	·0216	·0220	·0220	·0510

The above oils are noted to have contained about 2% of water.

The numerical measure of viscosity is often met with in terms of accepted forms of viscosimeters such as the Engler, Saybolt or Redwood, or again it is not infrequently stated in terms of the metric units, centimeter, dyne, second, or again in terms of the more recently proposed unit the "centipoise."

For the latter we have the following definition:

One absolute metric unit (c.g.s.)=100 centipoises.

Hence to convert viscosity in absolute units into centipoise units, multiply by 100. To convert viscosity in centipoises into absolute units, divide by 100.

To convert absolute metric units into absolute English units, or vice versa, we have the following relation: Viscosity (English units) =Viscosity (metric units)÷14·88 (poundal, foot, second) (dyne, centimeter, second).

Regarding the various forms of viscosimeters, the following

* "Journal, Am. Soc. Mech. Eng., 1915," p. 259.

† The units involved in these values are the poundal, foot, second.

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formulæ will serve for transforming the indications of these instruments into absolute English units :

$$\left.\begin{aligned}\frac{\mu}{\sigma}&=\cdot00000237t-\frac{\cdot00194}{t}\ \text{(Saybolt)*}\\ \frac{\mu}{\sigma}&=\cdot00000158t-\frac{\cdot00403}{t}\ \text{(Engler)}\\ \frac{\mu}{\sigma}&=\cdot00000280t-\frac{\cdot00185}{t}\ \text{(Redwood)}\end{aligned}\right\}\ldots\ldots\ldots(4)$$

Where μ=viscosity in absolute English units (poundal, foot, second).
σ=density in pounds per cubic foot.
t=time on instrument (Saybolt, Engler, Redwood) (seconds).

In illustration of the use of the equations of the present section, suppose we have given as follows :

Size of pipe . . .	8 (i).
Capacity per day . .	24,000 bbls.
Gravity of oil . .	18° B.
Average temperature .	100° Fah.
Viscosity (assumed) .	·12.

We find first v =4·47 (fs).
Then from (2) density at 60° F. =59·02
and from (3) density at 100° F. =58·10.
We then find the value of $Dv\sigma/\mu$=1443.

This implies stream line motion and (8) of Appendix I gives a value of f about ·0444.

Substituting this in (1) we find the pressure gradient 44·1 pounds per square inch per mile of length.

Again, if we should take the oil of gravity 16° B. at 60° F. with a working temperature of about 110° F. and a value of μ=·15 we should find similarly a value of $Dv\sigma/\mu$=1166, a value of f about ·055 and a pressure gradient G_m=55·2 pounds per square inch per mile of length.

Again, suppose a 10-inch line with oil of gravity 20° B. at 60° F. and at an average working temperature of 110° F. and with a velocity of 5 feet per second. We may take μ about ·06. We then find as above for the density at 110° F., σ=57·06.

This gives a value of $Dv\sigma/\mu$=3964, implying turbulent flow and for commercially smooth pipe a value of f about ·04 (see Table, Appendix I). If 10 per cent is added for a slightly roughened surface we

* The numerical constants in the Saybolt equation refer to the recently standardized form and proportions of the instrument. The indications of earlier instruments will not quite agree among themselves or with any one set of values. The values for the Redwood instrument are somewhat less well established than for the other two. See U.S. Bureau of Standards Technologic Papers, Nos. 100, 112.

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have $f=\cdot 044$ and substitution in (1) gives a pressure gradient of 42·95 pounds per square inch per mile of length.

Suppose again that we have a record of $t=1000$ for the Saybolt viscosity of a given oil at 100° F. Then from (4) we have $\mu/\sigma=\cdot 002368$ and $\sigma/\mu=422\cdot 3$. Again, if $t=4000$ we shall have $\mu/\sigma=\cdot 00948$ and $\sigma/\mu=105\cdot 5$. If in the first case the oil is one showing a Baumé gravity at 60° of 17·5 we shall have from (2) σ at $60°=59\cdot 22$ and from (3) σ at $100°=58\cdot 31$ and hence μ at $100°=58\cdot 31\times\cdot 002368=\cdot 1381$.

From an examination of the form of the curve $ABCD$, Appendix I, it is clear that the conditions of operation should, if possible, be so chosen so as to avoid a value of the abscissa $Dv\sigma/\mu$ at or just beyond the critical value. In the case of oil pipe lines the value will often fall close about this point. Obviously if the conditions admit of control they should be so adjusted as to give a value on the stream line branch AB, and as near the critical velocity as practicable without actually passing the limit. This will insure the lowest practicable value of the friction head coefficient f. Decrease in the abscissa value will mean a rapidly rising value of f on the branch AB, while a slight increase will mean rapid rise of the value to the branch CD.

82. Oil Pipe Lines

In the sense here employed the term oil pipe line is intended to refer to a line for the transportation of crude or fuel petroleum oil in bulk from the wells to convenient rail or water shipping points or to refinery locations. The principles involved in the discussion of pipe-line resistance are of course entirely general. The descriptive matter and the suggestions for design, however, are intended to more directly apply to the case of large and long pipe lines as above noted. Such pipe lines are usually of steel pipe of diameters from 6 to 12 inches. In the United States 8 inches is a common size. Such pipe is made of thickness suitable to stand a test pressure of 1200 (pi2) and with a safe working pressure of 800 (pi2). The pumps for handling the oil at the pumping stations are quite commonly made to meet the same pressure requirements, the number of such stations being determined by the capacity and size of the line, the length and the topographical characteristics.

The influence of viscosity on oil pipe line resistance has been noted in the preceding section, also the dependence of viscosity on temperature. To reduce the viscosity, especially with heavy oils, heaters are commonly employed, one at each station and often one intermediate between stations.

The station heaters are commonly formed of closed steel cylinders provided internally with headers and tubes through which the oil usually makes two passes on its way from the receiving tank to the pumps. The exhaust steam from the pumps passes between the headers and tubes and raises the oil to an initial temperature ranging usually from 125° to 150° F.

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The heaters used between stations consist of a by-pass manifold of 4-inch pipes lying at right angles to the main line, often 200 or 250 feet long, and with headers so arranged that the oil passes out and back a distance of 400 to 500 feet through the 4-inch pipes. These pipes are carried in a brick chamber or flue along which passes the hot gas from a furnace at one end, burning oil drawn from the line through a suitable reducing valve. With the pressure suitably reduced an atomizing burner of the usual type may be employed, giving with suitable air control a nearly smokeless combustion, the gasses from which pass along the flue as above noted. By these means the temperature of the oil may be raised some 30 to 50° F. at these midway points.

The pumps commonly used are of two types: (1) direct-acting duplex plunger with tandem compound or triple-expansion steam cylinders, and (2) the crank and flywheel, plunger fitted pumping engine with cross compound or triple-expansion steam cylinders fitted with Corliss valves. The design throughout must be exceptionally strong and rugged and the proportions such that with a steam pressure of 135 (pi2) at the boilers an effective pumping pressure of 800 to 1000 (pi2) may be realized. The plungers are usually from 6 to 9 inches diameter by about 36 inches stroke, those for direct acting pumps ranging larger than those for the flywheel type. Such pumps will handle about 1000 bbls. of oil per hour, the flywheel type at a somewhat higher number of strokes per minute.

The economy of the flywheel type is naturally superior to that of the direct acting, but the latter are found necessary for starting the oil in a long line after a shut-down. The steady direct thrust which can be realized by the direct-acting pump is found much better suited to overcome the resistance of a long line of cold oil in starting from a condition of rest, than the effort derived from a pump of the crank and flywheel type. In this connection it may be noted that a condition of rest is always avoided so far as possible, especially with cold weather or with heavy oil. In fact, the combination of the two may render starting up from a state of rest extremely difficult if not impossible. It is therefore a principle of operation that once the column of oil is in motion it must be kept moving at all hazards, so far as it is humanly possible to realize such end.

The construction of an oil pipe line offers but few points calling for special comment. The line is usually placed underground to a greater or less depth depending on the temperature conditions along the line. The joints are of the screw-collar type, usually with special design so far as length is concerned in order to insure a tight joint under the high pressures employed.

Protective coverings or coatings are employed, consisting usually of bitulithic enamels or hot asphaltum followed with a wrapping of roofing paper, which is again coated with asphaltum. Heat-insulating coverings to reduce the loss of heat from the oil would be desirable, but cannot usually be justified economically.

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The line is commonly tested by pumping water through under pressure from the station pumps, the line being carefully inspected in the meantime for leaks at the joints or flaws in the pipe.

Expansion joints are used to some extent, though not as much as good design would seem to indicate. Without such means for accommodating the variations due to changing temperatures, the stresses developed must be absorbed in the line itself; and, as noted in Sec. 69, this may cause serious stresses resulting in leaky joints and trouble.

The serious effect of low temperatures and consequent increased viscosity on pipe-line resistance, especially with heavy oils, is shown by the changing capacity of pipe lines dependent on the operating conditions. Thus an 8-inch line in California is stated to have had its capacity of 25,000 barrels per day with medium oil in summer reduced to about 3600 per day with heavy oil in winter.

The capacity of an 8-inch line is usually taken from 20,000 to 30,000 bbls. per day when working under conditions not excessively severe regarding temperature or gravity. Similarly for a 6-inch line, which is usually worked at a pressure somewhat higher than the 8-inch, the capacity is commonly taken from 12,000 to 18,000 bbls. per day.

83. Design of General Characteristics of Line

The factors which enter into the determination of the general characteristics of an oil pipe line are the following:

(1) The diameter of the line.
(2) The length of the line.
(3) The physical properties of the oil to be handled.
(4) The capacity of the line.
(5) The pressure to be realized by the pumps.
(6) The topography and profile along the proposed location of the line.
(7) The distribution of the pumping stations.

Very commonly the first six of these are known or assumed and it is required to find the last, the most suitable distribution of the pumping stations.

We shall outline briefly the steps whereby this problem may be readily investigated.

Conditions (1), (3), (4) will serve to determine the frictional resistance and hence the pressure gradient due to friction along the line from station to station.

If the physical conditions of the oil remained uniform throughout the run from one station to the next, the gradient would be a constant and the hydraulic grade line would be straight and inclined at the constant gradient value. Due, however, to the falling temperature, the viscosity and with it the friction coefficient

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f, will show constantly changing values, and hence a gradient changing with progress along the line. The hydraulic grade line is therefore no longer straight, but curved.

If data are at hand permitting an assumption of the probable temperature distance history, that is of the probable average temperature for each mile of run, the corresponding gradients may be determined and thus for any given distance the resulting hydraulic grade line determined.

In some cases the so-called logarithmic decrement law for relating temperature change to distance has been used. This is in the form

$$\frac{\log\left(\frac{t_0-T}{t-T}\right)}{\log\left(\frac{t_0-T}{t_1-T}\right)}=\frac{x}{x_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where t_0=initial temperature of oil.
t=temperature at distance x.
t_1=temperature at distance x_1.
T=temperature of earth or air surrounding pipe.
x=any distance along line.
x_1=a known length of run for which t_1 is known or assumed.

Hence if we know, in any given case, the length of run x, the final temperature at this point t_1, the initial temperature t and the temperature T, we may readily determine, on this hypothesis, a law for temperature change with distance, and determine the hydraulic grade line accordingly.

So long as the conditions of flow are either continuously stream line or continuously turbulent (see diagram Appendix I), falling temperature and increasing viscosity will result in a continuously increasing value of f and a continuously steeper and steeper gradient. If the conditions should change during the run from turbulent to stream line flow (as may readily be the case) the values of f may show first an increase, followed by a sudden decrease in passing from one condition to the other, and then followed by a continuous increase for further cooling under stream line conditions. The actual form of the grade line will be, therefore, more or less complex, depending on the history of the conditions of flow along the line.

The use of such a grade line, once determined, will be best illustrated by an example. Assume, therefore, data as follows:

1. Diameter of line: 8 (i).
2. Length of line: 144 miles.
3. Density of oil: Baumé gravity 18.
4. Capacity of line: 27,000 bbls. per day.
5. Pressure of pumps: 800 (pi2).

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In Fig. 132 let XX be a base line on which are laid off linear distances along the pipe line. Then at suitable intervals, and from the topographic data, altitudes above an arbitrary datum are erected. In this case we take station O as the datum and thus derive, as shown by the curve, a distance—altitude history $OABC$. . . . Note should be taken that this is not a profile in the more usual sense of the term. It is not a history of altitude on horizontal distance, but a history of altitude on linear distance along the line.

We next find $v=5{\cdot}03$, and then based on the best estimates which can be made regarding the temperature history with distance and regarding the variation of viscosity with temperature, we obtain by equation (1) values of the pressure gradient for successive miles of run. Suppose for illustration the first three of such gradients to be 49·5, 50·0, 50·7 pounds per square inch. Then the total head used

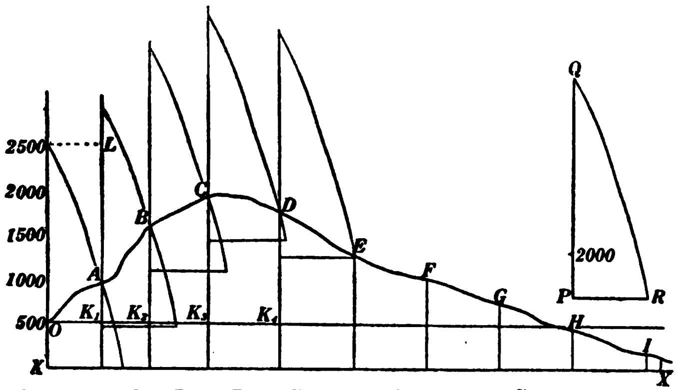

Fig. 132.—Oil Pipe Line Design—Graphical Construction.

will be 49·5 for one mile, 99·5 for two miles and 150·2 for three miles. In this manner we may prepare a continuous history of total pressure drop on distance. It will next be convenient to transform these pressure values into head of oil in feet. To this end we may use an average value of the density since the variation of density with temperature is relatively small. With such values we then plot (to the same scale as in Fig. 132) the hydraulic grade line on stiff paper and cut out as a template. Let PQR denote such a template covering a total pressure drop PQ of 1000 pounds, or about 2500 feet for the case in hand, and a total distance PR of 16 miles. The total head furnished by the pump is 800 (pi2), or about 2000 feet head of oil. We then place the template with head scale vertical and with the 2000 foot point on the head scale (counting downward from Q) at the home station O. The grade line QR intersects the distance-altitude line in A and at a distance from O measured by OK_1. At this point and for this distance it is clear that the height through which the oil has been lifted is measured by K_1A, while the head

used in pumping the given distance is AL. The sum of these make up K_1L the total head available from the pump. The point K_1 thus determined gives therefore the location of the next station beyond O. We next transfer the template to A as starting-point and repeat the operation, thus determining the successive points B, C, D, etc., and thence, the locations of the stations K_1, K_2, K_3, etc.*

When we pass the crest of the hill and descend as from D or K_4 on, we shall have a total head available made up of the head due to the pump plus the head due to the oil, and the head used in friction will then equal the sum of these two. The same procedure, however, will determine properly the location of the station as indicated for the run from D to E.

In this manner we continue throughout the length of the line, finding in this case the last station at I, or not quite at the end of the line. This implies, of course, a slight readjustment of values, either a slightly higher pumping pressure or a slightly lower velocity or a higher general temperature for the oil. Some readjustment may also be required in order to avoid undesirable locations topographically for the pumping stations. Such readjustments may involve extra heater provision, or the use of a larger pipe, or of a double line for a certain distance. These various problems of secondary adjustment do not, however, involve any new hydraulic principles, but call rather for the exercise of sound engineering judgment in the light of all the factors bearing on the problem in hand.

If the effect of changing temperature is neglected and an average temperature assumed, leading to a single value of the pressure gradient, the grade line will become straight and the template of Fig. 132 will become a right-angled triangle with the grade line as the hypothenuse. The general program of use is naturally the same as for the template of Fig. 132. In the case of oil of low density (high Baumé) no heating may be required, and the temperature will remain substantially uniform throughout the run. In such case the hydraulic gradient naturally becomes a straight line, with procedure as indicated above.

There are many other points of detail which will arise in connection with studies of this character, and the present chapter is only to be considered as a brief sketch of the hydraulic principles involved and of the general mode of treatment through which they may be effectively applied to the problem.

* In the diagram of Fig. 132 the scale is naturally very much reduced. In dealing with an actual problem such scales should be employed as will insure the degree of accuracy significant for the case in hand.

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APPENDIX I

GENERAL THEORY OF PIPE LINE FLOW

THE phenomena of pipe line flow will obviously depend on the following factors, or conditions defining the circumstances of the case :

Diameter of pipe denoted by	....	D
Density of liquid ,,	....	σ
Velocity of flow ,,	....	v
Viscosity of liquid ,,	....	μ
Length of pipe ,,	....	L
Character of pipe surface.		

From the first four of the above determining characteristics there will result :

Pressure gradient required to overcome resistance to flow, or otherwise, the loss of pressure per unit length due to resistance to flow, denoted by .... G

From G and L will result :

Total loss of pressure head in line, denoted by .... h

The theory of dimensions applied to the problem of pipe line flow shows that there must subsist between these quantities a relation of the form*

$$G=\frac{\sigma v^2}{D}\,\varphi\left(\frac{Dv\sigma}{\mu}\right) \qquad (1)$$

where φ denotes some function of the quantity $(Dv\sigma/\mu)$.

This equation takes cognizance of all factors in the problem except the character of the pipe surface. We have, moreover, for character of surface or degree of roughness, no definition nor unit of measure, and hence the influence due to this factor must remain to be allowed for within the numerical values of the function $\varphi(Dv\sigma/\mu)$.

With G known we have then, for the total loss of pressure head in the line :

$$h=\frac{GL}{\sigma} \qquad (2)$$

In the above equation the viscosity μ is the so-called absolute viscosity defined by the equation $R=\mu v/z$, where R is the force opposing the motion of a plane of unit area lying very near and moving with

* See among other references, Buckingham, "Trans. Am. Soc. Mech. Eng.," Vol. XXXVII, p. 263.

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a velocity v parallel to a large plane, the space of the thickness z between the two planes being filled with the liquid in question.

If, then, in equation (1) the form of the function φ could be determined, this equation would give a complete solution of the problem of pipe line flow, at least so far as determined by the quantities represented therein, and hence for any one value of roughness or condition of pipe surface.

In order to assimilate this formulæ to those more commonly employed in hydraulic problems we may take the Darcy formula (see Sec. 5).

$$h=\frac{fL}{D}\frac{v^2}{2g}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

equating the values in (2) and (3) we have

$$h=\frac{GL}{\sigma}=f\frac{L}{D}\frac{v^2}{2g}$$

$$\text{or}\quad G=\frac{f\sigma v^2}{2gD}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Combining this with (1) we find

$$\frac{f}{2g}=\varphi\left(\frac{Dv\sigma}{\mu}\right)$$

$$\text{or}\quad f=2g\varphi\left(\frac{Dv\sigma}{\mu}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

In this equation the value of the abscissa $(Dv\sigma/\mu)$ is independent of the system of units employed—English or Metric—so long as the system is homogeneous; that is, a given quantity always in terms of

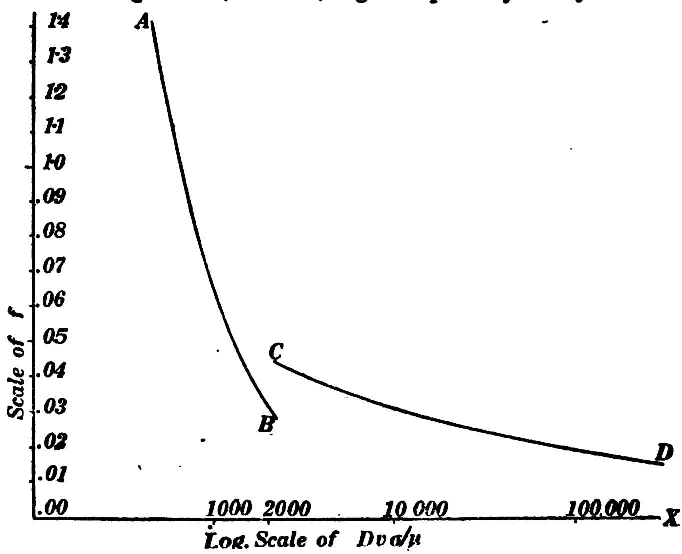

FIG. 133.—DIAGRAM OF VALUES OF f ON ABSCISSA OF $Dv\sigma/\mu$.

the same unit. We shall assume here, however, English units used throughout, feet, pounds and seconds.

We have, therefore, in any specific case, to take the values of D, v, σ and μ and find the value of $(Dv\sigma/\mu)$. If, then, we know or can estimate the value of the function for this value of the argument $(Dv\sigma/\mu)$ we may multiply by $2g$ and thus find the coefficient f in the familiar Darcy formula. If desired we may then readily find the Chézy coefficient C from the relation between f and C as developed in Sec. 5.

Now experiments with smooth brass and steel pipe made with such diverse substances as air, water and oil, agree in giving for the relation between f and $(Dv\sigma/\mu)$ a curve of the general form shown in Fig. 133.

The part of the curve from A to B corresponds to the so-called pure stream line or irrotational flow. In this mode of flow there is no turbulence or eddy formation and the paths of the liquid particles are smooth, open, straight or gently curving stream lines. The part of the curve from C to D corresponds to turbulent or rotational flow. In this mode of flow the liquid is turbulent with eddy formation and the paths of the particles are curving, twisted and contorted, as may result from the accidents of the turbulent flow.

Experience shows that the transition from one mode of flow to the other occurs abruptly at or near a so-called critical value of the abscissa $(Dv\sigma/\mu)$. It appears furthermore that this critical value is commonly found between 2000 and 2500 as indicated in the diagram. Further, as shown by the form of the curve, the transition from stream line to turbulent flow is accompanied by a sudden increase in the value of the ordinate f, followed later by a gradual decrease with further increasing values of the abscissa.

It appears further that for stream line flow the pressure gradient G varies directly with the velocity. Reference to equation (1) shows that this requires a form of relation

$$G=\frac{\sigma v^2}{D}K\left(\frac{\mu}{Dv\sigma}\right)\text{where } K \text{ is a constant.}$$

That is, the form of the function φ must be simply the reciprocal of the abscissa $(Dv\sigma/\mu)$. Hence from (5) we must have for this case

$$f=2gK\left(\frac{\mu}{Dv\sigma}\right)\dots\dots\dots\dots(6)$$

If, then, we denote the abscissa $(Dv\sigma/\mu)$ by x, we shall have

$$xf=2gK\dots\dots\dots\dots(7)$$

This equation shows, therefore, that for pure stream line flow the curve AB is a hyperbola determined as in (6).

Again, experience shows for turbulent flow that the gradient G varies nearly with the square of the speed. The index for ordinary values of the abscissa and for ordinary degrees of roughness of pipe is usually found slightly less than 2, often close about 1·85. For very rough pipe, however, and as the abscissa increases, the index approaches close to the value 2. It is instructive to note, so far as the value of the abscissa is concerned, that the approach to the index 2 is nearer as D is greater, as v is greater, and σ is greater and as μ is less.

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At the limit, when we may assume G to vary as v^2, we shall have from (1)

$$\varphi\left(\frac{Dv\sigma}{\mu}\right)=\text{constant.}$$

That is, the curve CD will reach and maintain itself, as a straight line parallel to the axis of X and giving an ordinate equal to f for this limiting condition.

It therefore appears that the curve CD must be considered as gradually approaching the horizontal as a limiting condition, at which point we shall have f=constant and $G \sim v^2$.

In Table XXXII are given values of f for various values of the abscissa $(Dv\sigma/\mu)$. Between abscissa values 200 to 2000 inclusive the values of f correspond to the branch AB representing stream line flow. These are derived from the hyperbola

$$(f)\ (Dv\sigma/\mu)=2gK=64.\quad \text{(See (7))} \ldots\ldots\ldots\ldots (8)$$

Intermediate values are readily determined from the equation.

From abscissa value 2500 upward the values correspond to the branch CD representing turbulent flow.

These values of f, which are closely accurate for such widely diverse substances as air, water and oil, are derived from experiments on brass or smooth steel pipe.

Regarding the influence of roughness on these values, it appears that for stream line flow the character of the surface of the pipe seems to have but slight influence. On the other hand, it does have a direct and important bearing on the phenomena of turbulent flow. It results that the values of f, or of $\varphi(Dv\sigma/\mu)$ within the stream line phase, are practically independent of roughness, at least so far as stream line flow prevails. There is, however, some evidence that the critical value for rough pipe occurs at smaller values of the abscissa $(Dv\sigma/\mu)$ than in the case of smooth pipe. On the other hand, for turbulent flow the values of f or of $\varphi(Dv\sigma/\mu)$ vary in marked degree with roughness. Referring to Fig. 133 which, as noted, shows the results for diverse substances with varying diameters and velocities, but all with smooth pipe, it is found for rough pipe that the curve beyond the critical velocity is of the same general form as for smooth but, with increasing roughness, located higher and higher and with increasing approach to parallelism with the axis of abscissa. This approach of f to a constant value implies, as previously noted, a corresponding approach to a law of variation of G or f with the square of the speed.

For ordinary iron or steel pipe, plain or galvanized, and with varying degrees of roughness as met with in actual practice, the values of f will range between those indicated in Table XXXII for smooth or new pipe, up to values 50 to 100 per cent greater for very rough and pitted pipe surfaces. The influence due to roughness must, therefore, be allowed for according to judgment and in accordance with the observed or assumed condition of the surface.*

For values of the abscissa close about the critical state, 2500 to 2000 or lower for rough pipe, the values of f will be very uncertain, due apparently to unsteady conditions of flow involving the irregular formation and disappearance of eddies and turbulence.

* Compare also discussion of values of f in Sec. 7.

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The entire field close about the critical value needs further examination, especially as to the dependence of the critical value upon roughness of pipe and, possibly, density of liquid.

It should be noted that in all usual cases of handling water, at least on an engineering scale, the conditions are such as to determine turbulent flow. This is readily verified by substituting usual numerical values in the abscissa $(Dv\sigma/\mu)$. On the other hand, with many liquids handled in industry (oils, syrups, etc.) the conditions are frequently such as to determine stream line flow.

TABLE XXXII

Abscissa	f	Abscissa	f
200	·3200	14,000	·0292
400	·1600	16,000	·0280
600	·1067	18,000	·0271
800	·0800	20,000	·0264
1,000	·0640	25,000	·0249
1,200	·0533	30,000	·0238
1,400	·0457	35,000	·0228
1,600	·0400	40,000	·0219
1,800	·0355	45,000	·0213
2,000	·0320	50,000	·0208
2,500	·0442	60,000	·0200
3,000	·0426	70,000	·0195
3,500	·0412	80,000	·0190
4,000	·0400	90,000	·0185
4,500	·0390	100,000	·0180
5,000	·0382	150,000	·0168
6,000	·0364	200,000	·0158
7,000	·0350	250,000	·0150
8,000	·0340	300,000	·0144
9,000	·0330	350,000	·0140
10,000	·0320	400,000	·0137
12,000	·0304	450,000	·0134

Values of Coefficient f on Abscissa $(Dv\sigma/\mu)$ for smooth brass and steel pipe.

It must not be assumed that the values of f in Table XXXII for turbulent flow are the least which may be obtained. It is simply a question of smoothness of surface. These values, as noted, refer to what may be termed "commercially smooth" pipe. It is possible that specially prepared or treated surfaces might show somewhat lower values; and, in fact, there is some evidence tending to indicate somewhat lower values in the case of water in new cast-iron pipe, asphaltum dipped. The tabular values are to be understood as applying to all cases of what may be termed "commercially smooth" surfaces, and thus form a general datum from which the effect of roughness may be estimated.

For the general problem of pipe line flow with any fluid whatever for which the density and viscosity are known or are determinable, it is therefore only necessary to find, for the conditions of operation,

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the value of the argument $Dv\sigma/\mu$, and thence, guided by judgment according to the factor of roughness, to select a suitable value of the coefficient f, and thence as in Sec. 5.

In Table XXXIII are given values of μ the absolute viscosity for water at varying temperatures between 0 and 100 C. or 32 to 212 F., likewise values of the density for the same temperature ranges. It should be especially noted that, with all other factors the same, the value of the abscissa $(Dv\sigma/\mu)$ will vary inversely with the ratio μ/σ. For handling water under widely varying temperature conditions, therefore, the influence of the latter on the value of $(Dv\sigma/\mu)$ should be allowed for in selecting the most appropriate value of f.

TABLE XXXIII

Absolute Viscosity and Density of Water

Temperature C.	Temperature F.	Absolute Viscosity, ft., lb., sec.	Density ft., lb.
0	32	·001204	62·42
5	41	·001021	62·42
10	50	·000879	62·41
15	59	·000766	62·38
20	68	·000673	62·33
25	77	·000601	62·26
30	86	·000538	62·17
35	95	·000486	62·08
40	104	·000441	61·97
45	113	·000402	61·85
50	122	·000369	61·70
55	131	·000340	61·54
60	140	·000315	61·37
65	149	·000293	61·20
70	158	·000273	61·02
75	167	·000255	60·83
80	176	·000240	60·64
85	185	·000225	60·44
90	194	·000213	60·22
95	203	·000201	60·00
100	212	·000191	59·76

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APPENDIX II

EXPRESSION FOR F CHAPTER III IN TERMS OF v_0 AND e

INSTEAD of expressing F in a form directly dependent on the quantities m and f, it will sometimes be more convenient to have its value in a form dependent rather on some steady motion velocity v_0 together with the ratio of the actual opening m to the opening m_0 for such steady motion.

Put the ratio $m/m_0 = e$. Then we may assume values of e, such as 1·0, ·9, ·8, ·7, ·6, etc., without knowing or assuming m or m_0 individually.

From p. 114 we have

$$F = \frac{(am)^2}{2M} \text{ and} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$M = \frac{1}{2gf} + \frac{Lm^2}{C^2r} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Using the above value of m in terms of m_0 and e we have

$$M = m^2\left[\frac{1}{2gfe^2m_0^2} + \frac{L}{C^2r}\right] \quad \ldots\ldots\ldots\ldots (3)$$

But from Chapter I equation (45) we readily derive for the present case

$$\frac{1}{2gfm_0^2} = \frac{H}{v_0^2} - \frac{L}{C^2r} \quad \ldots\ldots\ldots\ldots \quad (4)$$

Substituting this in (3) we find

$$M = m^2\left[\frac{H}{e^2v_0^2} - \left(\frac{1}{e^2} - 1\right)\frac{L}{C^2r}\right] \quad \ldots\ldots\ldots\ldots (5)$$

and putting this in (1) we have finally,

$$F = \frac{(ae)^2}{2\left[\frac{H}{v_0^2} - (1 - e^2)\frac{L}{C^2r}\right]} \quad \ldots\ldots\ldots\ldots (6)$$

We may then assume that there is some m_0 which gives v_0. Then, whatever it is, we assume that m has a value measured by some fraction of this as ·8, ·6, ·4, etc. These fractional ratios then represent values of e in the equations above and from (6) with v_0 and e the value of F is readily found.

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APPENDIX III

Proposition: In any hydraulic system or element containing water in motion, and where the dimensions are such that we may neglect the weight of the water as such, the force reaction of the water on the system will be given by the vector sum of the following systems of forces:

(*a*) The total pressures over the ideal sections bounding the system or element, reckoned from without inward and combined as vectors.

(*b*) The sum of the momenta per second at inflow and outflow, the former taken direct and the latter reversed and all combined as vectors.

This proposition may be established as follows:

Consider the hydraulic system of Fig. 91 comprising a pipe AD with water flowing through, entering at the section AB and discharging at CD. Let p_1, v_1, A_1, p_2, v_2, A_2 be respectively the pressure, velocity and area at these sections.

There enters the system in unit of time the volume A_1v_1 with velocity v_1, and hence the momentum $wA_1v_1^2/g$ directed along the line of flow at AB. There leaves the system in the same unit of time the momentum $wA_2v_2^2/g$ directed along the line of flow at CD. There is produced, therefore, per unit of time, a certain change in momentum. That is, under steady flow, there is produced in connection with this system a steady rate of change of momentum.

But a change of momentum is evidence of the operation of a force, and the rate of change is the measure of such force. Hence there must be in operation on the water, forces or systems of forces of which the resultant will be measured by the rate of change of momentum produced.

We now ask what forces or systems of forces can act on the water contained within the enclosure $ABCD$. First, considering that the element is substantially in one plane or that its dimensions are such that we may neglect the influence due to the weight of the water itself, we may classify the remaining forces as follows:

1. The end forces p_1A_1 and p_2A_2 acting from without inward and normal to the ideal sections A_1 and A_2.

2. The direct force reaction between the inner surfaces of the inclosure and the contained water and estimated from the inclosure to the water.

Adding now the rate of change of momentum as the third force system involved, we have

3. The rate of change of momentum produced between AB and CD.

Then as we have seen, system (3) will be a measure of the resultant of (1) and (2). We may express this by the vector equation:

$$S_1+S_2=S_3$$

where S_1, S_2 and S_3 denote the three systems of forces.

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Then by transposition we have:

$$S_1 - S_3 = -S_2.$$

Now with the system S_3, let the entering momentum be denoted by M_a and the issuing momentum by M_b. Then in a vector sense:

$$S_3 = M_b - M_a$$

and $-S_3 = -(M_b - M_a) = M_a + (-M_b)$.

This shows $-S_3$ as measured by the vector sum of the momenta per second at inflow and outflow, the former (M_a) taken direct and the latter (M_b) reversed.

Furthermore $-S_2$ means S_2 reversed: that is, the force reaction from the water to the inclosure.

But this is exactly what is wanted, and the above analysis therefore shows that this is measured by the vector sum of S_1 and $(-)$ S_3, and these are made up as in the statement of the proposition.

The proof is readily extended to include the case with any number of points of inflow and any number of outflow.

If the dimensions are such that the weight of the water cannot be neglected, we must then add this as a fourth system acting vertically downward through the centre of volume of the element. Denoting this by S_4 we shall then have as the vector equation for this case:

$$S_1 + S_4 - S_3 = -S_2.$$

This equation expresses $(-)$ S_2 (the force reaction desired) as the vector sum of three vector systems specified and defined as above.

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APPENDIX IV

ECONOMIC DESIGN

In the case of all carriers of energy in its various forms, whether pipe lines for water, steam or air, or metal lines for electricity, the same fundamental problem of economic design presents itself.

Broadly speaking the annual cost chargeable against such a line arises under two heads.

1. Fixed charges proportional generally to investment or to first cost.

2. Operating costs, resulting from the annual operating program.

Let X and Y denote respectively these two classes of cost and u the total. Then

$$u=X+Y \qquad (1)$$

Now in general it will result that a change in the size of the carrier (pipe or wire) will affect X and Y in opposite directions. Thus an increase in the size will increase the cost and hence the fixed charges while it will decrease, in general, the secondary losses (friction or electrical resistance) and thus decrease the operating costs for the same energy carried.

Similarly a decrease in the size of the carrier will produce changes in the opposite direction. It thus results that there may well be some value of the size of the carrier for which the total cost $X+Y$ will be a minimum. To investigate such a possibility we proceed in the usual manner. Thus let x denote in general diameter or size. Then we have

$$\frac{du}{dx}=\frac{dX}{dx}+\frac{dY}{dx} \qquad (2)$$

also

$$\frac{d^2u}{dx^2}=\frac{d^2X}{dx^2}+\frac{dY}{dx^2} \qquad (3)$$

For a maximum or minimum $dy/dx=O$ and hence

$$\frac{dX}{dx}=-\frac{dY}{dx} \qquad (4)$$

Likewise for a minimum, d^2u/dx^2 is positive in sign.

These general conditions are indicated geometrically in Fig. 134, where XX and YY denote the general character of the curves for X and Y plotted on x or size.

Then the condition of equation (4) may be put into words as follows:

A maximum or minimum value of u will be found for the value of x, for which the slopes X and Y are numerically the same, but opposite in direction. This will be at some point A in the curves of the diagram.

Again, to determine under what conditions this point will correspond to a minimum we have only to inquire as to the sign of d^2u/dx^2.

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The sign of d^2X/dx^2 or d^2Y/dx^2 is determined by the direction of rotation of the tangent for increasing x. This will be + for counter clockwise rotation and − in the inverse case. In the curves of Fig. 134, convex to the axis of x, it is seen that in both cases as x increases the tangent to the curve will rotate counter clockwise. Hence both d^2X/dx^2 and d^2Y/dx^2 are + and the sum will be +, and hence d^2u/dx^2 will be plus and the point determined as the sum of $AP+AQ$ will be a minimum. If either of these lines is straight, the second derivative is zero, but that of the other will be + and hence the sum will be plus

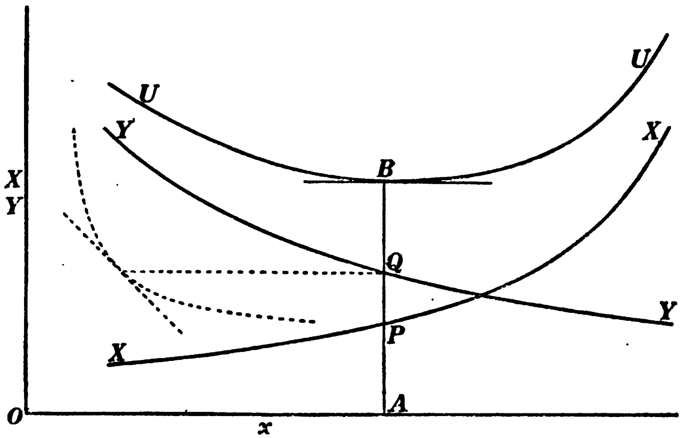

FIG. 134.—ECONOMIC VALUE, GRAPHICAL DETERMINATION.

and the point determined by (4) will give a minimum value. The reader will readily extend this analysis to forms which are concave to the axis of x and which will correspondingly determine a maximum for the sum of X and Y.

In all usual cases, however, in which pipes or wires are used as carriers of energy, one or both of the curves will be convex to the axis of x and hence the condition of (4) will determine a minimum.

As to the actual determination of the point A, various methods are open. If the two curves are plotted individually, an easy trial and error test will serve to find the points P and Q on the same ordinate where the slope is the same but in opposite directions.

Again, the sum of X and Y may be plotted, as indicated in the curve UU, and the minimum point determined by inspection.

Again, from (4) we have

$$\frac{dY}{dX}=-1;$$

Hence if Y and X are taken as the axes, or otherwise if we plot X on Y, the values of X and Y which will produce the minimum value of u will be where the slope of the resulting curve is 135°, or where it is 45° with the horizontal. See construction in dotted lines.

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